Finding the Impedance of a Coaxial Cable

Introduction

The coaxial cable (coax) is one of the most ubiquitous transmission line structures. It is composed of a central circular conductor, surrounded by an annular dielectric and shielded by an outer conductor; see Figure 1. In this example, you compute the electric and magnetic field distributions inside the coax. Using these fields, you then compute the characteristic impedance and compare the result with the known analytic expression.
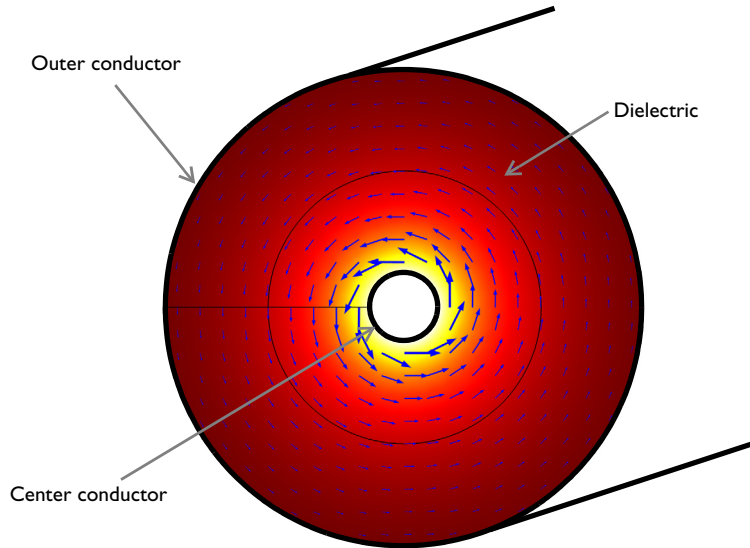

Figure 1: Cross section of a coaxial cable. The arrows visualize the magnetic field.

Model Definition

Because a coax operates in TEM mode — with the electric and magnetic fields normal to the direction of propagation along the cable — modeling a 2D cross section suffices to compute the fields and the impedance. For this example, assume perfect conductors and a lossless dielectric with relative permittivity $\varepsilon_r = 2.4$. The inner and outer radii are 0.5 mm and 3.43 mm, respectively.

The characteristic impedance, $Z_0 = V/I$, of a transmission line relates the voltage between the conductors to the current through the line. Although the model does not involve computing the potential field, the voltage of the TEM waveguide can be evaluated as a line integral of the electric field between the conductors:

$$V = V_i - V_o = -\int_{r_o}^{r_i} \mathbf{E} \cdot d\mathbf{r} \qquad (1)$$

Similarly, the current is obtained as a line integral of the magnetic field along the boundary of either conductor or any closed contour, C, bisecting the space between the conductors:

$$I = \oint_C \mathbf{H} \cdot d\mathbf{r}$$

The voltage and current in the direction out of the plane are positive for integration paths oriented as in Figure 2.

$$V = \int_1^2 \mathbf{E} \cdot d\mathbf{r}$$
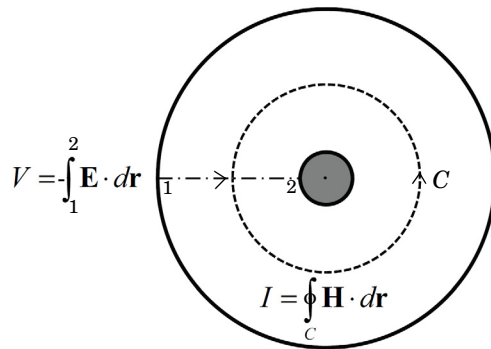
$$I = \oint_C \mathbf{H} \cdot d\mathbf{r}$$

Figure 2: The impedance of a coaxial cable can be found from the voltage, V, and current, I, which are computed via line integrals as shown.

The value of Z_0 obtained in this way, should be compared with the analytic result

$$Z_{0,\text{analytic}} = \frac{1}{2\pi}\sqrt{\frac{\mu_0}{\varepsilon_r \varepsilon_0}}\log\left(\frac{r_o}{r_i}\right) \approx 74.5 \ \Omega$$

Results and Discussion

Figure 3 is a combined plot of the electric field magnitude and the magnetic field visualized as an arrow plot.

