

Buckling of a Composite Cylinder

Introduction

Buckling is a structural instability that can lead to failure of a component even without initial material failure. Computation of the critical buckling loads and mode shapes can therefore be important from a design viewpoint, even though it has previously been determined that the loading of the component only causes elastic deformations. This applies to components made from laminated composite materials, where elastic properties, ply thicknesses and stacking sequence of a composite laminate will affect buckling loads and mode shapes.

This example illustrates a linear buckling analysis of a composite cylinder under compressive loading and fixed-end conditions. The composite cylinder is made up of eight layers (plies) of a carbon fiber reinforced epoxy material having different fiber orientations. An Equivalent Single Layer (ESL) theory based approach is used for this analysis. The effect of stacking sequence on the critical load factor is analyzed for different types of balanced laminates, such as a symmetric angle-ply laminate and an antisymmetric angle-ply laminate.

Model Definition

The model geometry consists of a composite cylinder with height of 0.4 m and radius of 0.15 m. The bottom end of the cylinder is fixed whereas the top end is free only to translate in the *z* direction, as shown in Figure 1. In order to perform a linear buckling analysis, a unit compressive load is applied on the top end of the cylinder in the downward direction.



Figure 1: Model geometry of the laminated composite cylinder.

LAMINA MATERIAL PROPERTIES

The composite lamina is assumed to be made of carbon fibers in an epoxy resin. The homogenized orthotropic material properties (Young's modulus, shear modulus, and Poisson's ratio) are given in Table 1.

Material property	Value
$\{E_1, E_2, E_3\}$	{134, 9.2, 9.2} GPa
{G ₁₂ , G ₂₃ , G ₁₃ }	{4.8, 4.8, 4.8} GPa
$\{v_{12}, v_{23}, v_{13}\}$	{0.28, 0.28, 0.28}

TABLE I: MATERIAL PROPERTIES OF A LAMINA

The density of the lamina is taken as 1700 kg/m^3 .

STACKING SEQUENCE

The composite laminate consists of eight layers where each layer (ply) has a thickness of 0.125 mm. This makes the total thickness of the composite laminate 1 mm. In order to study the effect of a stacking sequence on the buckling behavior of the composite cylinder, four different laminates are compared:

• Layered Material 1: [0/0/45/-45]_s (Symmetric angle-ply laminate)

- Layered Material 2: [90/90/45/-45]_s (Symmetric angle-ply laminate)
- Layered Material 3: [90/0/90/0]₈ (Symmetric cross-ply laminate)
- Layered Material 4: [45/45/45]_{as} (Antisymmetric angle-ply laminate)

The stacking sequence of each laminate is shown in Figure 2 and their corresponding fiber orientations are given in Table 2.

The fiber orientations are presented with respect to the first axis of the laminate coordinate system as shown in Figure 3.



Figure 2: Stacking sequence of laminated composite cylinder showing fiber orientation in each layer from bottom to top.

Layer Number	Fiber orientation in Layered material I (°)	Fiber orientation in Layered material 2 (°)	Fiber orientation in Layered material 3 (°)	Fiber orientation in Layered material 4 (°)
I	0	90	90	45
2	0	90	0	45
3	45	45	90	45
4	-45	-45	0	45
5	-45	-45	0	-45
6	45	45	90	-45
7	0	90	0	-45
8	0	90	90	-45

TABLE 2: STACKING SEQUENCES CONSIDERED.



Figure 3: The laminate coordinate system showing the first principal direction along the cylinder axis.



Material Switch 1(1)=Layered Material: [0/0/45/-45]_s Critical load factor=1.1705E5

Figure 4: First buckling mode shape and its corresponding critical load factor for Layered Material: $[0/0/45/-45]_{s}$.

The buckling analysis of a composite laminate with different stacking sequences shows that the critical buckling load and its corresponding mode shape is highly dependent on the stacking sequence of the individual laminae (plies).

For the first two stacking sequences, where the symmetric angle-ply arrangement is used, the first buckling mode is spiral-shaped as shown in Figure 4 and Figure 5. A notable difference between the two is the pitch of the spiraling.

