Created in COMSOL Multiphysics 6.1

Electrical Breakdown Between Spheres

Introduction

Whether electrical breakdown will occur in an electrical system depends on a number of parameters including geometry, applied voltage, fill gas, pressure and temperature. A fully self consistent plasma model of an electrical system can be very tricky, especially if the geometry is complicated. It is possible to estimate whether electrical breakdown will occur without solving a full blown plasma model, by integrating Townsend growth coefficients along electric field lines. This model shows how to do this for electrical breakdown between two spheres.

Model Definition

This tutorial model shows that the onset of electrical breakdown between two spheres separated by 2 cm, will occur at 51.8 kV for dry air at room temperature.The Electrical Breakdown Detection interface defines 3 different regimes which can occur in any given device. The breakdown condition for a self-sustaining discharge is given by the following:

$$\gamma_i\left(\exp\left(\int_0^D N\alpha ds\right) - 1\right) = 1 \tag{1}$$

where γ_i is the secondary emission coefficient (dimensionless), N is the number density (SI unit: $1/\mathrm{m}^3$), α is the reduced Townsend growth/decay coefficient (SI unit: m^2), s is the arc length along the particle trajectory, and D is the distance from the source boundary to any destination boundary. Using this, the following 3 regimes are defined.

NO DISCHARGE

Rearranging Equation 1, it is obvious that no discharge will occur if the following condition is true:

$$\int_0^D N\alpha ds < \ln(1 + 1/\gamma_i)\,. \tag{2}$$

SUSTAINED DISCHARGE

When the left hand side of Equation 1 is greater than 1, a self–sustaining discharge can occur. Another way of writing this condition is that a self sustained discharge can form when the Townsend condition is met:

$$\int_0^D N\alpha ds > \ln(1 + 1/\gamma_i)\,. \tag{3}$$

This is not necessarily catastrophic to an electrical design, since the current is usually limited in such a discharge. The third case however, can be catastrophic.

STREAMER

When the exponential of the left hand side is above around 10^8, a streamer will form across the gap. Mathematically, the streamer condition is given by:

$$\int_0^D N\alpha ds > 17.7 + \ln(d/(1[\mathrm{cm}])) \tag{4}$$

where d is the gap distance in cm.

The Electrical Breakdown Detection physics interface defines a variable, `ebd.bi` which takes the value of 0 for the no discharge case, 1 for the sustained discharge and 2 for the streamer. This variable is plotted by default when running a study.

All the information about which regime the system will operate is embedded in the reduced Townsend coefficient, α. The reduced Townsend coefficient is a strong function of the reduced electric field:

$$\alpha = \alpha\left(\frac{E}{N}\right) \tag{5}$$

where E is the electric field parallel to the streamlines (SI unit: V/m). The software computes the integral by solving an ordinary differential equation along the test particle trajectories:

$$\frac{d\alpha_D}{ds} = \alpha\left(\frac{N}{N_{\mathrm{stp}}}\right) \tag{6}$$

where N_{stp} is the number density at standard temperature and pressure. Another quantity of interest is the pressure multiplied by the path length. This is also computed by solving the following ordinary differential equation:

$$\left(\frac{1}{p}\right)\frac{d}{ds}(p_D) = 1 \tag{7}$$

where it is assumed the pressure is constant along the trajectory.

In this model, the Townsend growth coefficient is taken using the **Dry air** option, which uses an interpolation function for the growth coefficient vs. reduced electric field, from Ref. 2.

Results and Discussion

The electric field from the electrostatic study is shown in Figure 1. As expected, the electric field is strongest at the minimum distance between the two spheres. Any seed electrons exposed to this electric field have the potential to gain enough energy to ionize the background gas, possibly setting off a positive feedback loop whereby an electron avalanche can occur.

