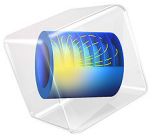


Quadrupole Lens

Introduction

Just like optical lenses focus light, electric and magnetic lenses can focus beams of charged particles. Systems of magnetic quadrupole lenses find a common use in focusing both ion and particle beams in accelerators at nuclear and particle physics centers such as CERN, SLAC, and ISIS. This COMSOL Multiphysics model shows the path of B^{5+} ions going through three consecutive magnetic quadrupole lenses. The model is set up in a cross section of the geometry.

Model Definition

The quadrupole consists of an assembly of four permanent magnets, as seen in Figure 1 below, where the magnets work together to give a good approximation of a quadrupole field. To strengthen the field and keep it contained within the system, the magnets are set in an iron cylinder.

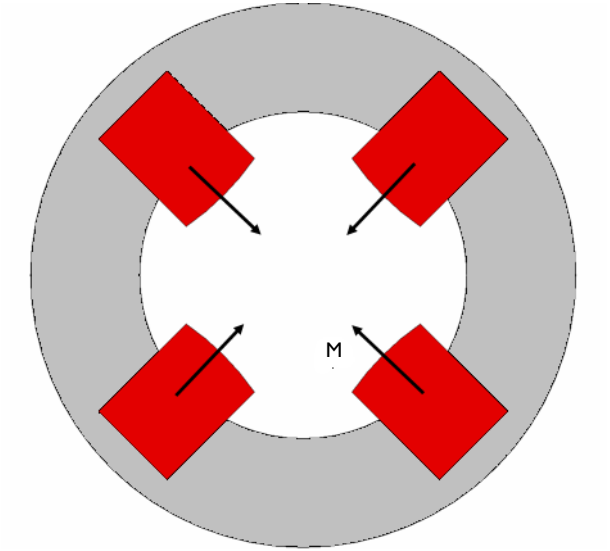


Figure 1: Cross-sectional view of one of the magnetic quadrupoles used in the lens.

The ions are sent through a system of three consecutive quadrupole assemblies. The middle one is twice as long as the other ones, and is rotated by 90 degrees around the

central axis. This means the polarity of its magnets is reversed. Figure 2 gives a full view of the magnetic quadrupole lens.

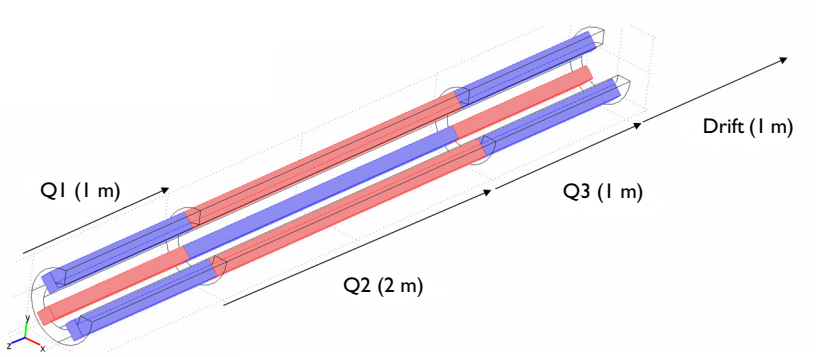


Figure 2: Cutout of the quadrupole lens. The second quadrupole (Q2) has its polarities reversed compared to Q1 and Q3. After traveling through the lens, the ions are left to drift 1 m.

An accelerator feeds the system with ions traveling with the velocity $0.01c$ along the central axis. To study the focusing effect of the quadrupoles, track a number of ions starting out from a distance of 3 cm from the central axis, evenly distributed along the circumference of a circle in the transverse plane. They are all assumed to have a zero initial transverse velocity. Each quadrupole focuses the ion beam along one of the transverse axes and defocuses it along the other one. The net effect after traveling through the system of the three quadrupoles and the drift length is focusing in all directions. As the ions exit the system, they are all contained within a 1 cm radius in the transverse plane.