For the third stacking sequence, where the symmetric cross-ply arrangement is used, the buckling mode is diamond-shaped, as shown in Figure 6. Last, for the fourth stacking sequence, in which an antisymmetric angle-ply arrangement is used, the buckling mode shape is axisymmetric, as shown in Figure 7.

Stacking sequence	Type of laminate	Critical buckling load (kN)	Mode shape
[0/0/45/-45]_s	Symmetric angle-ply	117.05	Spiral
[90/90/45/-45]_s	Symmetric angle-ply	101.86	Spiral

TABLE 3: CRITICAL BUCKLING LOAD FOR DIFFERENT LAMINATES.

Stacking sequence	Type of laminate	Critical buckling load (kN)	Mode shape
[90/0/90/0]_s	Symmetric cross-ply	92.21	Diamond
[45/45/45/45]_as	Antisymmetric angle-ply	64.56	Axisymmetric

The critical buckling loads for all the stacking sequences are listed in Table 3. The first stacking sequence, where the symmetric angle-ply arrangement is used and in which four out of eight ply angles are zero, has the highest critical load factor. Not surprisingly, the fourth stacking sequence, where an antisymmetric angle-ply arrangement is used, has the lowest critical load factor.

Interestingly, by *only* changing the stacking sequence from the fourth to the first, the critical buckling load can be increased by a factor of about two in the present model.



Material Switch 1(2)=Layered Material: [90/90/45/-45]_s Critical load factor=1.0186E5

Figure 5: First buckling mode shape and its corresponding critical load factor for Layered Material: [90/90/45/-45]_s.

Material Switch 1(3)=Layered Material: [90/0/90/0]_s Critical load factor=92205



Figure 6: First buckling mode shape and its corresponding critical load factor for Layered Material: [90/0/90/0]_s.



Material Switch 1(4)=Layered Material: [45/45/45/45]_as Critical load factor=64560

Figure 7: First buckling mode shape and its corresponding critical load factor for Layered Material: [45/45/45/45]_as.

The result of a stationary pre-study, in which the composite cylinder is subjected to the compressive load with fixed end condition, is shown in Figure 8.



Figure 8: von Mises stress distribution in Layered Material: [45/45/45/45]_as.

Notes About the COMSOL Implementation

- In order to perform a buckling analysis, a special **Linear Buckling** study is used. This consists of a **Stationary** study step and a **Linear Buckling** study step. The stationary study step performs the stress analysis for the applied load whereas buckling study uses eigenvalue solver and computes the critical load factors for the applied load.
- In order to run the analysis for various layered materials and compare the results, all the layered materials can be defined using a **Switch** node in **Global Materials**. This **Switch** node can be selected in the **Layered Material Link** node and a **Material Sweep** node is added in the study.
- Note that while performing material sweep over layered materials in a pres-stressed buckling analysis, pre-stress stationary study step results are stored only for the last layered material.
- Modeling a composite laminated shell requires a surface geometry (2D), in general called a base surface, and a **Layered Material** node which adds an extra dimension (1D) to the base surface geometry in the surface normal direction. You can use the **Layered**

Material functionality to model several layers stacked on top of each other having different thicknesses, material properties, and fiber orientations. You can also optionally specify the interface materials between the layers and control mesh elements in each layer.

- From a constitutive model point of view, you can either use the *Layerwise (LW)* theory based **Layered Shell** interface, or the *Equivalent Single Layer (ESL)* theory based **Layered Linear Elastic Material** node in the **Shell** interface. These interfaces are used in order to apply various loads and constraints on different layers of a composite shell, and to solve for stresses and other relevant variables in each layer of the composite shell.
- The laminated composite shell presented in the current model is modeled using a **Layered Linear Elastic Material** node in the **Shell** Interface.

Application Library path: Composite_Materials_Module/Buckling/ composite_cylinder_buckling

Modeling Instructions

From the File menu, choose New.

NEW

In the New window, click 🙆 Model Wizard.

MODEL WIZARD

- I In the Model Wizard window, click 间 3D.
- 2 In the Select Physics tree, select Structural Mechanics>Shell (shell).
- 3 Click Add.
- 4 Click 🔿 Study.
- 5 In the Select Study tree, select Preset Studies for Selected Physics Interfaces> Linear Buckling.
- 6 Click **M** Done.