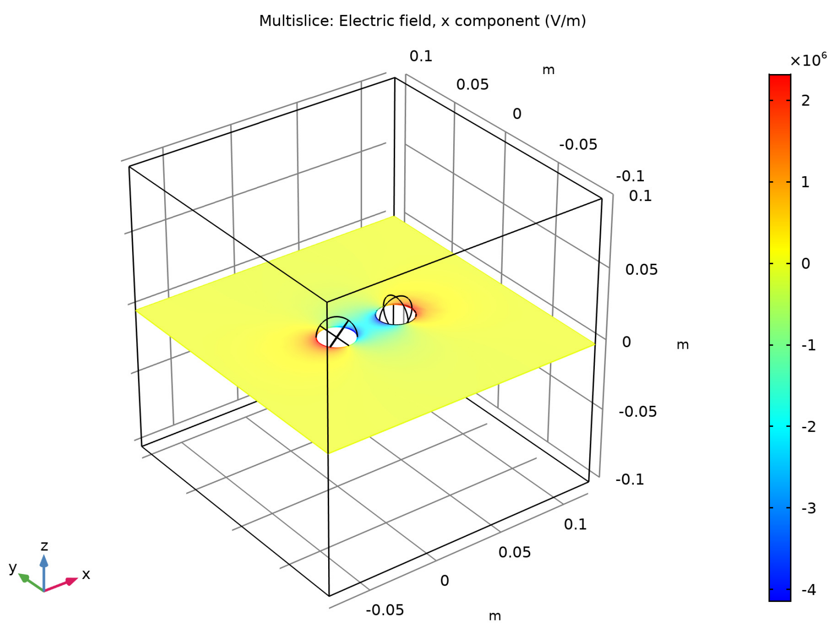

Figure 1: Plot of the electric field between two spheres.

The breakdown indicator for this configuration is shown in Figure 2 below. When the indicator is zero, no discharge will occur, when the indicator is one, a (current limited) Townsend discharge may occur. Since this is current limited, it is not necessarily a problem in the electrical system. However, when the indicator is two, as shown by the small red spot on the left sphere (at the minimum distance between the two spheres), a streamer may form which is not current limited, which can potentially be catastrophic to an electrical system. As expected, the region for streamer formation is a very small dot, meaning that onset is only just occurring. This is expected, since the applied voltage is right at the value (51.8 kV) where this should be the case.

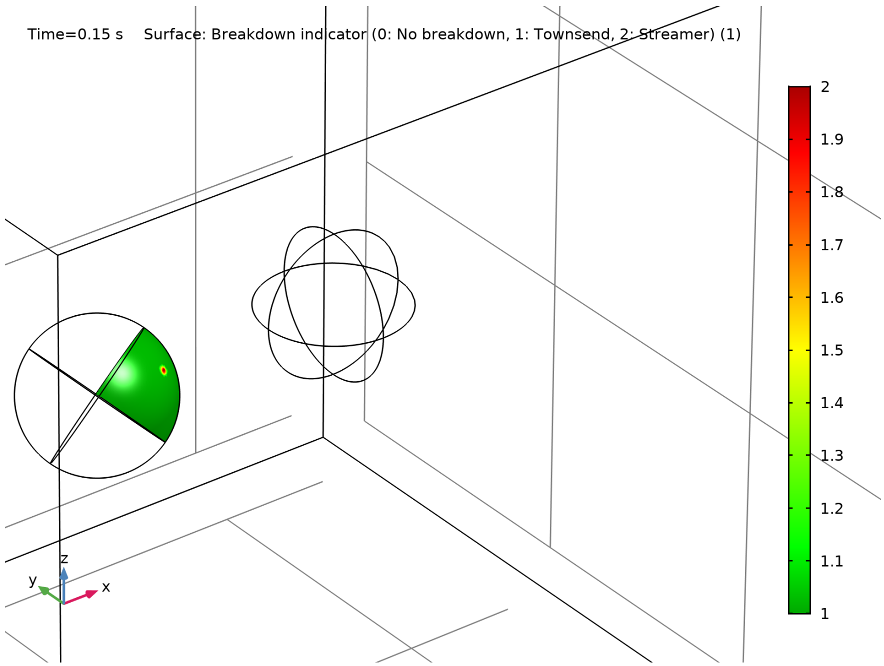

Figure 2: Electrical breakdown indicator on the cathode surface.

The integrated Townsend growth coefficient is show in Figure 3. When this is above around 18.3, streamer formation can occur. There is a substantial variation in this function over the cathode – away from the minimum distance between the two spheres, the integrated growth coefficient is substantially under the streamer threshold.