The model is set up in a 2D cross section of any of the two identical quadrupoles Q1 and Q3. Neglecting fringe fields, the transverse magnetic field at a given point in a transverse plane in Q2 automatically has the same magnitude as the corresponding in Q1 and Q3, but point in the opposite direction. It is therefore sufficient to model the fields in one of the quadrupoles.

DOMAIN EQUATIONS

The magnetic field is described using the Magnetostatics equation, solving for the z component of the magnetic potential $\mathbf{A}$ (Wb/m):

$$\nabla \times (\mu_0^{-1}(\nabla \times A_z - \mathbf{B}_r)) - \sigma \mathbf{v} \times (\nabla \times A_z) = J_z^e$$

Here $\mu_0 = 4\pi \cdot 10^{-7}$ H/m denotes the permeability of vacuum, $\mathbf{B}_r$ is the remanent flux density (T), σ the conductivity (S/m), and $\mathbf{v}$ the velocity of the medium (m/s). In this example, the medium is not moving. The right-hand side of the equation holds an

imposed current, specified in terms of an external current density $J_z^{\,e}$ (A/m^2). No currents are imposed. The iron domain uses a slightly different formulation of the same equation:

$$\left(\nabla \times \frac{1}{\mu_0 \mu_r} \nabla \times A_z\right) - \sigma \mathbf{v} \times (\nabla \times A_z) = J_z^{\,e}$$

where $\mu_r = 4000$ is the relative permeability. The magnetic potential is everywhere defined so that $\mathbf{B} = \nabla \times \mathbf{A}$.

BOUNDARY CONDITIONS

The magnetic field is approximately parallel to the exterior boundary of the iron cylinder. To enforce this, use the *magnetic insulation* boundary condition, stating that $A_z = 0$.

Results and Discussion

The magnetic field density and streamlines in a cross section of Q1 or Q3 appear in Figure 3 below.

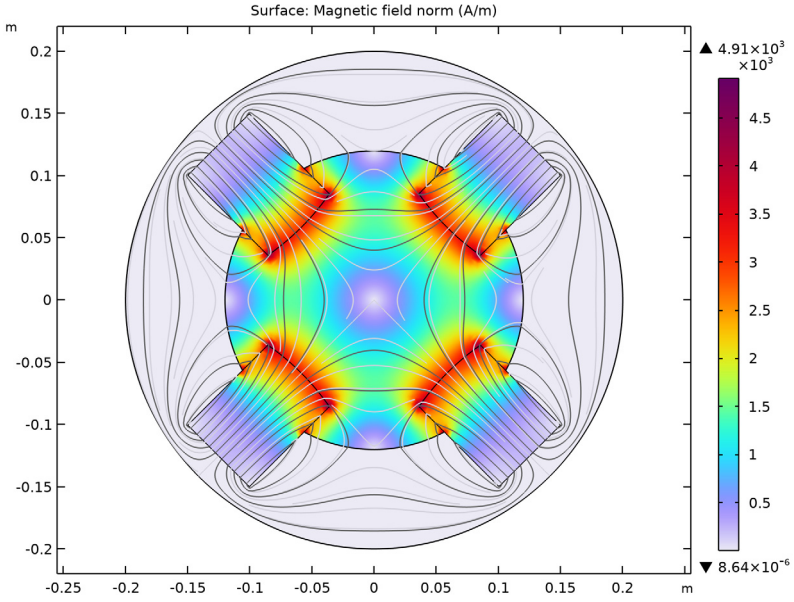


Figure 3: The magnetic field density and flow lines in the center of one of the quadrupole magnets.

