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Motion of Trapped Protons in Earth's Magnetic Field

Introduction

The Earth has a substantial magnetic field that extends outward for many thousands of kilometers. It is thought that this magnetic field is generated by circulating currents within a spinning metal liquid core — the dynamo effect. This magnetic field closely resembles a dipole field; however, there is a tilt between the spin axis of Earth and that of the magnetic dipole. There are also other asymmetries in Earth's magnetic field that require the use of software models to describe fully. These models are constructed using data obtained from orbiting spacecraft.

Having an accurate model of Earth's magnetic field is very important in studies of Earth's deep interior, its crust, and its ionosphere and magnetosphere. Therefore, a standard model is maintained by the International Association of Geomagnetism and Aeronomy (IAGA). This model is referred to as the International Geomagnetic Reference Field (IGRF). The IGRF model is updated every few years and is currently on its 12th generation (Ref. 1).

The influence of the solar wind means that the full extended shape of Earth's magnetosphere is very different from a dipole. However, within a few Earth radii, simpler models of Earth's magnetic field are sufficient.

Charged particles in Earth's magnetic field travel in helical paths around the field lines. The angle between the direction of the magnetic field and a particle's trajectory is referred to as the pitch angle. As particles move along the magnetic field lines, they approach a pole in the magnetic field. Near the poles, the particles get closer to earth, causing the magnetic field to increase in magnitude and thus causing the pitch angle to increase. If the pitch angle reaches 90 degrees while the particle is still outside of the atmosphere (>100 km), the particle motion reverses direction back along the field line. The point at which this occurs is referred to as a mirror, or bounce, point. The particle similarly bounces off the other mirror point in the other hemisphere.

Particles are considered trapped if they are not lost to Earth's atmosphere during this motion. Particles can also be liberated due to scattering by electromagnetic waves.

The latitude of a mirror point (λ_m) is related to the particle's pitch angle at the equatorial plane, or its equatorial pitch angle (α_{eq}). As the equatorial pitch angle increases, the mirror point latitude decreases. Any particles that have equatorial pitch angles that give mirror points outside the atmosphere are said to be outside the atmospheric loss cone and are trapped.

Particles that bounce from mirror point to mirror point also exhibit a drift motion around Earth, switching from field line to field line. This is due to the fact that the magnetic field magnitude is increasing as the particle moves toward Earth, so that its gyration is not circular but has in fact a smaller radius on the side closer to Earth.

As electrons and protons drift in opposite directions around Earth, an electric current (ring current) is set up. The magnitude of the ring current increases during solar storms, and its effect can be measured on the ground as a weakening of the measured magnetic field. The disturbance storm time index, Dst, is one such measure of this ring current and is used to assess the severity of magnetic storms from the Sun.

These motions of trapped charge particles lead to “belts” of energetic charged particles in the near Earth environment and are referred to as the Van Allen radiation belts. They extend from about 1000 to 60,000 km (~0.15 to ~10 Earth radii) above Earth’s surface and therefore pose a real threat to space-based microelectronics.

Model Definition

Mathematically, the IGRF model consists of the Gauss coefficients, which define a spherical harmonic expansion of the magnetic scalar potential:

$$V(r, \Phi, \theta) = a\sum_{l=1}^{L}\sum_{m=-l}^{l}\left(\frac{R_e}{r}\right)^{l-1}(g_l^m\cos m\Phi + h_l^m\sin m\Phi)P_l^m(\cos\theta)$$

where r is the radial distance from Earth’s center, L is the maximum degree of the expansion, Φ is East longitude, θ is colatitude (polar angle), R_e is Earth’s radius, g_l^m and h_l^m are Gauss coefficients, and $P_l^m\cos\theta$ are the Schmidt normalized associated Legendre functions of degree l and order m.

The model uses a simple sphere of radius R_e to represent Earth within a larger spherical simulation domain of radius $5R_e$ where this particle trajectories are computed. The geometry is shown in Figure 1.

The Magnetic Force feature in the Charged Particle Tracing interface and the Particle Tracing for Fluid Flow interface includes a built-in option to compute magnetic fields using the data from the IGRF model. To access this data, in a 3D model select **Earth’s magnetic field** from the **Magnetic flux density** list in the settings window for the Magnetic Force feature.

