

Isoelectric Separation

Introduction

This modeling example applies the Electrophoretic Transport and the Laminar Flow interfaces to model isoelectric separation in a free-flow electrophoresis device. A stream containing four different proteins is divided into separated component streams by means of migrative transport in an electric field.

Free flow electrophoresis can be used to separate macromolecules, such as proteins perpendicular to the flow of the carrier fluid. If a pH and potential gradient is applied across the carrier flow, then molecules can be focused along their isoelectric points. The isoelectric point is the pH at which a molecule has zero net charge. The concept of isoelectric focusing is illustrated in Figure 1.



Figure 1: Ampholytic molecules can be focused around their isoelectric point by means of migrative transport in an electric field.

Molecules with a positive net charge travel in the direction of the electric field, along the pH gradient, until they reach the isoelectric point. At this instance the migrative transport

is switched off as the molecules net charge is zero. Similarly, negative net charge species travel in the direction opposite of the electric field.

Model Definition

The model geometry is shown in Figure 2. It represents the separation region in an isoelectric focusing chip. A laminar carrier stream transports a mixture of four proteins, one weak acid and one weak base, injected at the bottom of the cell.



Figure 2: Model geometry.

The Electrophoretic Transport interface supports ionic species transport by diffusion, convection, and migration in electric fields. The mass balance that is solved in the interface is as follows:

$$\nabla \cdot (-D_i \nabla c_i - z_i u_{m,i} F c_i \nabla \phi) + \mathbf{u} \cdot \nabla c_i = 0$$

In the equation, c_i denotes the concentration of species i (SI unit: mol/m³), D_i is the diffusion coefficient of species i (SI unit: m²/s), **u** is the fluid velocity (SI unit: m/s), F refers to Faraday's constant (SI unit: s·A/mol), ϕ denotes the electric potential (SI unit: V), z_i is the charge number of the ionic species (unitless), and $u_{m,i}$ its ionic mobility (SI unit: s·mol/kg). The Electrophoretic Transport interface also adds an equation for the

charge transport in the electrolyte, based on an assumption of electroneutrality, and a set of chemical equilibria describing the water self- ionization reaction and the dissociation reactions of weak acids and bases.

The fluid flow is set up with a Laminar Flow interface, solving for the Navier–Stokes equations.

Results and Discussion

Figure 3 shows the pH in the cell. The pH gradients in the x direction increase toward the outlet.



Figure 3: Surface plot of pH in the cell.



Figure 4 shows the electrolyte conductivity. The conductivity gets lower close to the electrode surfaces and toward the outlet due to depletion of charge carriers.

Figure 4: Surface plot of electrolyte conductivity in the cell.

Figure 5 shows the electrolyte potential. Outside the electrode gap, the potential gradients are low, which means that the migrative flux close to the inlet and outlet is low.



Figure 5: Surface plot of electrolyte potential in the cell.

Figure 6 shows the concentration of Protein 1, which has an isoelectric point of 4.7. The protein is concentrated toward the right electrode.



Figure 6: Surface plot of molar concentration of Protein 1 in the cell.





Figure 7: Surface plot of molar concentration of Protein 2 in the cell.

Figure 8 and Figure 9 show the weak acid and base concentrations, respectively. The weak acid, with a negative average charge, is transported to the right and the base, with a positive average charge, is transported to the left in the electric field.



Figure 8: Surface plot of weak acid concentration in the cell.



Figure 9: Surface plot of weak base concentration in the cell.

Finally, Figure 10 and Figure 11 show the pH and protein concentration profiles at the outlet.



Figure 10: pH profile along the outlet boundary of the cell.



Figure 11: Protein concentration profiles along the outlet boundary of the cell.

Application Library path: Chemical_Reaction_Engineering_Module/ Electrokinetic_Effects/isoelectric_separation

Modeling Instructions

In this tutorial we couple Electrophoretic Transport to Laminar Flow in 2D. Use the **Model Wizard** to select the space dimension and the physics interfaces. (Studies will be added to the model at a later stage.)

From the File menu, choose New.

NEW

In the New window, click 🔗 Model Wizard.

MODEL WIZARD

I In the Model Wizard window, click 🍳 2D.