Each ion passing through the assembly experiences a Maxwell force equal to $\mathbf{F} = q\mathbf{v} \times \mathbf{B}$, where $\mathbf{v}$ (m/s) is the ion's velocity. Next, assume that the z component of the velocity is

constant and much larger than the x and y (transverse) components. Thus consider only the force contributions from the z component of the velocity. To find the transverse position as a function of time, you need to solve Newton's second law for each ion, $q\mathbf{v} \times \mathbf{B} = m\mathbf{a}$, where m is the ion mass (kg), and $\mathbf{a}$ denotes its acceleration (m/s^2). If the computed magnetic flux density in Q1 equals $\mathbf{B}'$, and the length of quadrupole i is L_i (m), the flux density that the ion experiences is given by

$$\mathbf{B} = \begin{cases} \mathbf{B}' \text{ if } t < \dfrac{L_1}{v_z} \\ -\mathbf{B}' \text{ if } \dfrac{L_1}{v_z} < t < \dfrac{L_1 + L_2}{v_z} \\ \mathbf{B}' \text{ if } \dfrac{L_1 + L_2}{v_z} < t < \dfrac{L_1 + L_2 + L_3}{v_z} \\ 0 \text{ if } t > \dfrac{L_1 + L_2 + L_3}{v_z} \end{cases}$$

where t (s) is the time of flight. This dependency of the magnetic flux density on the time of flight is fed to the particle-tracing algorithm as a logical expression. Figure 4 below shows how the ions travel in the transverse plane.

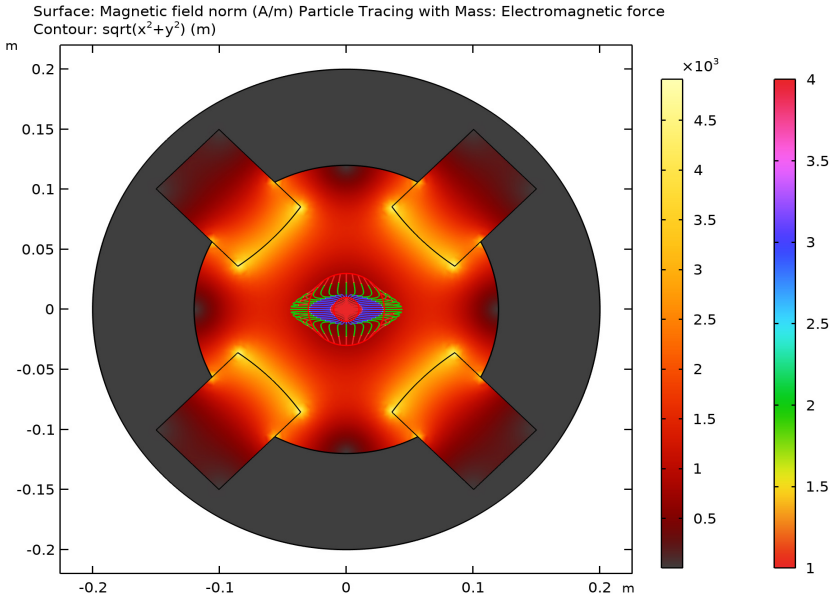


Figure 4: As the ions enter Q1, they start out evenly distributed around the larger circle, 3 cm from the z-axis. Q1 focuses along the x-axis and defocuses along the y-axis. The force on each ion is approximately proportional to its distance from the z-axis, so as the ions enter Q2, those that are far out on the x-axis rapidly turn around and move toward the center. Q3 stabilizes the motion and gets all ions on the right track. Finally the ions are left to drift toward a waist situated a little bit more than 1 m beyond Q3.

Application Library path: `COMSOL_Multiphysics/Electromagnetics/quadrupole`

Modeling Instructions

From the **File** menu, choose **New**.

NEW

In the **New** window, click **Model Wizard**.