Figure 2 shows the IGRF magnetic field lines. The difference from a simple dipole is not really evident in this figure, but asymmetries exist in both latitude and longitude.

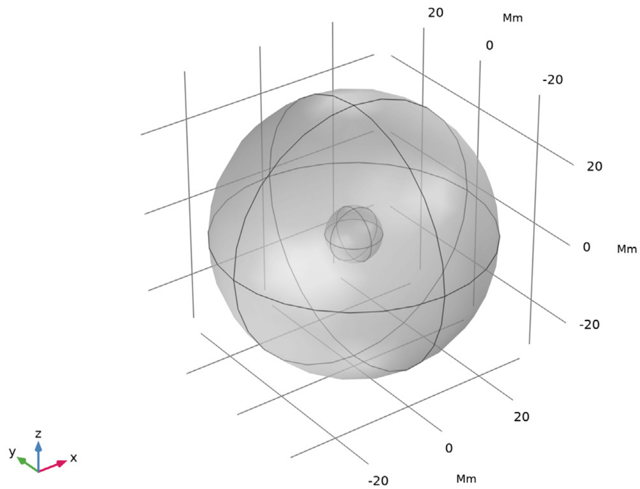

Figure 1: Geometry of the simulation domain, which extends from radius Re to 5Re.

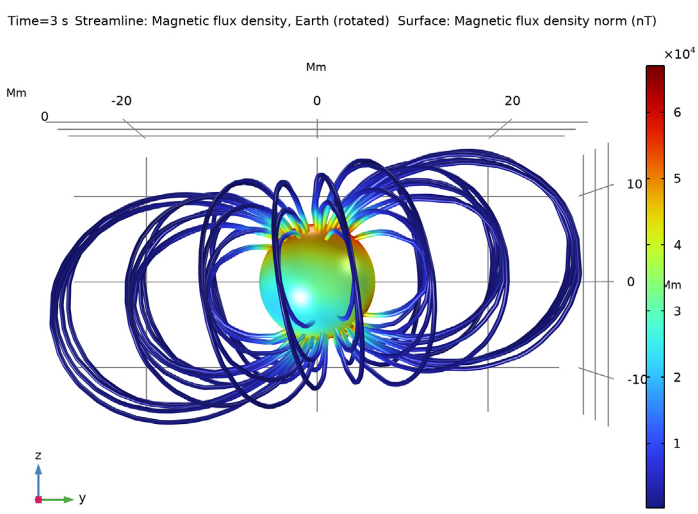

Figure 2: IGRF magnetic field lines.

The model uses the Charged Particle Tracing interface only, together with a Time Dependent study.

Results and Discussion

A single 10 MeV proton is released from the equatorial plane at a distance of $2R_e$ from the center of Earth. It is released with an equatorial pitch angle of 30 degrees. The three components of its motion (gyration, bounce, and drift) are clearly visible in Figure 3. The timescales for the drift motion are much longer than that of the bounce motion, which in turn is much longer than the gyration period.

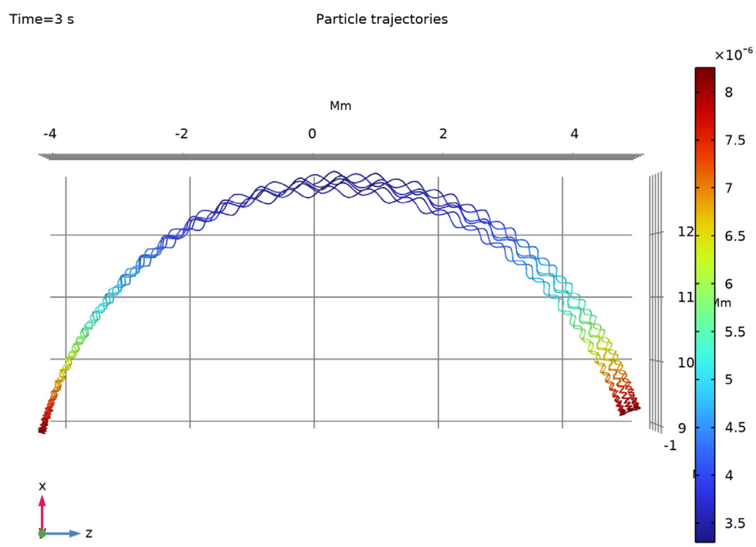

Figure 3: Particle trajectory of a single 10 MeV proton in Earth's magnetic field. Particle gyration, bounce and drift motion are clearly visible.