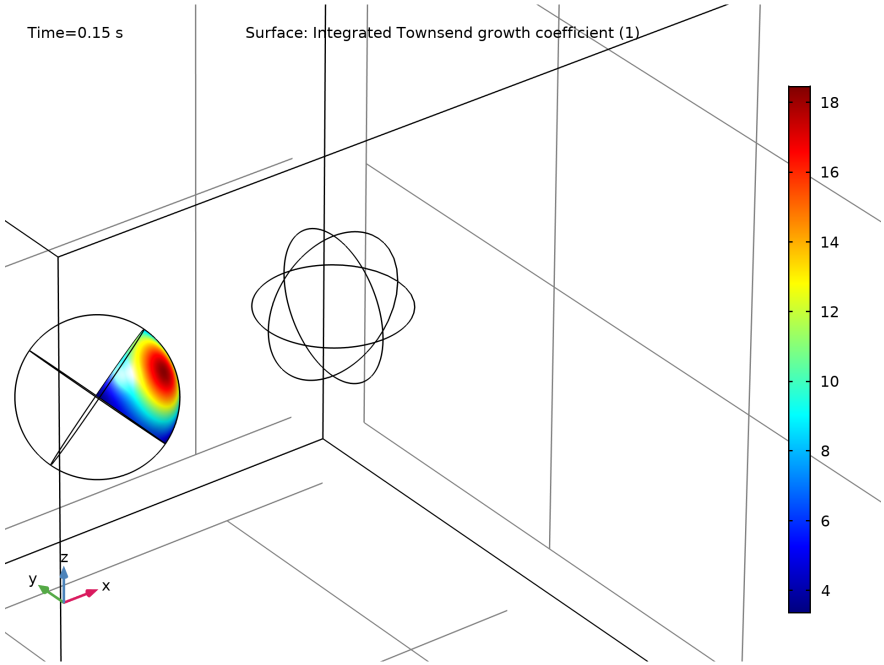

Figure 3: Integrated Townsend growth coefficient on the cathode surface. Once this value gets above around 18, electrical breakdown will occur.

Finally, in Figure 4 the pressure times the path length is plotted in units of `torr-cm`. This quantity can be of interest, since Paschen curves usually plot the breakdown voltage vs. pressure times gap. Providing this as a default plot makes it easy to verify that breakdown is occurring at the expected voltage (for a given pressure times gap length), since experimental Paschen curve data is often available.

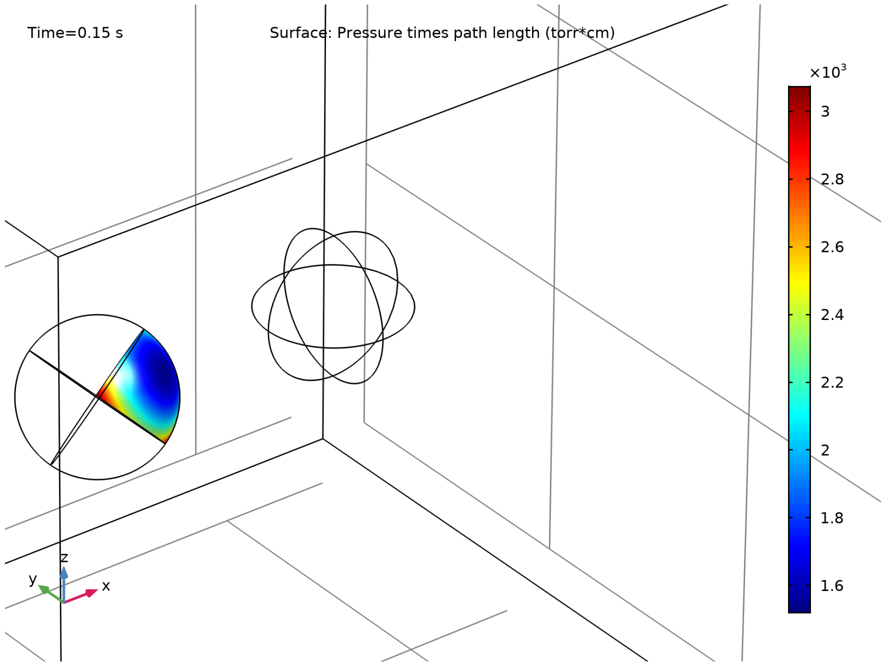

Figure 4: Plot of the pressure times the path length, in units of torr cm. The values are within the range in which the approximate method used to detect electrical breakdown is valid.

Reference

1. Larry K. Warne, Roy E. Jorgenson and Scott D. Nicolaysen, *Ionization Coefficient Approach to Modeling Breakdown in Nonuniform Geometries*, Sandia Report (2003).

2. J. Dutton, *A survey of Electron Swarm Data*, J. Phys. Chem. Ref. Data, Vol 4, No 3, 1975.

Application Library path: `Plasma_Module/Electrical_Breakdown_Detection/breakdown_between_spheres`

Modeling Instructions

From the **File** menu, choose **New**.

NEW

In the **New** window, click **Model Wizard**.

MODEL WIZARD

1 In the **Model Wizard** window, click **3D**.
2 In the **Select Physics** tree, select **AC/DC>Electric Fields and Currents>Electrostatics (es)**.
3 Click **Add**.
4 Click **Study**.
5 In the **Select Study** tree, select **General Studies>Stationary**.
6 Click **Done**.