MODEL WIZARD

1 In the **Model Wizard** window, click **2D**.

2 In the **Select Physics** tree, select **AC/DC>Electromagnetic Fields>Magnetic Fields (mf)**.

3 Click **Add**.

4 Click **Study**.

5 In the **Select Study** tree, select **General Studies>Stationary**.

6 Click **Done**.

GLOBAL DEFINITIONS

Parameters 1

1 In the **Model Builder** window, under **Global Definitions** click **Parameters 1**.

2 In the **Settings** window for **Parameters**, locate the **Parameters** section.

3 In the table, enter the following settings:

Name	Expression	Value	Description
M	11	11	Ion mass number
Z	5	5	Ion charge number
L1	1[m]	1 m	Length of first quadrupole
L2	2[m]	2 m	Length of second quadrupole
L3	1[m]	1 m	Length of third quadrupole
vz	0.01*c_const	2.9979E6 m/s	Ion velocity
m	M*mp_const	1.8399E-26 kg	Ion mass
q	Z*e_const	8.0109E-19 C	Ion charge
Br	8[mT]	0.008 T	Quadrupole remanent flux density

Here, `c_const`, `mp_const` and `e_const` are predefined constants for, respectively, the speed of light, the mass of the proton and the elementary charge.

GEOMETRY 1

Rectangle 1 (r1)

1 In the **Geometry** toolbar, click **Rectangle**.

2 In the **Settings** window for **Rectangle**, locate the **Size and Shape** section.

3 In the **Width** text field, type `0.177`.

4 In the **Height** text field, type `0.07`.

5 Locate the **Position** section. In the **y** text field, type `-0.035`.

6 Click **Build Selected**.

Rotate 1 (rot1)

1 In the **Geometry** toolbar, click **Transforms** and choose **Rotate**.

2 Select the object **r1** only.

3 In the **Settings** window for **Rotate**, locate the **Rotation** section.

4 In the **Angle** text field, type `45`.

5 Click **Build Selected**.

6 Click the **Zoom Extents** button in the **Graphics** toolbar.

Circle 1 (c1)

1 In the **Geometry** toolbar, click **Circle**.

2 In the **Settings** window for **Circle**, locate the **Size and Shape** section.

3 In the **Radius** text field, type `0.2`.

4 Locate the **Position** section. In the **x** text field, type `0.2`.

5 In the **y** text field, type `0.2`.

6 Click **Build Selected**.

7 Click the **Zoom Extents** button in the **Graphics** toolbar.

Intersection 1 (int1)

1 In the **Geometry** toolbar, click **Booleans and Partitions** and choose **Intersection**.

2 Click in the **Graphics** window and then press Ctrl+A to select both objects.

3 In the **Settings** window for **Intersection**, click **Build Selected**.

Rotate 2 (rot2)

1 In the **Geometry** toolbar, click **Transforms** and choose **Rotate**.

2 Select the object **int1** only.

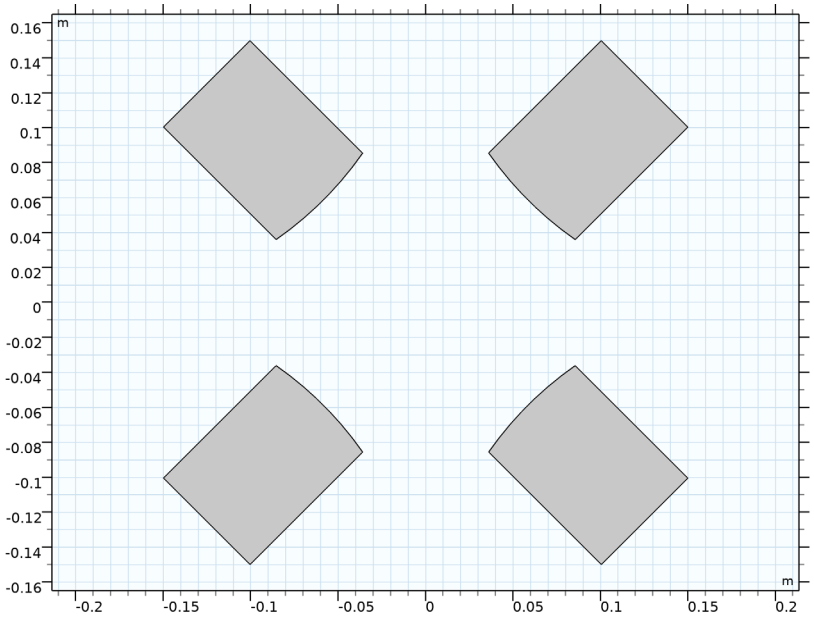
3 In the **Settings** window for **Rotate**, locate the **Rotation** section.