- 2 In the Select Physics tree, select Chemical Species Transport> Electrophoretic Transport (el).
- 3 Click Add.
- 4 In the Select Physics tree, select Fluid Flow>Single-Phase Flow>Laminar Flow (spf).
- 5 Click Add.
- 6 Click 🗹 Done.

GLOBAL DEFINITIONS

Load the model parameters from a text file.

Parameters 1

- I In the Model Builder window, under Global Definitions click Parameters I.
- 2 In the Settings window for Parameters, locate the Parameters section.
- 3 Click 📂 Load from File.
- **4** Browse to the model's Application Libraries folder and double-click the file isoelectric_separation_parameters.txt.

GEOMETRY I

Now draw the geometry as a union of three rectangles.

Rectangle 1 (r1)

- I In the **Geometry** toolbar, click **Rectangle**.
- 2 In the Settings window for Rectangle, locate the Size and Shape section.
- 3 In the Width text field, type W.
- **4** In the **Height** text field, type H.

Rectangle 2 (r2)

- I In the Geometry toolbar, click Rectangle.
- 2 In the Settings window for Rectangle, locate the Size and Shape section.
- 3 In the Width text field, type W.
- 4 In the Height text field, type W.
- 5 Locate the Position section. In the y text field, type -W.
- 6 Click 틤 Build Selected.

Rectangle 3 (r3)

- I Right-click Rectangle 2 (r2) and choose Duplicate.
- 2 In the Settings window for Rectangle, locate the Position section.

3 In the **y** text field, type H.

4 Click 틤 Build Selected.

Inlet

Add selections for the inlet and outlet boundaries. These will be used later when setting up the physics.

- I In the Geometry toolbar, click 🔓 Selections and choose Explicit Selection.
- 2 In the Settings window for Explicit Selection, type Inlet in the Label text field.
- **3** Locate the **Entities to Select** section. From the **Geometric entity level** list, choose **Boundary**.
- 4 On the object r2, select Boundary 1 only.

Outlet

- I In the Geometry toolbar, click 🝖 Selections and choose Explicit Selection.
- 2 In the Settings window for Explicit Selection, type Outlet in the Label text field.
- **3** Locate the **Entities to Select** section. From the **Geometric entity level** list, choose **Boundary**.
- 4 On the object r3, select Boundary 3 only.

The interior boundaries are not part of the physical geometry. Assign them as **Mesh Control Edges** to make them available in the mesh only.

Mesh Control Edges 1 (mcel)

- I In the Geometry toolbar, click 🏠 Virtual Operations and choose Mesh Control Edges.
- 2 On the object fin, select Boundaries 4 and 6 only.
- 3 In the Settings window for Mesh Control Edges, locate the Input section.
- 4 Clear the Include adjacent vertices check box.
- 5 In the Geometry toolbar, click 🟢 Build All.

The model is slender. Change **View scale** to more easily see variations over the entire geometry.

DEFINITIONS

In the Model Builder window, expand the Component I (compl)>Definitions node.

Axis

I In the Model Builder window, expand the Component I (compl)>Definitions>View I node, then click Axis.

- 2 In the Settings window for Axis, locate the Axis section.
- 3 From the View scale list, choose Manual.
- 4 In the **x scale** text field, type 3.
- 5 Click 🚺 Update.
- 6 Click the + Zoom Extents button in the Graphics toolbar.

The finished geometry should now look like this:



MATERIALS

To set up flow equations for the electrolyte we will use some material parameters for water from the **Material Library**.

ADD MATERIAL

- I In the Home toolbar, click 🙀 Add Material to open the Add Material window.
- 2 Go to the Add Material window.
- 3 In the tree, select Built-in>Water, liquid.
- 4 Right-click and choose Add to Component I (compl).
- 5 In the Home toolbar, click 🙀 Add Material to close the Add Material window.

ELECTROPHORETIC TRANSPORT (EL)

Now start setting up the Electrophoretic Transport.

- I In the Settings window for Electrophoretic Transport, locate the Transport Mechanisms section.
- **2** Select the **Convection** check box.

Solvent I

- I In the Model Builder window, under Component I (compl)>Electrophoretic Transport (el) click Solvent I.
- 2 In the Settings window for Solvent, locate the Convection section.
- **3** From the **u** list, choose **Velocity field (spf)**.