GLOBAL DEFINITIONS

Parameters 1

I In the Model Builder window, under Global Definitions click Parameters I.

2 In the Settings window for Parameters, locate the Parameters section.

3 In the table, enter the following settings:

Name	Expression	Value	Description
r	0.15[m]	0.15 m	Cylinder radius
1	0.4[m]	0.4 m	Cylinder length

DEFINITIONS

Variables I

- I In the Model Builder window, under Component I (compl) right-click Definitions and choose Variables.
- 2 In the Settings window for Variables, locate the Variables section.
- **3** In the table, enter the following settings:

Name	Expression	Unit	Description
Fc	shell.LFcrit*1[N]		Critical buckling load
un	u*nX+v*nY+w*nZ	m	Normal displacement

GEOMETRY I

Cylinder I (cyl1)

- I In the **Geometry** toolbar, click **Cylinder**.
- 2 In the Settings window for Cylinder, locate the Object Type section.
- 3 From the Type list, choose Surface.
- 4 Locate the Size and Shape section. In the Radius text field, type r.
- 5 In the **Height** text field, type 1.
- 6 Click 틤 Build Selected.

You may want to import material data from a different file and use it while modeling. In the present example, the material properties are loaded from the file composite_cylinder_buckling_material.mph stored in the model's Application Libraries folder.

MATERIALS

In the Model Builder window, under Component I (compl) right-click Materials and choose Browse Materials.

MATERIAL BROWSER

- I In the Material Browser window, click 🔚 Import Material Library.
- 2 Browse to the model's Application Libraries folder and double-click the file composite_cylinder_buckling_material.mph.
- 3 Click **M** Done.

ADD MATERIAL

- I In the Home toolbar, click 🚼 Add Material to open the Add Material window.
- 2 Go to the Add Material window.
- 3 In the tree, select composite cylinder buckling material>Layered Material: [0/0/45/-45]_s.
- 4 Click Add to Global Materials in the window toolbar.
- 5 In the Home toolbar, click 🙀 Add Material to close the Add Material window.

GLOBAL DEFINITIONS

Layered Material: [0/0/45/-45]_s (Imat I)

- I In the Settings window for Layered Material, locate the Layer Definition section.
- 2 Click Layer Stack Preview in the upper-right corner of the section.

Material Switch 1 (sw1)

In the Model Builder window, right-click Materials and choose Material Switch.

Drag the Layered Material: [0/0/45/-45]_s node to Material Switch I node.

Layered Material: [0/0/45/-45]_s (sw1.lmat1)

Drag and drop Layered Material: [0/0/45/-45]_s (Imat I) on Material Switch I (swI).

Layered Material: [90/90/45/-45]_s

- I Right-click Layered Material: [0/0/45/-45]_s (sw1.lmat1) and choose Duplicate.
- 2 In the Settings window for Layered Material, type Layered Material: [90/90/45/-45]_s in the Label text field.
- **3** Find the **Layer Definition** section and change the rotation angles in the **Rotation** column as summarized in the table below.

Layer	Rotation
Layer I	90
Layer 2	90
Layer 3	45

Layer	Rotation
Layer 4	- 45
Layer 5	- 45
Layer 6	45
Layer 7	90
Layer 8	90

4 Locate the **Layer Definition** section. Click **Layer Stack Preview** in the upper-right corner of the section.

Layered Material: [90/0/90/0]_s

- I Right-click Layered Material: [90/90/45/-45]_s and choose Duplicate.
- 2 In the Settings window for Layered Material, type Layered Material: [90/0/90/0]_s in the Label text field.
- **3** Find the **Layer Definition** section and change the rotation angles in the **Rotation** column as summarized in the table below.

Layer	Rotation
Layer I	90
Layer 2	0
Layer 3	90
Layer 4	0
Layer 5	0
Layer 6	90
Layer 7	0
Layer 8	90

4 Locate the **Layer Definition** section. Click **Layer Stack Preview** in the upper-right corner of the section.

Layered Material: [45/45/45]_as

- I Right-click Layered Material: [90/0/90/0]_s and choose Duplicate.
- 2 In the Settings window for Layered Material, type Layered Material: [45/45/45/45]_as in the Label text field.