4 In the **Angle** text field, type `90, 180, 270`.

5 Locate the **Input** section. Select the **Keep input objects** check box.

6 Click **Build Selected**.

7 Click the **Zoom Extents** button in the **Graphics** toolbar.



Now make two circles centered at the origin.

Circle 2 (c2)

1 In the **Geometry** toolbar, click **Circle**.

2 In the **Settings** window for **Circle**, locate the **Size and Shape** section.

3 In the **Radius** text field, type `0.2`.

4 Click **Build Selected**.

5 Click the **Zoom Extents** button in the **Graphics** toolbar.

Circle 3 (c3)

1 In the **Geometry** toolbar, click **Circle**.

2 In the **Settings** window for **Circle**, locate the **Size and Shape** section.

3 In the **Radius** text field, type `0.12`.

4 Click **Build Selected**.

Create a composite object subtracting the magnets from the union of the two circles. This complex operation can be done in one step by using the Compose geometry operation and specifying the formula.

Compose 1 (co1)

1 In the **Geometry** toolbar, click **Booleans and Partitions** and choose **Compose**.

2 Click in the **Graphics** window and then press Ctrl+A to select all objects.

3 In the **Settings** window for **Compose**, locate the **Compose** section.

4 In the **Set formula** text field, type `c2+c3-(int1+rot2(1)+rot2(2)+rot2(3))`.

5 Click **Build Selected**.

Make one more circle centered at the origin.

Circle 4 (c4)

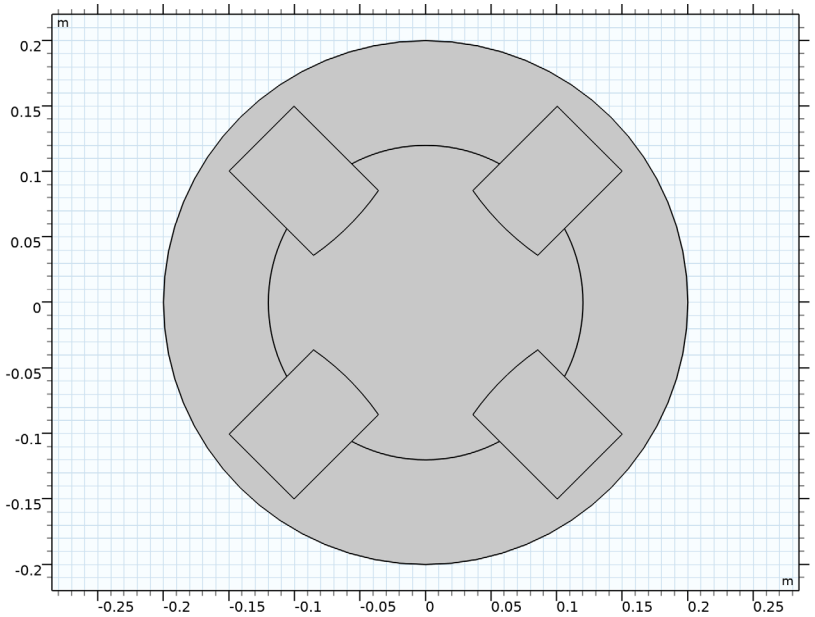
1 In the **Geometry** toolbar, click **Circle**.