Due to the large changes in the magnitude of the magnetic field in different parts of the simulation domain, care has to be taken to ensure an appropriate time step is used for the particles. An good indication of how well the time steps are resolving the gyration of the particles is to plot the particle energy as a function of time and observe little or no change. Figure 4 plots the relative change in the particle kinetic energy as a function of time. This relative error is of the order 10^{-3} and so you can be certain you are resolving most of the particle motion.

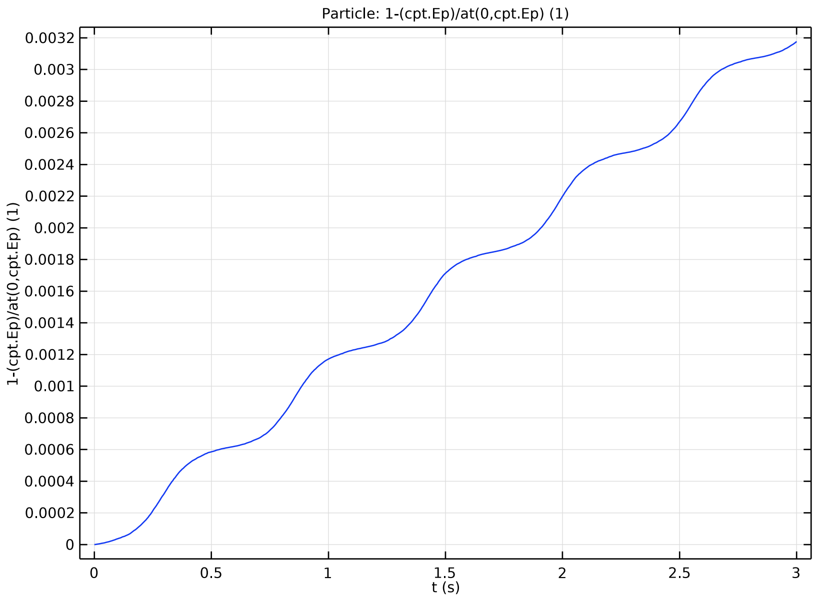

Figure 4: Relative error in particle kinetic energy.

There are many aspects of the particle motion that can be investigated using this model, but the scope of this tutorial is limited to investigating how the mirror point latitude is affected by the equatorial pitch angle of the released protons.

As mentioned above, as the equatorial pitch angle of a particle decreases, the mirror point latitude is expected to increase. At the limits, a particle with an equatorial pitch angle of 90 degrees would remain in the equatorial plane whereas one with a pitch angle of zero degrees would travel directly along the field line without bouncing.

Figure 6 shows the mirror point latitude versus equatorial pitch angle for 15 particles with equatorial pitch angles ranging from 5 degrees to 80 degrees.

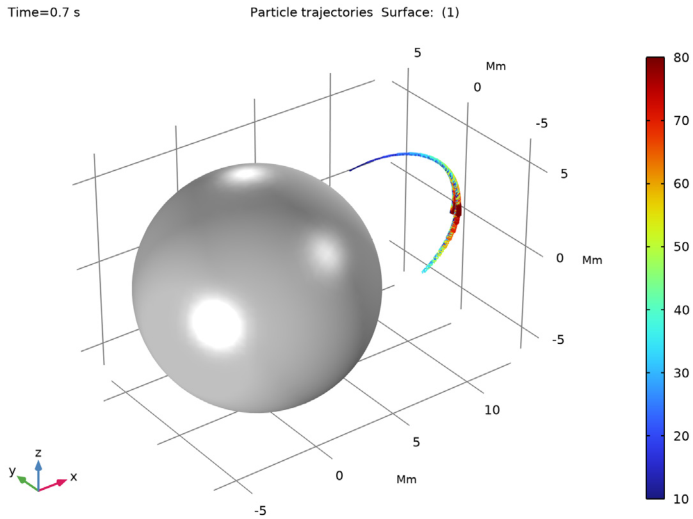

Figure 5: Trajectories of several trapped protons. The color expression corresponds to the equatorial pitch angle of each proton.