Layer	Rotation
Layer I	45
Layer 2	45
Layer 3	45
Layer 4	45
Layer 5	- 45
Layer 6	- 45
Layer 7	- 45
Layer 8	- 45

3 Find the **Layer Definition** section and change the rotation angles in the **Rotation** column as summarized in the table below.

4 Locate the **Layer Definition** section. Click **Layer Stack Preview** in the upper-right corner of the section.

MATERIALS

Layered Material Link 1 (Ilmat1)

In the Model Builder window, under Component I (compl) right-click Materials and choose Layers>Layered Material Link.

SHELL (SHELL)

Layered Linear Elastic Material I

- I In the Model Builder window, under Component I (compl) right-click Shell (shell) and choose Material Models>Layered Linear Elastic Material.
- **2** In the Settings window for Layered Linear Elastic Material, locate the Boundary Selection section.
- 3 From the Selection list, choose All boundaries.
- 4 Locate the Linear Elastic Material section. From the Material symmetry list, choose Orthotropic.

Fixed Constraint I

- I In the Physics toolbar, click 🔚 Edges and choose Fixed Constraint.
- **2** Select Edges 2, 3, 7, and 10 only.

Prescribed Displacement/Rotation 1

I In the Physics toolbar, click 🔚 Edges and choose Prescribed Displacement/Rotation.

- **2** Select Edges 4, 5, 8, and 11 only.
- **3** In the Settings window for Prescribed Displacement/Rotation, locate the Prescribed Displacement section.
- **4** Select the **Prescribed in x direction** check box.
- **5** Select the **Prescribed in y direction** check box.
- 6 Locate the Prescribed Rotation section. From the By list, choose Rotation.

Edge Load I

- I In the Physics toolbar, click 📄 Edges and choose Edge Load.
- **2** Select Edges 4, 5, 8, and 11 only.
- 3 In the Settings window for Edge Load, locate the Force section.
- 4 From the Load type list, choose Total force.
- **5** Specify the **F**_{tot} vector as

0	x
0	у
-1[N]	z

MESH I

I In the Model Builder window, under Component I (compl) click Mesh I.

2 In the Settings window for Mesh, locate the Physics-Controlled Mesh section.

3 From the Element size list, choose Extra fine.

4 Click 📗 Build All.

STUDY I

In the Model Builder window, collapse the Study I node.

Material Sweep

- I In the Study toolbar, click 🚦 Material Sweep.
- 2 In the Settings window for Material Sweep, locate the Study Settings section.
- 3 Click + Add.
- **4** In the **Study** toolbar, click **= Compute**.

RESULTS

Mode Shape: [0/0/45/-45]_s

I Click the yz Go to YZ View button in the Graphics toolbar.

2 Click the **F Zoom Extents** button in the **Graphics** toolbar.

Use the following instructions to plot the first critical buckling mode shape as shown in Figure 4.

- 3 In the Settings window for 3D Plot Group, type Mode Shape: [0/0/45/-45]_s in the Label text field.
- 4 Locate the Data section. From the Material Switch I list, choose Layered Material: [0/0/45/ -45]_s.
- 5 Click to expand the Title section. From the Title type list, choose Custom.
- 6 Find the Type and data subsection. Clear the Unit check box.
- 7 Clear the **Description** check box.
- 8 Clear the **Type** check box.
- 9 Locate the Plot Settings section. Clear the Plot dataset edges check box.

Surface 1

- I In the Model Builder window, expand the Mode Shape: [0/0/45/-45]_s node, then click Surface I.
- 2 In the Settings window for Surface, locate the Expression section.
- 3 In the Expression text field, type un.
- 4 In the Mode Shape: [0/0/45/-45]_s toolbar, click 💿 Plot.
- 5 In the Home toolbar, click 📕 Add Predefined Plot.