2 In the **Settings** window for **Circle**, locate the **Size and Shape** section.

3 In the **Radius** text field, type `0.2`.

4 Click **Build Selected**.

5 Click the **Zoom Extents** button in the **Graphics** toolbar.



ADD MATERIAL

1 In the **Home** toolbar, click **Add Material** to open the **Add Material** window.

2 Go to the **Add Material** window.

3 In the tree, select **Built-in>Iron**.

4 Click **Add to Component** in the window toolbar.

5 In the **Home** toolbar, click **Add Material** to close the **Add Material** window.

MATERIALS

Iron (mat1)

Select Domain 2 only.

Magnets

1 In the **Model Builder** window, right-click **Materials** and choose **Blank Material**.

2 In the **Settings** window for **Material**, type `Magnets` in the **Label** text field.

3 Locate the **Geometric Entity Selection** section. Click **Paste Selection**.

4 In the **Paste Selection** dialog box, type `1, 3, 5-6` in the **Selection** text field.

5 Click **OK**.

6 In the **Settings** window for **Material**, locate the **Material Properties** section.

7 In the **Material properties** tree, select **Electromagnetic Models>Remanent Flux Density**.

8 Click **Add to Material**.

9 Locate the **Material Contents** section. In the table, enter the following settings:

Property	Variable	Value	Unit	Property group
Recoil permeability	murec_iso ; murecii = murec_iso, murecij = 0	1.05	1	Remanent flux density
Remanent flux density norm	normBr	Br	T	Remanent flux density

MAGNETIC FIELDS (MF)

Use the default condition, **Magnetic Insulation**, on all exterior boundaries.

Ampère's Law 2

1 In the **Model Builder** window, under **Component 1 (comp1)** right-click **Magnetic Fields (mf)** and choose **Ampère's Law**.

2 Select Domain 6 only.

3 In the **Settings** window for **Ampère's Law**, locate the **Constitutive Relation B-H** section.

4 From the **Magnetization model** list, choose **Remanent flux density**.

5 From the μ_{rec} list, choose **From material**.

6 Specify the **e** vector as

-1/sqrt(2)	x
-1/sqrt(2)	y
0	z

7 Locate the **Constitutive Relation Jc-E** section. From the σ list, choose **User defined**. Locate the **Constitutive Relation D-E** section. From the ε_r list, choose **User defined**.

Ampère's Law 3

1 In the **Physics** toolbar, click **Domains** and choose **Ampère's Law**.

2 Select Domain 3 only.

3 In the **Settings** window for **Ampère's Law**, locate the **Constitutive Relation B-H** section.

4 From the **Magnetization model** list, choose **Remanent flux density**.

5 Specify the **e** vector as

-1/sqrt(2)	x
1/sqrt(2)	y
0	z

6 Locate the **Constitutive Relation Jc-E** section. From the σ list, choose **User defined**. Locate the **Constitutive Relation D-E** section. From the ε_r list, choose **User defined**.

Ampère's Law 4

1 In the **Physics** toolbar, click **Domains** and choose **Ampère's Law**.

2 Select Domain 1 only.

3 In the **Settings** window for **Ampère's Law**, locate the **Constitutive Relation B-H** section.

4 From the **Magnetization model** list, choose **Remanent flux density**.

5 Specify the **e** vector as

1/sqrt(2)	x
1/sqrt(2)	y
0	z

6 Locate the **Constitutive Relation Jc-E** section. From the σ list, choose **User defined**. Locate the **Constitutive Relation D-E** section. From the ε_r list, choose **User defined**.

Ampère's Law 5

1 In the **Physics** toolbar, click **Domains** and choose **Ampère's Law**.

2 Select Domain 5 only.

3 In the **Settings** window for **Ampère's Law**, locate the **Constitutive Relation B-H** section.

4 From the **Magnetization model** list, choose **Remanent flux density**.

5 Specify the **e** vector as

`1/sqrt(2)`	x
`-1/sqrt(2)`	y
0	z

6 Locate the **Constitutive Relation Jc-E** section. From the σ list, choose **User defined**. Locate the **Constitutive Relation D-E** section. From the ε_r list, choose **User defined**.

Ampère's Law 6

1 In the **Physics** toolbar, click **Domains** and choose **Ampère's Law**.

2 Select Domain 4 only.

3 In the **Settings** window for **Ampère's Law**, locate the **Constitutive Relation B-H** section.

4 From the μ_r list, choose **User defined**. Locate the **Constitutive Relation Jc-E** section. From the σ list, choose **User defined**. Locate the **Constitutive Relation D-E** section. From the ε_r list, choose **User defined**.

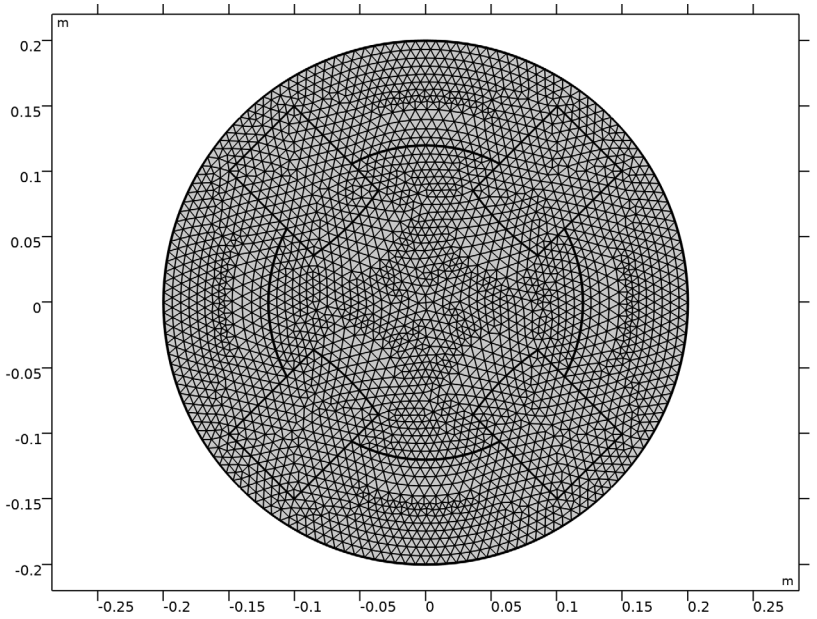
MESH 1

1 In the **Model Builder** window, under **Component 1 (comp1)** click **Mesh 1**.

2 In the **Settings** window for **Mesh**, locate the **Physics-Controlled Mesh** section.

3 From the **Element size** list, choose **Extra fine**.

4 Click **Build All**.



STUDY 1

In the **Home** toolbar, click **Compute**.

RESULTS

Magnetic Flux Density Norm (mf)

The default plot shows the norm of the magnetic flux density. Follow the instructions below to view the magnetic field.

Surface 1

1 In the **Model Builder** window, expand the **Magnetic Flux Density Norm (mf)** node, then click **Surface 1**.

2 In the **Settings** window for **Surface**, click **Replace Expression** in the upper-right corner of the **Expression** section. From the menu, choose **Component 1 (comp1)>Magnetic Fields> Magnetic>mf.normH - Magnetic field norm - A/m**.

3 In the **Magnetic Flux Density Norm (mf)** toolbar, click **Plot**.

4 Click the **Zoom Extents** button in the **Graphics** toolbar.

To see how the ions travel through the system of quadrupoles, do the following:

Ions Trajectories

1 In the **Home** toolbar, click **Add Plot Group** and choose **2D Plot Group**.

2 In the **Settings** window for **2D Plot Group**, type `Ions Trajectories` in the **Label** text field.

Surface 1

1 Right-click **Ions Trajectories** and choose **Surface**.

2 In the **Settings** window for **Surface**, click **Replace Expression** in the upper-right corner of the **Expression** section. From the menu, choose **Component 1 (comp1)>Magnetic Fields>Magnetic>mf.normH - Magnetic field norm - A/m**.