Potential I

This cell is operated potentiostatically (at a constant potential).

- I In the Physics toolbar, click Boundaries and choose Potential.
- **2** Select Boundary **3** only.

Potential 2

- I In the Physics toolbar, click Boundaries and choose Potential.
- 2 Select Boundary 7 only.
- 3 In the Settings window for Potential, locate the Electrolyte Potential section.
- **4** In the $\phi_{l,bnd}$ text field, type V0.

Protein I

Now add the various species in the electrolyte: 4 proteins, one weak acid and one weak base.

- I In the Physics toolbar, click **Domains** and choose **Protein**.
- 2 In the Settings window for Protein, locate the Protein section.
- 3 In the Species name text field, type p1.
- 4 In the Average charge text field, type iep_1-el.pH.
- **5** Locate the **Diffusion and Migration** section. In the *D* text field, type D_p.

Inflow I

- I In the Model Builder window, expand the Protein I node.
- 2 Right-click Protein I and choose Inflow.
- 3 In the Settings window for Inflow, locate the Boundary Selection section.

- 4 From the Selection list, choose Inlet.
- **5** Locate the **Concentration** section. In the c_0 text field, type cp_in.
- 6 Locate the Boundary Condition Type section. From the list, choose Flux (Danckwerts).

Protein I

In the Model Builder window, click Protein I.

Outflow I

- I In the Physics toolbar, click Attributes and choose Outflow.
- 2 In the Settings window for Outflow, locate the Boundary Selection section.
- 3 From the Selection list, choose Outlet.

Protein 2

In this tutorial we assume that only the isoelectric point varies between the proteins. Therefore you can just duplicate the first protein to create the second protein, and then and change the average charge.

- I Right-click Protein I and choose Duplicate.
- 2 In the Settings window for Protein, locate the Protein section.
- **3** In the **Species name** text field, type p2.
- 4 In the Average charge text field, type iep_2-el.pH.

Protein 3

- I Right-click Protein 2 and choose Duplicate.
- 2 In the Settings window for Protein, locate the Protein section.
- 3 In the Species name text field, type p3.
- 4 In the Average charge text field, type iep_3-el.pH.

Protein 4

- I Right-click Protein 3 and choose Duplicate.
- 2 In the Settings window for Protein, locate the Protein section.
- **3** In the **Species name** text field, type p4.
- 4 In the Average charge text field, type iep_4-el.pH.

Weak Acid I

Finish the electrophoretic transport settings by adding one weak acid and one weak base.

- I In the Physics toolbar, click **Domains** and choose Weak Acid.
- 2 In the Settings window for Weak Acid, locate the Weak Acid section.

- 3 In the Species name text field, type wa.
- **4** In the pK_a text field, type pKa_wa .
- 5 Locate the Diffusion and Migration section. In the $u_{\rm m}$ text field, type mob_wa.

Initial Concentration 1

- I In the Model Builder window, expand the Weak Acid I node, then click Initial Concentration I.
- 2 In the Settings window for Initial Concentration, locate the Initial Concentration section.
- 3 In the *c* text field, type cwa_in.

Weak Acid I

In the Model Builder window, click Weak Acid I.

Inflow I

- I In the Physics toolbar, click Attributes and choose Inflow.
- 2 In the Settings window for Inflow, locate the Boundary Selection section.
- 3 From the Selection list, choose Inlet.
- **4** Locate the **Concentration** section. In the c_0 text field, type cwa_in.
- 5 Locate the Boundary Condition Type section. From the list, choose Flux (Danckwerts).

Weak Acid I

In the Model Builder window, click Weak Acid I.

Outflow I

- I In the Physics toolbar, click Attributes and choose Outflow.
- 2 In the Settings window for Outflow, locate the Boundary Selection section.
- **3** From the **Selection** list, choose **Outlet**.

Weak Base I

- I In the Physics toolbar, click **Domains** and choose Weak Base.
- 2 In the Settings window for Weak Base, locate the Weak Base section.
- 3 In the Species name text field, type wb.
- **4** In the pK_a text field, type pKa_wb .
- 5 Locate the Diffusion and Migration section. In the $u_{\rm m}$ text field, type mob_wb.