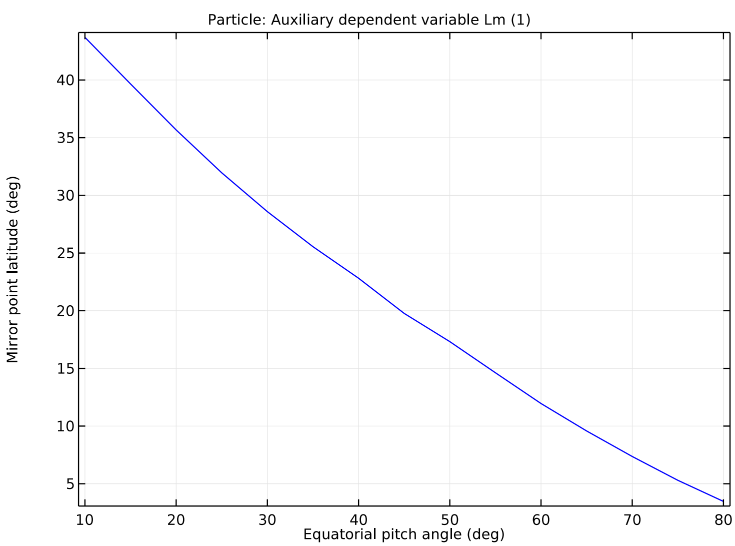

Figure 6: Mirror point latitude (λ_m) against particle equatorial pitch angle (Ea).

References

1. International Geomagnetic Reference Field website, http://www.ngdc.noaa.gov/IAGA/vmod/igrf.html.

Application Library path: `Particle_Tracing_Module/Charged_Particle_Tracing/trapped_protons`

Modeling Instructions

From the **File** menu, choose **New**.

NEW

In the **New** window, click **Model Wizard**.

MODEL WIZARD

1 In the **Model Wizard** window, click **3D**.
2 In the **Select Physics** tree, select **AC/DC>Particle Tracing>Charged Particle Tracing (cpt)**.
3 Click **Add**.
4 Click **Study**.
5 In the **Select Study** tree, select **General Studies>Time Dependent**.
6 Click **Done**.

GLOBAL DEFINITIONS

Parameters 1

1 In the **Model Builder** window, under **Global Definitions** click **Parameters 1**.
2 In the **Settings** window for **Parameters**, locate the **Parameters** section.
3 In the table, enter the following settings:

Name	Expression	Value	Description
Re	6371.2[km]	6.3712E6 m	Radius of the Earth
E0	10[MeV]	1.6022E-12 J	Initial particle energy
alpha	30[deg]	0.5236 rad	Equatorial pitch angle

GEOMETRY 1

Add a sphere with radius `Re` within a larger simulation domain of radius `5*Re`.

1 In the **Model Builder** window, under **Component 1 (comp1)** click **Geometry 1**.

2 In the **Settings** window for **Geometry**, locate the **Units** section.

3 From the **Length unit** list, choose **Mm**.

Sphere 1 (sph1)

1 In the **Geometry** toolbar, click **Sphere**.

2 In the **Settings** window for **Sphere**, locate the **Size** section.

3 In the **Radius** text field, type `Re`.

Sphere 2 (sph2)

1 In the **Geometry** toolbar, click **Sphere**.

2 In the **Settings** window for **Sphere**, locate the **Size** section.

3 In the **Radius** text field, type `5*Re`.

Difference 1 (dif1)

1 In the **Geometry** toolbar, click **Booleans and Partitions** and choose **Difference**.

2 Select the object **sph2** only.

3 In the **Settings** window for **Difference**, locate the **Difference** section.

4 Find the **Objects to subtract** subsection. Click to select the **Activate Selection** toggle button.

5 Click the **Transparency** button in the **Graphics** toolbar.

6 Select the object **sph1** only.

7 Click **Build All Objects**.

8 Click the **Zoom Extents** button in the **Graphics** toolbar.

9 Click the **Transparency** button in the **Graphics** toolbar.

DEFINITIONS

Hide the outermost boundary to facilitate the setup of the Charged Particle Tracing interface.