3 Locate the **Coloring and Style** section. Click **Change Color Table**.

4 In the **Color Table** dialog box, select **Thermal>GrayBody** in the tree.

5 Click **OK**.

6 Click the **Show More Options** button in the **Model Builder** toolbar.

7 In the **Show More Options** dialog box, in the tree, select the check box for the node **Results>All Plot Types**.

8 Click **OK**.

Ions Trajectories

In the **Model Builder** window, click **Ions Trajectories**.

Particle Tracing with Mass 1

1 In the **Ions Trajectories** toolbar, click **More Plots** and choose **Particle Tracing with Mass**.

Enter the expressions for the Lorentz force acting on the particles.

The position of the particle along the z-axis can be computed using the particle time variable `partt`.

2 In the **Settings** window for **Particle Tracing with Mass**, locate the **Total Force** section.

3 In the **Fx** text field, type `-q*vz*mf.By*(1-2*(partt>L1/vz)+2*(partt>(L1+L2)/vz)-(partt>(L1+L2+L3)/vz))`.

4 In the **Fy** text field, type `q*vz*mf.Bx*(1-2*(partt>L1/vz)+2*(partt>(L1+L2)/vz)-(partt>(L1+L2+L3)/vz))`.

5 Click to expand the **Mass and Velocity** section. In the **Mass** text field, type `m`.

6 Locate the **Particle Positioning** section. In the **x** text field, type `0.03*cos(range(0,0.05*pi,2*pi))`.

7 In the **y** text field, type `0.03*sin(range(0,0.05*pi,2*pi))`.

Color Expression 1

1 Right-click **Particle Tracing with Mass 1** and choose **Color Expression**.

2 In the **Settings** window for **Color Expression**, locate the **Expression** section.

3 In the **Expression** text field, type `1+(partt>L1/vz)+(partt>(L1+L2)/vz)+(partt>(L1+L2+L3)/vz)`.

4 Locate the **Coloring and Style** section. Click  **Change Color Table**.

5 In the **Color Table** dialog box, select **Rainbow>Cyclic** in the tree.

6 Click **OK**.

Particle Tracing with Mass 1

1 In the **Model Builder** window, click **Particle Tracing with Mass 1**.

2 In the **Settings** window for **Particle Tracing with Mass**, click to expand the **Release** section.

3 Click to expand the **Coloring and Style** section. Click to expand the **Quality** section. Find the **ODE solver settings** subsection. In the **Relative tolerance** text field, type `1e-6`.

4 Click to expand the **Advanced** section. Find the **Termination** subsection.

5 Select the **Maximum number of steps** check box. In the associated text field, type `1e5`.

6 Find the **Instantaneous flow field** subsection.

7 Select the **End time** check box. In the associated text field, type `5/3e6`.

8 In the **Ions Trajectories** toolbar, click  **Plot**.

Contour 1

1 In the **Model Builder** window, right-click **Ions Trajectories** and choose **Contour**.

2 In the **Settings** window for **Contour**, locate the **Expression** section.

3 In the **Expression** text field, type `sqrt(x^2+y^2)`.

4 Locate the **Levels** section. From the **Entry method** list, choose **Levels**.

5 In the **Levels** text field, type `0.01 0.03`.

6 Locate the **Coloring and Style** section. From the **Coloring** list, choose **Uniform**.

7 From the **Color** list, choose **Custom**.

8 On Windows, click the colored bar underneath, or — if you are running the cross-platform desktop — the **Color** button.

9 Clear the **Color legend** check box.

10 In the **Ions Trajectories** toolbar, click  **Plot**.

Zoom in on the center of the model geometry.