Effective mode index=1.5492 Surface: Electric field norm (V/m) Arrow Surface: Magnetic field
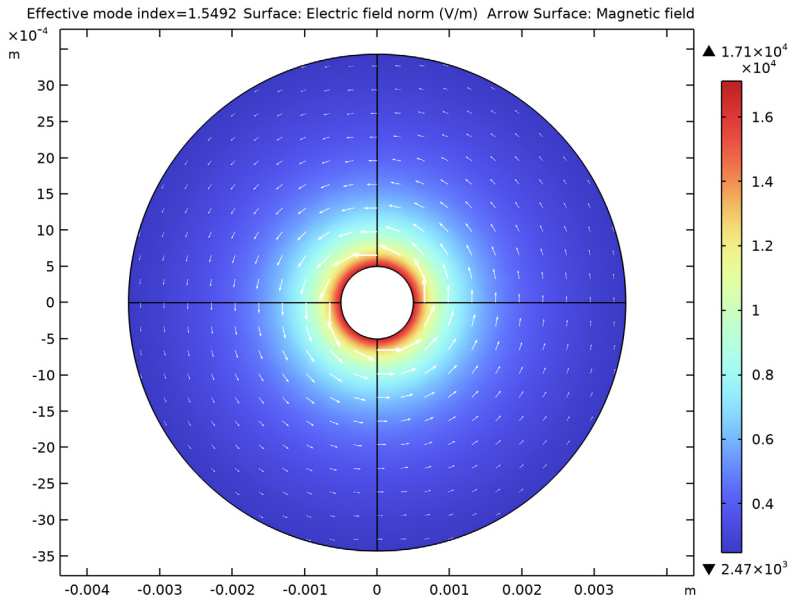
Figure 3: Electric field magnitude (surface) and magnetic field (arrows) inside the coaxial cable.

The impedance computed with the default mesh is $Z_0 = 74.65 \ \Omega$. When the mesh is refined, the result approaches the analytic value of $74.5 \ \Omega$.

Notes About the COMSOL Implementation

Solve this example using a Mode Analysis study. The effective mode index for the propagating TEM mode is $n_{\text{eff}} = \sqrt{\varepsilon_r} \approx 1.5$. Use the default frequency, $f = 1$ GHz, which is well below the cutoff frequency for TE modes and TM modes for the chosen cable diameter.

Application Library path: `RF_Module/Verification_Examples/`
`coaxial_cable_impedance`

Modeling Instructions

From the **File** menu, choose **New**.

N E W

In the **New** window, click 🔷 **Model Wizard**.

M O D E L   W I Z A R D

1 In the **Model Wizard** window, click ⚛ **2D**.

2 In the **Select Physics** tree, select **Radio Frequency>Electromagnetic Waves, Frequency Domain (emw)**.

3 Click **Add**.

4 Click ➡ **Study**.

5 In the **Select Study** tree, select **Preset Studies for Selected Physics Interfaces> Mode Analysis**.

6 Click ☑ **Done**.

G L O B A L   D E F I N I T I O N S

Parameters 1

1 In the **Model Builder** window, under **Global Definitions** click **Parameters 1**.

2 In the **Settings** window for **Parameters**, locate the **Parameters** section.

3 In the table, enter the following settings:

Name	Expression	Value	Description
r_i	0.5[mm]	5E-4 m	Coax inner radius
r_o	3.43[mm]	0.00343 m	Coax outer radius
eps_r	2.4	2.4	Relative dielectric constant
Z0_analytic	(Z0_const/(2*pi* sqrt(eps_r)))* log(r_o/r_i)	74.531 Ω	Characteristic impedance, analytic

Here Z0_const is a predefined COMSOL constant for the characteristic impedance of vacuum, $Z_0 = sqrt(\mu_0/\varepsilon_0)$. From the Value column you can read off the value Z0, analytic = 74.53 Ω.

Create the geometry using a single circle node with the radius of the outer conductor and an extra layer representing the inner conductor.

Circle 1 (c1)

1 In the **Geometry** toolbar, click ⊙ **Circle**.

2 In the **Settings** window for **Circle**, locate the **Object Type** section.

3 From the **Type** list, choose **Curve**.

4 Locate the **Size and Shape** section. In the **Radius** text field, type r_o.

5 Click to expand the **Layers** section. In the table, enter the following settings:

Layer name	Thickness (m)
Layer 1	r_o-r_i

6 Click 🗐 **Build Selected**.

An advantage of using layers is that you automatically get a radial line to use for computing the voltage as a line integral of the electric field.