Initial Concentration 1

I In the Model Builder window, expand the Weak Base I node, then click Initial Concentration I.

2 In the Settings window for Initial Concentration, locate the Initial Concentration section.

3 In the *c* text field, type cwb_in.

Weak Base I

In the Model Builder window, click Weak Base I.

Inflow I

- I In the Physics toolbar, click Attributes and choose Inflow.
- 2 In the Settings window for Inflow, locate the Boundary Selection section.
- **3** From the **Selection** list, choose **Inlet**.
- **4** Locate the **Concentration** section. In the c_0 text field, type cwb_in.

Weak Base I

In the Model Builder window, click Weak Base I.

Outflow I

- I In the Physics toolbar, click Attributes and choose Outflow.
- 2 In the Settings window for Outflow, locate the Boundary Selection section.
- 3 From the Selection list, choose Outlet.

LAMINAR FLOW (SPF)

Now set up the Laminar Flow interface. Only the boundary conditions need to be specified here since most settings are taken from the Materials node (Water), o.

I In the Model Builder window, under Component I (compl) click Laminar Flow (spf).

Inlet 1

- I In the Physics toolbar, click Boundaries and choose Inlet.
- 2 Select Boundary 2 only.
- 3 In the Settings window for Inlet, locate the Boundary Condition section.
- 4 From the list, choose Fully developed flow.
- 5 Locate the Fully Developed Flow section. In the $U_{\rm av}$ text field, type Uave.

Outlet I

- I In the Physics toolbar, click Boundaries and choose Outlet.
- 2 In the Settings window for Outlet, locate the Boundary Selection section.
- **3** From the **Selection** list, choose **Outlet**.
- 4 Locate the Pressure Conditions section. Select the Normal flow check box.

5 Right-click Outlet I and choose Build All.

MESH I

Now set up the mesh. A mapped mesh is suitable since we are using a rectangular geometry.

Mapped 1 In the **Mesh** toolbar, click **Mapped**.

Distribution I

- I Right-click Mapped I and choose Distribution.
- **2** Select Boundaries 2, 5, 9, and 10 only.
- 3 In the Settings window for Distribution, locate the Distribution section.
- 4 In the Number of elements text field, type 60.

Size

- I In the Model Builder window, under Component I (compl)>Mesh I click Size.
- 2 In the Settings window for Size, locate the Element Size section.
- **3** Click the **Custom** button.
- 4 Locate the **Element Size Parameters** section. In the **Maximum element size** text field, type 5e-4.
- 5 Click 📗 Build All.

The finished mesh should look like this:



ROOT

The model is now ready for solving. In this model the flow is not affected the physics of the **Electrophoretic Transport** interface. Therefore solve the **Laminar Flow** first in a separate study.

ADD STUDY

- I In the Home toolbar, click $\stackrel{\sim}{\sim}$ Add Study to open the Add Study window.
- 2 Go to the Add Study window.
- 3 Find the Studies subsection. In the Select Study tree, select General Studies>Stationary.
- 4 Find the Physics interfaces in study subsection. In the table, clear the Solve check box for Electrophoretic Transport (el).
- 5 Click Add Study in the window toolbar.
- 6 In the Home toolbar, click \sim Add Study to close the Add Study window.

STUDY I - FLOW CALCULATION

I In the Model Builder window, click Study I.

- **2** In the **Settings** window for **Study**, type **Study 1 Flow Calculation** in the **Label** text field.
- **3** In the **Home** toolbar, click **= Compute**.

RESULTS

Default plots for the velocity and pressure are generated automatically as follows:

Velocity (spf)



ROOT

Add a second study to solve for the **Electrophoretic Transport**, using the solution of the first Study as input for the velocity (and pressure).

ADD STUDY

- I In the Home toolbar, click 2 Add Study to open the Add Study window.
- 2 Go to the Add Study window.
- **3** Find the **Physics interfaces in study** subsection. In the table, clear the **Solve** check box for **Laminar Flow (spf)**.
- 4 Find the Studies subsection. In the Select Study tree, select Preset Studies for Selected Physics Interfaces>Stationary with Initialization.
- 5 Click Add Study in the window toolbar.
- 6 In the Home toolbar, click 2 Add Study to close the Add Study window.