GLOBAL DEFINITIONS

Parameters 1

1 In the **Model Builder** window, under **Global Definitions** click **Parameters 1**.
2 In the **Settings** window for **Parameters**, locate the **Parameters** section.
3 In the table, enter the following settings:

Name	Expression	Value	Description
Vapp	-51.8[kV]	-51800 V	Applied voltage
a	1.25[cm]	0.0125 m	Sphere diameter
hg	2.25[cm]	0.0225 m	Gap 1
d	2*hg-2*a	0.02 m	Distance between spheres

GEOMETRY 1

Sphere 1 (sph1)

1 In the **Model Builder** window, expand the **Component 1 (comp1)>Geometry 1** node.
2 Right-click **Geometry 1** and choose **Sphere**.
3 In the **Settings** window for **Sphere**, locate the **Size** section.
4 In the **Radius** text field, type a.
5 Locate the **Rotation Angle** section. In the **Rotation** text field, type 45.

Rotate 1 (rot1)

1 In the **Geometry** toolbar, click **Transforms** and choose **Rotate**.

2 Select the object **sph1** only.

3 In the **Settings** window for **Rotate**, locate the **Rotation** section.

4 From the **Axis type** list, choose **Cartesian**.

5 In the **y** text field, type `1`.

6 In the **z** text field, type `0`.

7 In the **Angle** text field, type `45`.

8 Click **Build All Objects**.

Sphere 2 (sph2)

1 In the **Geometry** toolbar, click **Sphere**.

2 In the **Settings** window for **Sphere**, locate the **Size** section.

3 In the **Radius** text field, type `a`.

4 Locate the **Position** section. In the **x** text field, type `2*hg`.

Block 1 (blk1)

1 In the **Geometry** toolbar, click **Block**.

2 In the **Settings** window for **Block**, locate the **Size and Shape** section.

3 In the **Width** text field, type `20[cm]`.

4 In the **Depth** text field, type `20[cm]`.

5 In the **Height** text field, type `20[cm]`.

6 Locate the **Position** section. In the **x** text field, type `-8[cm]`.

7 In the **y** text field, type `-10[cm]`.

8 In the **z** text field, type `-10[cm]`.

9 Click **Build All Objects**.

10 Click the **Go to Default View** button in the **Graphics** toolbar.

11 Click the **Wireframe Rendering** button in the **Graphics** toolbar.

Difference 1 (dif1)

1 In the **Geometry** toolbar, click **Booleans and Partitions** and choose **Difference**.

2 Select the object **blk1** only.

3 In the **Settings** window for **Difference**, locate the **Difference** section.

4 Find the **Objects to subtract** subsection. Click to select the **Activate Selection** toggle button.

5 Select the objects **rot1** and **sph2** only.

6 Click **Build All Objects**.

MATERIALS

Material 1 (mat1)

1 In the **Model Builder** window, under **Component 1 (comp1)** right-click **Materials** and choose **Blank Material**.

2 In the **Settings** window for **Material**, locate the **Material Contents** section.

3 In the table, enter the following settings:

Property	**Variable**	**Value**	**Unit**	**Property group**
Relative permittivity	epsilonr_iso ; epsilonrii = epsilonr_iso, epsilonrij = 0	1	1	Basic

ELECTROSTATICS (ES)

Ground 1

1 In the **Model Builder** window, under **Component 1 (comp1)** right-click **Electrostatics (es)** and choose **Ground**.

2 Click the **Select Box** button in the **Graphics** toolbar.

3 Select Boundaries 14–21 only.

Electric Potential 1

1 In the **Physics** toolbar, click **Boundaries** and choose **Electric Potential**.

2 Click the **Select Box** button in the **Graphics** toolbar.

3 Select Boundaries 6–13 only.

4 In the **Settings** window for **Electric Potential**, locate the **Electric Potential** section.

5 In the V_0 text field, type `Vapp`.

MESH 1

Size 1

1 In the **Model Builder** window, under **Component 1 (comp1)** right-click **Mesh 1** and choose **Size**.
2 In the **Settings** window for **Size**, locate the **Geometric Entity Selection** section.
3 From the **Geometric entity level** list, choose **Boundary**.
4 Click the **Select Box** button in the **Graphics** toolbar.
5 Select Boundaries 6–17 only.
6 Locate the **Element Size** section. Click the **Custom** button.
7 Click to collapse the **Element Size Parameters** section. Click to expand the **Element Size Parameters** section.
8 Select the **Maximum element size** check box. In the associated text field, type `5E-4`.
9 Locate the **Geometric Entity Selection** section. Click **Copy Selection**.