ADD PREDEFINED PLOT

- I Go to the Add Predefined Plot window.
- 2 In the tree, select Study I/Solution I (soll)>Shell>Thickness and Orientation (shell).
- **3** Click **Add Plot** in the window toolbar.
- 4 In the Home toolbar, click 💻 Add Predefined Plot.

RESULTS

Thickness and Orientation (shell)

- I In the Model Builder window, under Results click Thickness and Orientation (shell).
- 2 In the Thickness and Orientation (shell) toolbar, click 💿 Plot.

Follow the instructions below to plot the von Mises stress distribution as shown in Figure 8.

Stress: [45/45/45/45]_as

- I In the Home toolbar, click 🚛 Add Plot Group and choose 3D Plot Group.
- 2 In the Settings window for 3D Plot Group, type Stress: [45/45/45/45]_as in the Label text field.
- 3 Locate the Data section. From the Dataset list, choose Study I/Solution Store I (sol2).
- 4 Locate the Plot Settings section. From the Frame list, choose Spatial (x, y, z).

Layered Material Slice I

- I In the Stress: [45/45/45]_as toolbar, click i More Plots and choose Layered Material Slice.
- 2 In the Settings window for Layered Material Slice, locate the Expression section.
- 3 In the **Expression** text field, type shell.mises.
- 4 Locate the Coloring and Style section. Click Change Color Table.
- 5 In the Color Table dialog box, select Rainbow>RainbowLight in the tree.
- 6 Click OK.

Deformation I

- I Right-click Layered Material Slice I and choose Deformation.
- 2 In the Settings window for Deformation, locate the Expression section.
- 3 In the X-component text field, type shell.u.
- 4 In the **Y-component** text field, type shell.v.
- 5 In the **Z-component** text field, type shell.w.

Stress: [45/45/45/45]_as

- I In the Model Builder window, under Results click Stress: [45/45/45]_as.
- 2 In the Stress: [45/45/45]_as toolbar, click 💽 Plot.

Follow the instructions below to plot the first critical buckling mode shape as shown in Figure 5.

Mode Shape: [90/90/45/-45]_s

- I In the Model Builder window, right-click Mode Shape: [0/0/45/-45]_s and choose Duplicate.
- 2 In the Settings window for 3D Plot Group, type Mode Shape: [90/90/45/-45]_s in the Label text field.
- 3 Locate the Data section. From the Material Switch I list, choose Layered Material: [90/90/ 45/-45]_s.

Follow the instructions below to plot the first critical buckling mode shape as shown in Figure 6.

Mode Shape: [90/0/90/0]_s

- I Right-click Mode Shape: [90/90/45/-45]_s and choose Duplicate.
- 2 In the Settings window for 3D Plot Group, type Mode Shape: [90/0/90/0]_s in the Label text field.
- 3 Locate the Data section. From the Material Switch I list, choose Layered Material: [90/0/ 90/0]_s.
- 4 In the Mode Shape: [90/0/90/0]_s toolbar, click 💿 Plot.

Follow the instructions below to plot the first critical buckling mode shape as shown in Figure 7.

Mode Shape: [45/45/45/45]_as

- I Right-click Mode Shape: [90/0/90/0]_s and choose Duplicate.
- 2 In the Settings window for 3D Plot Group, type Mode Shape: [45/45/45]_as in the Label text field.
- 3 Locate the Data section. From the Material Switch I list, choose Layered Material: [45/45/ 45/45]_as.
- 4 In the Mode Shape: [45/45/45]_as toolbar, click 💿 Plot.

Follow the instructions below to compare the buckling mode shape for all four laminates.

Mode Shape: Comparison

- I Right-click Mode Shape: [45/45/45]_as and choose Duplicate.
- 2 In the Settings window for 3D Plot Group, type Mode Shape: Comparison in the Label text field.
- **3** Locate the **Title** section. Find the **Solution** subsection. Clear the **Solution** check box.
- 4 Find the Type and data subsection. Select the Type check box.
- **5** Select the **Description** check box.
- 6 Select the Unit check box.

Surface 1

- I In the Model Builder window, expand the Mode Shape: Comparison node, then click Surface I.
- 2 In the Settings window for Surface, locate the Data section.
- 3 From the Dataset list, choose Layered Material.

4 From the Material Switch I list, choose Layered Material: [0/0/45/-45]_s.

Deformation

- I In the Model Builder window, expand the Surface I node, then click Deformation.
- 2 In the Settings window for Deformation, locate the Scale section.
- 3 Select the Scale factor check box. In the associated text field, type 1.

Surface 2

- I In the Model Builder window, under Results>Mode Shape: Comparison right-click Surface I and choose Duplicate.
- 2 In the Settings window for Surface, locate the Data section.
- 3 From the Material Switch I list, choose Layered Material: [90/90/45/-45]_s.
- 4 Click to expand the Title section. From the Title type list, choose None.

Deformation

- I In the Model Builder window, expand the Surface 2 node, then click Deformation.
- 2 In the Settings window for Deformation, locate the Expression section.
- 3 In the y-component text field, type v+1.3*1.

Surface 3

- I In the Model Builder window, under Results>Mode Shape: Comparison right-click Surface 2 and choose Duplicate.
- 2 In the Settings window for Surface, locate the Data section.
- 3 From the Material Switch I list, choose Layered Material: [90/0/90/0]_s.

Deformation

- I In the Model Builder window, expand the Surface 3 node, then click Deformation.
- 2 In the Settings window for Deformation, locate the Expression section.
- **3** In the **y-component** text field, type v.
- 4 In the **z-component** text field, type w+1.3*1.

Surface 4

- I In the Model Builder window, under Results>Mode Shape: Comparison right-click Surface 3 and choose Duplicate.
- 2 In the Settings window for Surface, locate the Data section.
- 3 From the Material Switch I list, choose Layered Material: [45/45/45/45]_as.

Deformation

- I In the Model Builder window, expand the Surface 4 node, then click Deformation.
- 2 In the Settings window for Deformation, locate the Expression section.
- 3 In the y-component text field, type v+1.3*1.

Table Annotation 1

- I In the Model Builder window, right-click Mode Shape: Comparison and choose Table Annotation.
- 2 In the Settings window for Table Annotation, locate the Data section.
- 3 From the Source list, choose Local table.
- **4** In the table, enter the following settings:

x-coordinate	y-coordinate	z-coordinate	Annotation
0	0	-0.15*1	[0/0/45/-45]_s
0	1.3*1	-0.15*1	[90/90/45/-45]_s
0	0	-0.15*1+1.3*1	[90/0/90/0]_s
0	1.3*1	-0.15*1+1.3*1	[45/45/45/45]_as

- 5 Locate the Coloring and Style section. Clear the Show point check box.
- 6 From the Anchor point list, choose Lower middle.

Mode Shape: Comparison

- I In the Model Builder window, click Mode Shape: Comparison.
- 2 In the Settings window for 3D Plot Group, locate the Plot Settings section.
- **3** From the **View** list, choose **New view**.
- **4** Click the Y^Z **Go to YZ View** button in the **Graphics** toolbar.
- **5** Click the **F Zoom Extents** button in the **Graphics** toolbar.
- 6 In the Mode Shape: Comparison toolbar, click 💽 Plot.

Use an **Evaluation Group** instead of **Derived Values** nodes to compute the critical buckling load.

Critical Buckling Load

- I In the **Results** toolbar, click **Evaluation Group**.
- 2 In the Settings window for Evaluation Group, type Critical Buckling Load in the Label text field.
- 3 Locate the Data section. From the Dataset list, choose Study I/ Parametric Solutions I (sol3).

Global Evaluation 1

- I Right-click Critical Buckling Load and choose Global Evaluation.
- 2 In the Settings window for Global Evaluation, locate the Expressions section.
- **3** In the table, enter the following settings:

Expression	Unit	Description
Fc	kN	Critical buckling load

4 In the **Critical Buckling Load** toolbar, click **= Evaluate**.

Enable automatic reevaluation of evaluation groups when the model is re-solved.

- 5 In the Model Builder window, click Results.
- 6 In the Settings window for Results, locate the Update of Results section.
- 7 Select the Reevaluate all evaluation groups after solving check box.