7 Click the ⬍ **Zoom Extents** button in the **Graphics** toolbar.
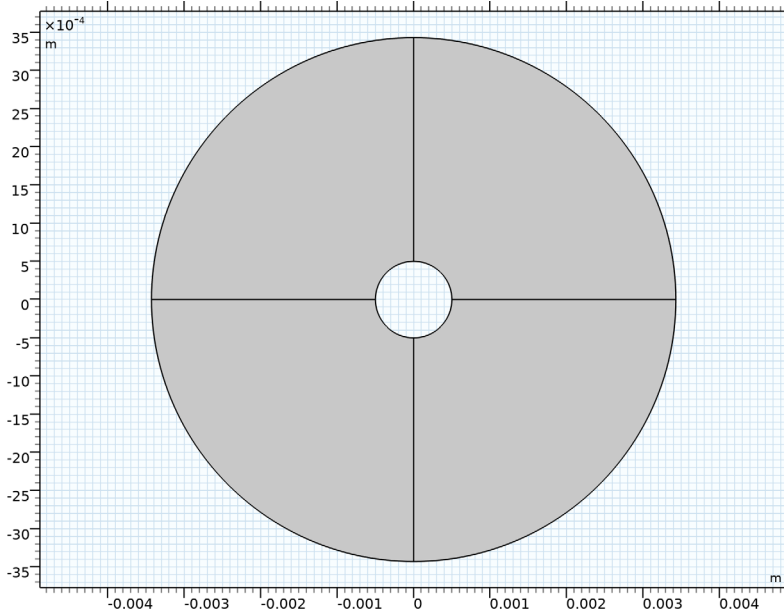
MATERIALS

Define a dielectric material for the region between the conductors.

Insulator

1 In the **Model Builder** window, under **Component 1 (comp1)** right-click **Materials** and choose **Blank Material**.

2 In the **Settings** window for **Material**, type Insulator in the **Label** text field.

3 Locate the **Material Contents** section. In the table, enter the following settings:

| Property	Variable	Value	Unit	Property group
Relative permittivity	epsilonr_iso ; epsilonrii = epsilonr_iso, epsilonrij = 0	eps_r	1	Basic
Relative permeability	mur_iso ; murii = mur_iso, murij = 0	1	1	Basic
Electrical conductivity	sigma_iso ; sigmaii = sigma_iso, sigmaij = 0	0	S/m	Basic

DEFINITIONS

Add a variable for the characteristic impedance computed as the voltage between the conductors divided by the current through the cable. For this purpose, you need two nonlocal integration couplings.

Integration 1 (intop1)

1 In the **Definitions** toolbar, click 🔗 **Nonlocal Couplings** and choose **Integration**.

2 In the **Settings** window for **Integration**, type int_rad in the **Operator name** text field.

3 Locate the **Source Selection** section. From the **Geometric entity level** list, choose **Boundary**.

4 Select Boundary 1 only.

Any of the four interior boundaries that connect the two conductors would do.
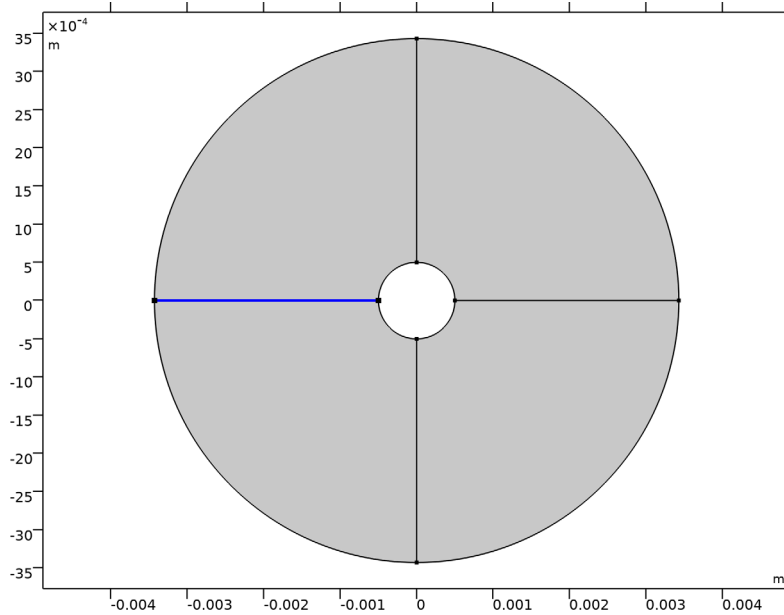
Integration 2 (intop2)