Hide for Geometry 1

1 In the **Model Builder** window, expand the **Definitions** node.

2 Right-click **View 1** and choose **Hide for Geometry**.

3 In the **Settings** window for **Hide for Geometry**, locate the **Selection** section.

4 From the **Geometric entity level** list, choose **Boundary**.

5 On the object **dif1**, select Boundaries 1–4, 9, 10, 13, and 16 only.

Ball 1

Add a **Ball** selection containing all boundaries on the Earth's surface.

1 In the **Definitions** toolbar, click **Ball/Disk**.

2 In the **Settings** window for **Ball**, locate the **Geometric Entity Level** section.

3 From the **Level** list, choose **Boundary**.

4 Locate the **Ball Radius** section. In the **Radius** text field, type `Re`.

CHARGED PARTICLE TRACING (CPT)

1 In the **Model Builder** window, under **Component 1 (comp1)** click **Charged Particle Tracing (cpt)**.

2 In the **Settings** window for **Charged Particle Tracing**, locate the **Particle Release and Propagation** section.

3 From the **Formulation** list, choose **Newtonian, first order**.

In this example, the particles are relativistic protons.

4 Select the **Relativistic correction** check box.

Particle Properties 1

1 In the **Model Builder** window, under **Component 1 (comp1)>Charged Particle Tracing (cpt)** click **Particle Properties 1**.

2 In the **Settings** window for **Particle Properties**, locate the **Particle Species** section.

3 From the **Particle species** list, choose **Proton**.

Magnetic Force 1

Exert a magnetic force on the particles using the Earth's magnetic field.

1 In the **Physics** toolbar, click **Domains** and choose **Magnetic Force**.

2 In the **Settings** window for **Magnetic Force**, locate the **Domain Selection** section.

3 From the **Selection** list, choose **All domains**.

4 Locate the **Magnetic Force** section. From the $\mathbf{B}$ list, choose **Earth's magnetic field**.

Release from Grid 1

Set up the **Release from Grid** feature to release a single 10 MeV proton at a distance of 2 Earth radii along the x-axis. Set its velocity x-component to zero, and its y- and z-components using the equatorial pitch angle.

1 In the **Physics** toolbar, click **Global** and choose **Release from Grid**.
2 In the **Settings** window for **Release from Grid**, locate the **Initial Coordinates** section.
3 In the $q_{x,0}$ text field, type `2*Re`.
4 Locate the **Initial Velocity** section. From the **Initial velocity** list, choose **Kinetic energy and direction**.
5 In the E_0 text field, type `E0`.
6 Specify the $\mathbf{L}_0$ vector as

`0`	x
`sin(alpha)`	y
`cos(alpha)`	z

Auxiliary Dependent Variable 1

Define two auxiliary dependent variables for the mirror point latitude and equatorial pitch angle. These auxiliary dependent variables will be disabled in Study 1 but will be needed in Study 2.

1 In the **Physics** toolbar, click **Global** and choose **Auxiliary Dependent Variable**.
2 In the **Settings** window for **Auxiliary Dependent Variable**, locate the **Auxiliary Dependent Variable** section.
3 In the **Field variable name** text field, type `Lm`.

Auxiliary Dependent Variable 2

1 In the **Physics** toolbar, click **Global** and choose **Auxiliary Dependent Variable**.
2 In the **Settings** window for **Auxiliary Dependent Variable**, locate the **Auxiliary Dependent Variable** section.
3 In the **Field variable name** text field, type `Ea`.

Velocity Reinitialization 1

The **Velocity Reinitialization** feature can be used to detect when a particle bounces, and set the auxiliary dependent variable `Lm` to the instantaneous value of the latitude at that point.

1 In the **Physics** toolbar, click **Domains** and choose **Velocity Reinitialization**.
2 In the **Settings** window for **Velocity Reinitialization**, locate the **Domain Selection** section.
3 From the **Selection** list, choose **All domains**.