Free Triangular 1

1 In the **Mesh** toolbar, click **Boundary** and choose **Free Triangular**.
2 In the **Settings** window for **Free Triangular**, locate the **Boundary Selection** section.
3 Click **Paste Selection**.
4 In the **Paste Selection** dialog box, type `6-17` in the **Selection** text field.
5 Click **OK**.

Free Tetrahedral 1

1 In the **Mesh** toolbar, click **Free Tetrahedral**.
2 In the **Settings** window for **Free Tetrahedral**, click **Build All**.
3 Click the **Go to Default View** button in the **Graphics** toolbar.

STUDY 1

In the **Home** toolbar, click **Compute**.

RESULTS

Multislice 1

1 In the **Model Builder** window, expand the **Results>Electric Field Norm (es)** node, then click **Multislice 1**.
2 In the **Settings** window for **Multislice**, locate the **Multiplane Data** section.
3 Find the **x-planes** subsection. From the **Entry method** list, choose **Number of planes**.

4 In the **Planes** text field, type 0.

5 Find the **y-planes** subsection. From the **Entry method** list, choose **Number of planes**.

6 In the **Planes** text field, type 0.

Streamline Multislice 1

1 In the **Model Builder** window, click **Streamline Multislice 1**.

2 In the **Settings** window for **Streamline Multislice**, locate the **Multiplane Data** section.

3 Find the **x-planes** subsection. From the **Entry method** list, choose **Number of planes**.

4 In the **Planes** text field, type 0.

5 Find the **y-planes** subsection. From the **Entry method** list, choose **Number of planes**.

6 In the **Planes** text field, type 0.

7 In the **Electric Field Norm (es)** toolbar, click **Plot**.

ADD PHYSICS

1 In the **Home** toolbar, click **Add Physics** to open the **Add Physics** window.

2 Go to the **Add Physics** window.

3 In the tree, select **Plasma>Electric Discharges>Electrical Breakdown Detection (ebd)**.

4 Find the **Physics interfaces in study** subsection. In the table, clear the **Solve** check box for **Study 1**.

5 Click **Add to Component 1** in the window toolbar.

6 In the **Home** toolbar, click **Add Physics** to close the **Add Physics** window.

ELECTRICAL BREAKDOWN DETECTION (EBD)

Cathode 1

1 Right-click **Component 1 (comp1)>Electrical Breakdown Detection (ebd)** and choose **Cathode**.

2 Select Boundary 13 only.

3 Click the **Go to Default View** button in the **Graphics** toolbar.

Particle Counter 1

1 In the **Physics** toolbar, click **Boundaries** and choose **Particle Counter**.

2 Select Boundaries 14–17 only.

Electrical Breakdown Detection 1

1 In the **Model Builder** window, click **Electrical Breakdown Detection 1**.

2 In the **Settings** window for **Electrical Breakdown Detection**, locate the **Electric Field** section.

3 From the **E** list, choose **Electric field (es/ccn1)**.

ADD STUDY

1 In the **Home** toolbar, click **Add Study** to open the **Add Study** window.

2 Go to the **Add Study** window.

3 Find the **Physics interfaces in study** subsection. In the table, clear the **Solve** check box for **Electrostatics (es)**.

4 Find the **Studies** subsection. In the **Select Study** tree, select **General Studies> Time Dependent**.

5 Click **Add Study** in the window toolbar.

6 In the **Home** toolbar, click **Add Study** to close the **Add Study** window.

STUDY 2

Step 1: Time Dependent

1 In the **Settings** window for **Time Dependent**, locate the **Study Settings** section.

2 In the **Output times** text field, type `range(0,0.002,0.15)`.

3 Click to expand the **Values of Dependent Variables** section. Find the **Values of variables not solved for** subsection. From the **Settings** list, choose **User controlled**.

4 From the **Method** list, choose **Solution**.

5 From the **Study** list, choose **Study 1, Stationary**.

6 In the **Home** toolbar, click **Compute**.

RESULTS

Breakdown Indicator (ebd)

1 In the **Model Builder** window, under **Results** click **Breakdown Indicator (ebd)**.

2 In the **Breakdown Indicator (ebd)** toolbar, click **Plot**.

Integrated Townsend Growth Coefficient (ebd)

1 In the **Model Builder** window, click **Integrated Townsend Growth Coefficient (ebd)**.

2 In the **Integrated Townsend Growth Coefficient (ebd)** toolbar, click **Plot**.

Pressure Times Path Length (ebd)

1 In the **Model Builder** window, click **Pressure Times Path Length (ebd)**.

2 In the **Pressure Times Path Length (ebd)** toolbar, click **Plot**.