1 In the **Definitions** toolbar, click ✐ **Nonlocal Couplings** and choose **Integration**.

2 In the **Settings** window for **Integration**, type int_circ in the **Operator name** text field.

3 Locate the **Source Selection** section. From the **Geometric entity level** list, choose **Boundary**.

4 Select Boundaries 5, 6, 9, and 12 only, (the outer conductor boundaries).
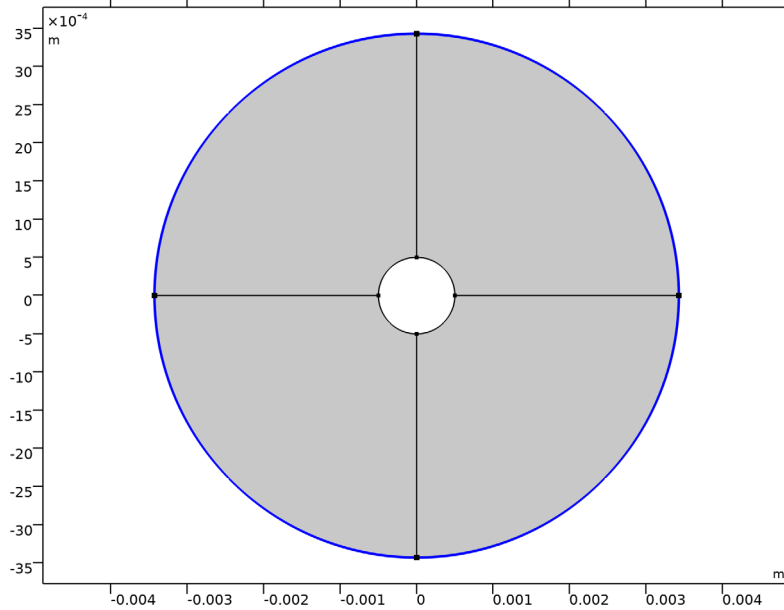
Now define the variable for the characteristic impedance computed from the simulation.

Variables 1

1 In the **Definitions** toolbar, click $a=$ **Local Variables**.

2 In the **Settings** window for **Variables**, locate the **Variables** section.

3 In the table, enter the following settings:

Name	Expression	Unit	Description
V	int_rad(-emw.Ex*t1x-emw.Ey*t1y)	V	Voltage
I	-int_circ(emw.Hx*t1x+emw.Hy*t1y)	A	Current
ZO_model	V/I	Ω	Characteristic impedance

Here, t1x and t1y are the tangential vector components along the integration boundaries (1 refers to the boundary dimension). Shortly, you will determine the tangential vector directions along the boundaries using an arrow plot of t1. The signs in the definitions above are chosen such that $V = V_i - V_o$ (see Equation 1) and to have a positive current value correspond to a current in the positive z direction. The emw.

prefix gives the correct physics-interface scope for the electric and magnetic field vector components.

ELECTROMAGNETIC WAVES, FREQUENCY DOMAIN (EMW)

Keep the default physics settings, which include perfect electric conductor conditions for the outer boundaries.

MESH 1

Use the default mesh.

1 In the **Model Builder** window, under **Component 1 (comp1)** click **Mesh 1**.

2 In the **Settings** window for **Mesh**, locate the **Physics-Controlled Mesh** section.

3 In the table, clear the **Use** check box for **Electromagnetic Waves, Frequency Domain (emw)**.

4 Click ▦ **Build All**.
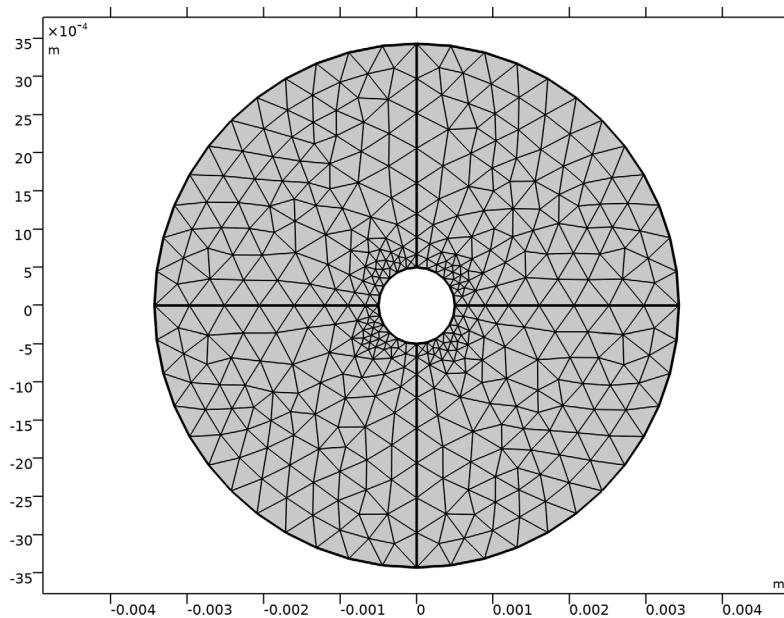
STUDY 1

Step 1: Mode Analysis

1 In the **Model Builder** window, under **Study 1** click **Step 1: Mode Analysis**.

2 In the **Settings** window for **Mode Analysis**, locate the **Study Settings** section.

3 Select the **Desired number of modes** check box. In the associated text field, type 1.

4 Select the **Search for modes around** check box. In the associated text field, type
   sqrt(eps_r).

5 In the **Home** toolbar, click = **Compute**.

RESULTS

Electric Field (emw)
The default plot shows the distribution of the norm of the electric field. Add an arrow plot
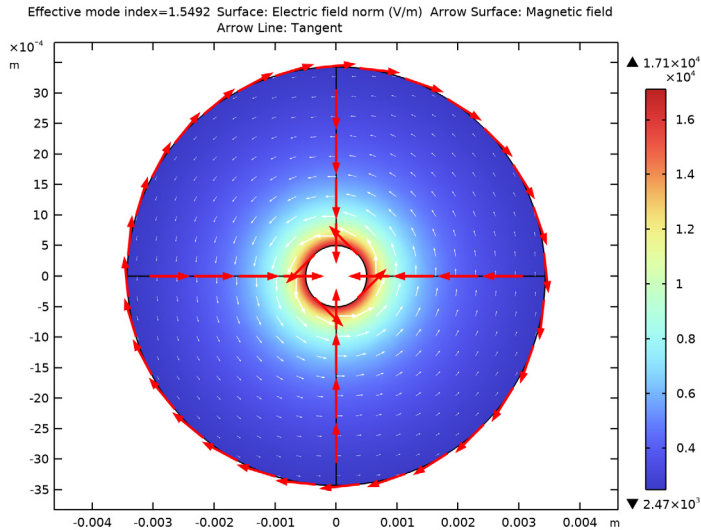of the magnetic field.

Arrow Surface 1
1 Right-click **Electric Field (emw)** and choose **Arrow Surface**.

2 In the **Settings** window for **Arrow Surface**, locate the **Arrow Positioning** section.

3 Find the **X grid points** subsection. In the **Points** text field, type 21.

4 Find the **Y grid points** subsection. In the **Points** text field, type 21.

5 Locate the **Coloring and Style** section.

6 Select the **Scale factor** check box. In the associated text field, type 7e-6.

   You can use the slider to adjust the arrow lengths.

7 From the **Color** list, choose **White**.

8 In the **Electric Field (emw)** toolbar, click Plot **Plot**.

9 Click the   **Zoom Extents** button in the **Graphics** toolbar.

   Compare the resulting plot with that shown in Figure 2.

To find out the integration contour orientations, plot the tangent vector, t1, along the
boundaries as follows:

Arrow Line 1
1 Right-click **Electric Field (emw)** and choose **Arrow Line**.

2 In the **Settings** window for **Arrow Line**, click **Replace Expression** in the upper-right corner
   of the **Expression** section. From the menu, choose **Component 1 (comp1)>Geometry>tx,
   ty - Tangent**.

3 Locate the **Arrow Positioning** section. In the **Number of arrows** text field, type 50.

4 In the **Electric Field (emw)** toolbar, click **Plot**.

Effective mode index=1.5492 Surface: Electric field norm (V/m) Arrow Surface: Magnetic field
Arrow Line: Tangent

A comparison with Equation 1 reveals that the line integral for the voltage computes the potential difference V_i - V_o. When computing the line integral for the current, the clockwise orientation of the integration contour would mean that a positive current is directed in the negative z direction, that is, into the modeling plane. The minus sign added in the definition of I reverses this direction.

5 Right-click **Arrow Line 1** and choose **Disable**, to retrieve the result plot.

Finish by computing the characteristic impedance.

Global Evaluation 1

1 In the **Results** toolbar, click (8.5) **Global Evaluation**.

2 In the **Settings** window for **Global Evaluation**, click **Replace Expression** in the upper-right corner of the **Expressions** section. From the menu, choose **Component 1 (comp1)> Definitions>Variables>Z0_model - Characteristic impedance -** Ω.

3 Click ▬ **Evaluate**.

TABLE

1 Go to the **Table** window.

The result, roughly 74.65 Ω, is within 0.2% of the analytic value, 74.53 Ω.