4 Locate the **Velocity Reinitialization** section. In the *e* text field, type `(cpt.vx*cpt.mf1.Berx+cpt.vy*cpt.mf1.Bery+cpt.vz*cpt.mf1.Berz)<0 && Lm==0`.

This expression is true for the first time step at which the angle between the particle velocity and the Earth's magnetic field is greater than 90 degrees.

5 From the **Effect on primary particle** list, choose **None**.

6 Click to expand the **New Value of Auxiliary Dependent Variables** section. Select the **Assign new value to auxiliary variable : Lm** check box.

7 In the Lm_{new} text field, type `(-acos(qz/sqrt(qx^2+qy^2+qz^2))+pi/2)*(180/pi)`.

Release from Grid 1

Duplicate the release feature to create a separate release feature for the second study.

Release from Grid 2

1 In the **Model Builder** window, under **Component 1 (comp1)>Charged Particle Tracing (cpt)** right-click **Release from Grid 1** and choose **Duplicate**.

2 In the **Settings** window for **Release from Grid**, locate the **Initial Velocity** section.

3 Specify the $\mathbf{L}_0$ vector as

`0`	x
`sin(Ea)`	y
`cos(Ea)`	z

The second **Release from Grid** feature stores the initial equatorial pitch angle of each particle.

4 Locate the **Initial Value of Auxiliary Dependent Variables** section. From the **Distribution function** list, choose **List of values**.

5 Click **Range**.

6 In the **Range** dialog box, type `10[deg]` in the **Start** text field.

7 In the **Step** text field, type `5[deg]`.

8 In the **Stop** text field, type `80[deg]`.

9 Click **Replace**.

Since the initial direction will be defined in terms of the equatorial pitch angle, specify that this auxiliary variable must be initialized before the particle momentum.

10 In the Settings window for Release from Grid, locate the **Initial Value of Auxiliary Dependent Variables** section.

11 Select the second **Initialize before particle momentum** check box, which corresponds to the variable for equatorial pitch angle Ea.

STUDY 1

Step 1: Time Dependent

Disable the features that are not needed for the first study.

1 In the **Model Builder** window, under **Study 1** click **Step 1: Time Dependent**.

2 In the **Settings** window for **Time Dependent**, locate the **Physics and Variables Selection** section.

3 Select the **Modify model configuration for study step** check box.

4 In the tree, select **Component 1 (comp1)>Charged Particle Tracing (cpt)>Auxiliary Dependent Variable 1**.

5 Right-click and choose **Disable**.

6 In the tree, select **Component 1 (comp1)>Charged Particle Tracing (cpt)>Auxiliary Dependent Variable 2**.

7 Right-click and choose **Disable**.

8 In the tree, select **Component 1 (comp1)>Charged Particle Tracing (cpt)>Velocity Reinitialization 1**.

9 Right-click and choose **Disable**.

10 In the tree, select **Component 1 (comp1)>Charged Particle Tracing (cpt)>Release from Grid 2**.

11 Right-click and choose **Disable**.

12 Locate the **Study Settings** section. Click **Range**.

13 In the **Range** dialog box, type 0.005 in the **Step** text field.

14 In the **Stop** text field, type 3.

15 Click **Replace**.

16 In the **Home** toolbar, click **Compute**.

RESULTS

Particle Trajectories (cpt)

1 In the **Settings** window for **3D Plot Group**, locate the **Plot Settings** section.

2 Clear the **Plot dataset edges** check box.

Particle Trajectories 1

1. In the **Model Builder** window, expand the **Particle Trajectories (cpt)** node, then click **Particle Trajectories 1**.
2. In the **Settings** window for **Particle Trajectories**, locate the **Coloring and Style** section.
3. Find the **Line style** subsection. From the **Type** list, choose **Tube**.
4. Select the **Radius scale factor** check box. In the associated text field, type `0.01`.
5. From the **Interpolation** list, choose **Uniform**.
6. In the **Number of interpolated times** text field, type `2000`.
7. Find the **Point style** subsection. From the **Type** list, choose **None**.

Color Expression 1

1. In the **Model Builder** window, expand the **Particle Trajectories 1** node, then click **Color Expression 1**.
2. In the **Settings** window for **Color Expression**, click **Replace Expression** in the upper-right corner of the **Expression** section. From the menu, choose **Component 1 (comp1)>Charged Particle Tracing>Fields>cpt.mf1.normB - Magnetic flux density norm - T**.
3. In the **Particle Trajectories (cpt)** toolbar, click **Plot**.
4. In the **Graphics** window toolbar, click ▾ next to **Go to Default View**, then choose **Go to ZX View**. Compare the resulting plot with Figure 3.

   Create some additional datasets to visualize the magnetic field using a **Streamline** plot.

Cut Plane 1

1. In the **Results** toolbar, click **Cut Plane**.
2. In the **Settings** window for **Cut Plane**, locate the **Plane Data** section.
3. From the **Plane** list, choose **XY-planes**.

Study 1/Solution 1 (2) (sol1)

In the **Model Builder** window, under **Results>Datasets** right-click **Study 1/Solution 1 (sol1)** and choose **Duplicate**.

Selection

1. In the **Results** toolbar, click **Attributes** and choose **Selection**.
2. In the **Settings** window for **Selection**, locate the **Geometric Entity Selection** section.
3. From the **Geometric entity level** list, choose **Boundary**.
4. From the **Selection** list, choose **Ball 1**.

Magnetic Flux Density

1 In the **Results** toolbar, click **3D Plot Group**.
2 In the **Settings** window for **3D Plot Group**, type `Magnetic Flux Density` in the **Label** text field.
3 Locate the **Plot Settings** section. Clear the **Plot dataset edges** check box.

Streamline 1

1 Right-click **Magnetic Flux Density** and choose **Streamline**.
2 In the **Settings** window for **Streamline**, click **Replace Expression** in the upper-right corner of the **Expression** section. From the menu, choose **Component 1 (comp1)> Charged Particle Tracing>Fields>cpt.mf1.Berx,...,cpt.mf1.Berz - Magnetic flux density, Earth (rotated)**.
3 Locate the **Streamline Positioning** section. From the **Positioning** list, choose **Starting-point controlled**.
4 In the **Points** text field, type `50`.
5 From the **Along curve or surface** list, choose **Cut Plane 1**.
6 Locate the **Coloring and Style** section. Find the **Line style** subsection. From the **Type** list, choose **Tube**.

Color Expression 1

1 Right-click **Streamline 1** and choose **Color Expression**.
2 In the **Settings** window for **Color Expression**, click **Replace Expression** in the upper-right corner of the **Expression** section. From the menu, choose **Component 1 (comp1)> Charged Particle Tracing>Fields>cpt.mf1.normB - Magnetic flux density norm - T**.
3 Locate the **Expression** section. From the **Unit** list, choose **nT**.

Surface 1

1 In the **Model Builder** window, right-click **Magnetic Flux Density** and choose **Surface**.
2 In the **Settings** window for **Surface**, locate the **Data** section.
3 From the **Dataset** list, choose **Study 1/Solution 1 (2) (sol1)**.
4 Click **Replace Expression** in the upper-right corner of the **Expression** section. From the menu, choose **Component 1 (comp1)>Charged Particle Tracing>Fields>cpt.mf1.normB - Magnetic flux density norm - T**.
5 Locate the **Expression** section. From the **Unit** list, choose **nT**.
6 Click to expand the **Inherit Style** section. From the **Plot** list, choose **Streamline 1**.
7 In the **Magnetic Flux Density** toolbar, click **Plot**.

8 Click the **Go to YZ View** button in the **Graphics** toolbar. Compare the resulting plot with Figure 2.

Energy Loss

1 In the **Home** toolbar, click **Add Plot Group** and choose **1D Plot Group**.

2 In the **Settings** window for **1D Plot Group**, type `Energy Loss` in the **Label** text field.

3 Locate the **Data** section. From the **Dataset** list, choose **Particle 1**.

Particle 1

1 In the **Energy Loss** toolbar, click **More Plots** and choose **Particle**.

Verify that energy is conserved throughout the study.

2 In the **Settings** window for **Particle**, locate the **y-Axis Data** section.

3 In the **Expression** text field, type `1-(cpt.Ep)/at(0,cpt.Ep)`.

4 In the **Energy Loss** toolbar, click **Plot**. Compare the resulting plot with Figure 4.

ADD STUDY

1 In the **Home** toolbar, click **Add Study** to open the **Add Study** window.

2 Go to the **Add Study** window.

3 Find the **Studies** subsection. In the **Select Study** tree, select **General Studies> Time Dependent**.

4 Click **Add Study** in the window toolbar.

5 In the **Home** toolbar, click **Add Study** to close the **Add Study** window.

STUDY 2

Step 1: Time Dependent

1 In the **Settings** window for **Time Dependent**, locate the **Physics and Variables Selection** section.

2 Select the **Modify model configuration for study step** check box.

3 In the tree, select **Component 1 (comp1)>Charged Particle Tracing (cpt)> Release from Grid 1**.

4 Click **Disable**.

5 Locate the **Study Settings** section. Click **Range**.

6 In the **Range** dialog box, type `0.001` in the **Step** text field.

7 In the **Stop** text field, type `0.7`.

8 Click **Replace**.

9 In the **Home** toolbar, click **Compute**.

RESULTS

Particle Trajectories (cpt) 1

1 In the **Settings** window for **3D Plot Group**, locate the **Plot Settings** section.

2 Clear the **Plot dataset edges** check box.

Particle Trajectories 1

1 In the **Model Builder** window, expand the **Particle Trajectories (cpt) 1** node, then click **Particle Trajectories 1**.

2 In the **Settings** window for **Particle Trajectories**, locate the **Coloring and Style** section.

3 Find the **Line style** subsection. From the **Type** list, choose **Line**.

4 Find the **Point style** subsection. From the **Type** list, choose **None**.

Color Expression 1

1 In the **Model Builder** window, expand the **Particle Trajectories 1** node, then click **Color Expression 1**.

2 In the **Settings** window for **Color Expression**, locate the **Expression** section.

3 In the **Expression** text field, type `Ea*180/pi`.

Surface 1

1 In the **Model Builder** window, right-click **Particle Trajectories (cpt) 1** and choose **Surface**.

2 In the **Settings** window for **Surface**, locate the **Data** section.

3 From the **Dataset** list, choose **Study 1/Solution 1 (2) (sol1)**.

4 Locate the **Coloring and Style** section. From the **Coloring** list, choose **Uniform**.

5 From the **Color** list, choose **Gray**.

6 In the **Particle Trajectories (cpt) 1** toolbar, click **Plot**.

7 Click the **Go to Default View** button in the **Graphics** toolbar. Compare the resulting plot with Figure 5.

Mirror Point Latitude

1 In the **Home** toolbar, click **Add Plot Group** and choose **1D Plot Group**.

2 In the **Settings** window for **1D Plot Group**, type `Mirror Point Latitude` in the **Label** text field.

Plot the particle mirror point latitude against equatorial pitch angle.

3 Locate the **Data** section. From the **Dataset** list, choose **Particle 2**.

4 From the **Time selection** list, choose **Last**.

5 Locate the **Plot Settings** section.

6 Select the **x-axis label** check box. In the associated text field, type `Equatorial pitch angle (deg)`.

7 Select the **y-axis label** check box. In the associated text field, type `Mirror point latitude (deg)`.

Particle 1

1 In the **Mirror Point Latitude** toolbar, click **More Plots** and choose **Particle**.

2 In the **Settings** window for **Particle**, click **Replace Expression** in the upper-right corner of the **y-Axis Data** section. From the menu, choose **Component 1 (comp1)>Charged Particle Tracing>Auxiliary dependent variables>Lm - Auxiliary dependent variable Lm**.

3 Locate the **x-Axis Data** section. From the **Parameter** list, choose **Expression**.

4 In the **Expression** text field, type `Ea*180/pi`.

5 In the **Mirror Point Latitude** toolbar, click **Plot**.

6 Click the **Zoom Extents** button in the **Graphics** toolbar. Compare the resulting plot with Figure 6.