STUDY 2 - SEPARATION CALCULATION

- I In the Model Builder window, click Study 2.
- 2 In the Settings window for Study, type Study 2 Separation Calculation in the Label text field.

Step 1: Current Distribution Initialization

- I In the Model Builder window, under Study 2 Separation Calculation click Step I: Current Distribution Initialization.
- 2 In the Settings window for Current Distribution Initialization, click to expand the Values of Dependent Variables section.
- **3** Find the Values of variables not solved for subsection. From the Settings list, choose User controlled.
- 4 From the Method list, choose Solution.
- 5 From the Study list, choose Study I Flow Calculation, Stationary.
- 6 In the Home toolbar, click **=** Compute.

RESULTS

pH (el)

- I In the Settings window for 2D Plot Group, click to expand the Title section.
- 2 From the Title type list, choose None.

3 Locate the Color Legend section. Select the Show units check box.

A default pH plot should now have been generated, looking like this:



Electrolyte Conductivity (el) The default electrolyte conductivity plot should look like this:

- I In the Model Builder window, click Electrolyte Conductivity (el).
- 2 In the Settings window for 2D Plot Group, locate the Title section.
- **3** From the **Title type** list, choose **None**.



4 Locate the Color Legend section. Select the Show units check box.

The default electrolyte potential plot should look like this:

- I In the Model Builder window, click Electrolyte Potential (el).
- 2 In the Settings window for 2D Plot Group, locate the Title section.
- **3** From the **Title type** list, choose **None**.

Electrolyte Potential (el)



4 Locate the Color Legend section. Select the Show units check box.

There should also be six default molar concentration plots present; one for each species in the **Electrophoretic Transport** interface:

Molar Concentration - pl (el)

- I In the Model Builder window, click Molar Concentration pl (el).
- 2 In the Settings window for 2D Plot Group, locate the Title section.
- **3** From the **Title type** list, choose **None**.



4 Locate the Color Legend section. Select the Show units check box.

Molar Concentration - p2 (el)

- I In the Model Builder window, click Molar Concentration p2 (el).
- 2 In the Settings window for 2D Plot Group, locate the Title section.
- **3** From the **Title type** list, choose **None**.



4 Locate the Color Legend section. Select the Show units check box.

Molar Concentration - p3 (el)



Molar Concentration - wa (el)

- I In the Model Builder window, click Molar Concentration wa (el).
- 2 In the Settings window for 2D Plot Group, locate the Title section.

3 From the **Title type** list, choose **None**.



4 Locate the Color Legend section. Select the Show units check box.

- I In the Model Builder window, click Molar Concentration wb (el).
- 2 In the Settings window for 2D Plot Group, locate the Title section.
- **3** From the **Title type** list, choose **None**.

Molar Concentration - wb (el)



4 Locate the Color Legend section. Select the Show units check box.

Molar Concentration - p2 (el)

Now, to visualize the transport in radial direction, add **Arrow lines** and **Streamlines** to the plot for Protein 2.

Arrow Surface I

- I In the Model Builder window, expand the Molar Concentration p2 (el) node.
- 2 Right-click Results>Molar Concentration p2 (el)>Arrow Surface I and choose Delete.

Arrow Line 1

- I In the Model Builder window, right-click Molar Concentration p2 (el) and choose Arrow Line.
- 2 In the Settings window for Arrow Line, click Replace Expression in the upper-right corner of the Expression section. From the menu, choose Component I (compl)>Laminar Flow> Velocity and pressure>u,v Velocity field.
- 3 Locate the Arrow Positioning section. In the Number of arrows text field, type 20.
- 4 Locate the Coloring and Style section. From the Color list, choose Custom.
- **5** On Windows, click the colored bar underneath, or if you are running the cross-platform desktop the **Color** button.
- 6 Click Define custom colors.
- 7 Set the RGB values to 5, 132, and 133, respectively.

- 8 Click Add to custom colors.
- 9 Click Show color palette only or OK on the cross-platform desktop.
- 10 In the Molar Concentration p2 (el) toolbar, click 💽 Plot.

Selection 1

- I Right-click Arrow Line I and choose Selection.
- 2 Select Boundaries 2 and 5 only.
- 3 In the Molar Concentration p2 (el) toolbar, click 🗿 Plot.

Streamline 1

- I In the Model Builder window, right-click Molar Concentration p2 (el) and choose Streamline.
- 2 In the Settings window for Streamline, click Replace Expression in the upper-right corner of the Expression section. From the menu, choose Component I (compl)> Electrophoretic Transport>Protein 2>el.tflux_p2x,el.tflux_p2y Total flux.
- 3 Locate the Streamline Positioning section. In the Number text field, type 10.
- **4** Select Boundary 5 only.
- 5 Locate the Coloring and Style section. Find the Point style subsection. From the Type list, choose Arrow.
- 6 Select the Number of arrows check box. In the associated text field, type 90.
- 7 Select the Scale factor check box. In the associated text field, type 10.
- 8 In the Molar Concentration p2 (el) toolbar, click **O** Plot.

The generated plot should look like this:



pH at Outlet

Now proceed to plot the pH at the outlet.

- I In the Home toolbar, click 🚛 Add Plot Group and choose ID Plot Group.
- 2 In the Settings window for ID Plot Group, type pH at Outlet in the Label text field.
- 3 Locate the Data section. From the Dataset list, choose Study 2 Separation Calculation/ Solution 2 (sol2).
- 4 Click to expand the Title section. From the Title type list, choose None.

Line Graph 1

- I Right-click **pH at Outlet** and choose **Line Graph**.
- 2 In the Settings window for Line Graph, locate the Selection section.
- 3 From the Selection list, choose Outlet.
- 4 Locate the x-Axis Data section. From the Parameter list, choose Expression.
- **5** In the **Expression** text field, type **x**.
- 6 In the pH at Outlet toolbar, click **Plot**.

The pH plot should look like this:



Protein Concentrations at Outlet

Finally, create plot of the protein concentrations at the outlet as follows:

- I In the Home toolbar, click 🚛 Add Plot Group and choose ID Plot Group.
- 2 In the Settings window for ID Plot Group, type Protein Concentrations at Outlet in the Label text field.
- 3 Locate the Data section. From the Dataset list, choose Study 2 Separation Calculation/ Solution 2 (sol2).
- 4 Click to expand the **Title** section. From the **Title type** list, choose **None**.
- 5 Locate the Legend section. From the Layout list, choose Outside graph axis area.

Line Graph 1

- I Right-click Protein Concentrations at Outlet and choose Line Graph.
- 2 In the Settings window for Line Graph, locate the Selection section.
- 3 From the Selection list, choose Outlet.
- 4 Click Replace Expression in the upper-right corner of the y-Axis Data section. From the menu, choose Component I (comp1)>Electrophoretic Transport>Protein I>el.c_p1 Concentration mol/m³.
- 5 Locate the x-Axis Data section. From the Parameter list, choose Expression.
- 6 In the Expression text field, type x.

- 7 Click to expand the Legends section. Select the Show legends check box.
- 8 From the Legends list, choose Manual.
- **9** In the table, enter the following settings:

Legends

Protein 1

IO In the **Protein Concentrations at Outlet** toolbar, click **ID Plot**.

Line Graph 2

- I Right-click Line Graph I and choose Duplicate.
- 2 In the Settings window for Line Graph, locate the y-Axis Data section.
- **3** In the **Expression** text field, type el.c_p2.
- 4 Locate the Legends section. In the table, enter the following settings:

Legends

Protein 2

Line Graph 3

- I Right-click Line Graph 2 and choose Duplicate.
- 2 In the Settings window for Line Graph, locate the y-Axis Data section.
- **3** In the **Expression** text field, type el.c_p3.
- 4 Locate the Legends section. In the table, enter the following settings:

Legends

Protein 3

Line Graph 4

- I Right-click Line Graph 3 and choose Duplicate.
- 2 In the Settings window for Line Graph, locate the y-Axis Data section.
- **3** In the **Expression** text field, type el.c_p4.
- 4 Locate the Legends section. In the table, enter the following settings:

Legends

Protein 4

Protein Concentrations at Outlet

I In the Model Builder window, click Protein Concentrations at Outlet.

2 In the Protein Concentrations at Outlet toolbar, click 💿 Plot.

The plot should look like this:

