Created in COMSOL Multiphysics 6.1

Gaussian Pulse Absorption by Perfectly Matched Layers: Pressure Acoustics, Transient

Introduction

This tutorial model is a test and benchmark model for perfectly matched layers (PML) in the time domain. PMLs are widely used for problems of wave propagation in open domains. A perfectly matched layer is supposed to meet two main requirements. First, a wave must propagate without spurious reflections from the interface between the physical domain and the PML. Second, a PML must ensure long time stability of the numerical solution.

The model solves the same problem as in *Gaussian Pulse in 2D Uniform Flow: Convected Wave Equation and Absorbing Layers* in the absence of the background flow. The Pressure Acoustics, Transient interface solves the wave equation in a squared computational domain and the Perfectly Matched Layers surround the computational domain suppressing the reflections from the outer boundary. An acoustic pulse is generated by an initial Gaussian distribution at the center of the computational domain. An analytical solution to the problem exists and is used to validate the solution. The model shows how to set up and use the PMLs.

For more information about PMLs in acoustics, see the section *Modeling with the Pressure Acoustics Branch (FEM-Based Interfaces)* in the *Acoustics Module User's Guide*.

Model Definition

The computational domain with perfectly matched layers is depicted in Figure 1. The model is set up in a dimensionless system where the speed of sound $c_0 = 1$ and the density $\rho_0 = 1$. The Gaussian pulse is emitted at the origin $\mathbf{x} = \mathbf{0}$ with initial values

$$\begin{bmatrix} p \\ u \\ v \end{bmatrix} = \begin{bmatrix} 1 \\ \beta x \\ \beta y \end{bmatrix} e^{-\alpha(x^2 + y^2)} \qquad \text{for} \qquad t = 0 \tag{1}$$

where $\alpha = \ln(2)/9$ and $\beta = 0.04$. Since the velocity $\mathbf{u} = (u, v)$ does not enter the wave equation explicitly, the two last initial values are reformulated in terms of the first time derivative of the pressure. The use of the mass conservation law yields

$$\frac{\partial p}{\partial t} = -\rho_0 c_0^2\left(\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y}\right) \qquad \text{for} \qquad t = 0.$$

The Gaussian pulse parameters and the expressions for the initial values are included to the model as parameters and variables.

The analytical solution to Equation 1 is given by (see Ref. 1)

$$p(\mathbf{x}, t) = \frac{1}{2\alpha}\int_0^{\infty}\left[\cos(\lambda t) - \frac{\beta}{2\alpha}\lambda\sin(\lambda t)\right]\lambda J_0(\lambda r)e^{-\frac{\lambda^2}{4\alpha}}d\lambda,\ r = \sqrt{x^2 + y^2}$$

In the model, the `integrate()` operator is used to express the analytical solution. The integration is performed on a finite interval.

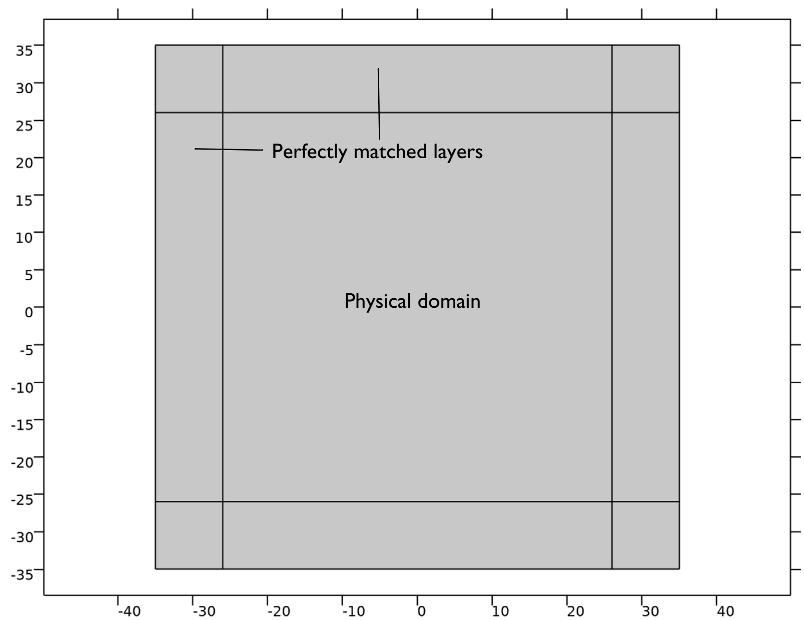

Figure 1: Geometry: physical domain and perfectly matched layers.

Results and Discussion

The propagation of the acoustic pulse is depicted in Figure 2. It shows the distribution of the acoustic pressure across the computational domain at four time steps. By the time $t = 30$, the pulse has entered the PML domain without any reflections from the interface between the physical domain and the PMLs. The pressure at the final simulated time $t = 120$ is depicted in Figure 3. By this time, the pulse has left the computational domain. There are no visible signals propagating back to the physical domain, which asserts the long-time stability of the perfectly matched layers.

In the next two figures, the simulated results are compared with the analytical solution. In Figure 4, the pressure at point (20, 10) is depicted as function of time. In Figure 5, the pressure is depicted along the x-axis at t = 40. Both show very good agreement with the analytical solution.

Figure 6 shows the frequency spectrum of the signal at point (20, 10).

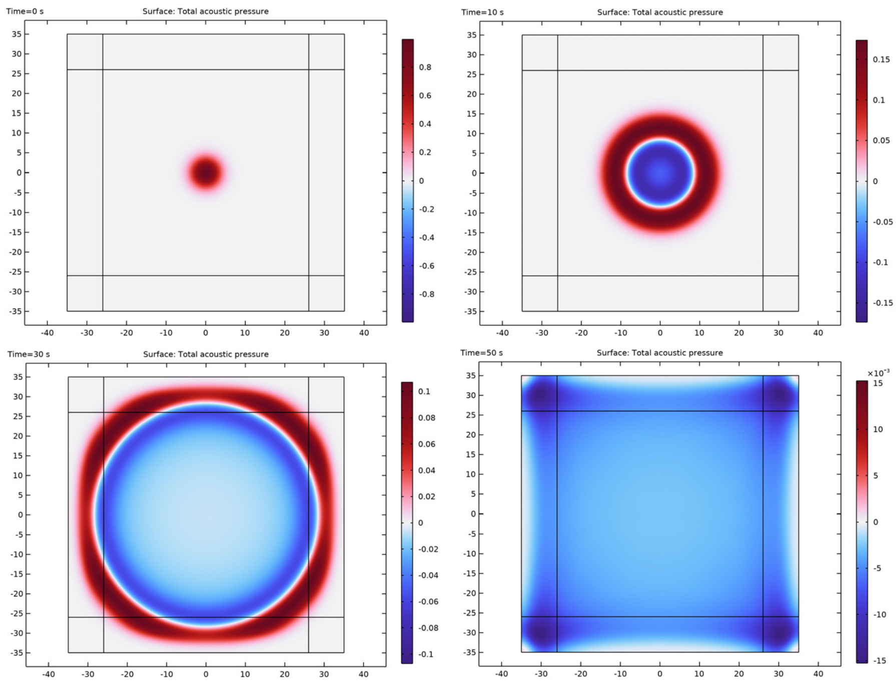

Figure 2: The pressure profile at different time steps.

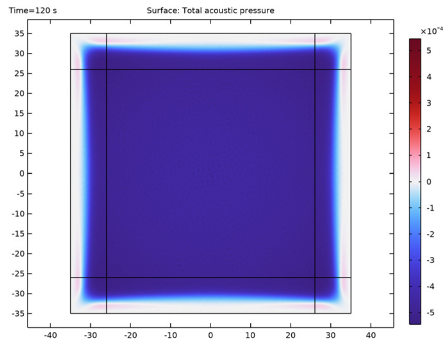

Figure 3: The pressure profile at the final simulated time t = 120.

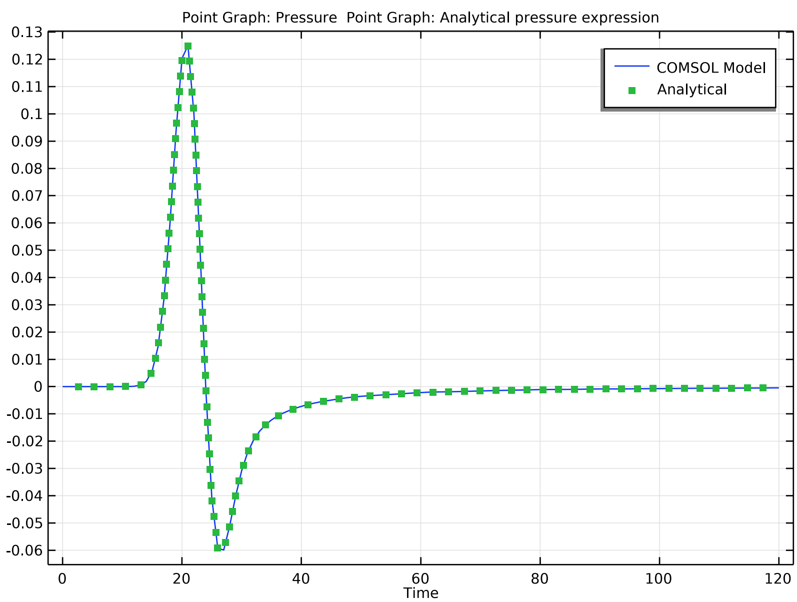

Figure 4: The pressure as function of time at point (x, y) = (20, 10). The model solution compared with the analytical solution.

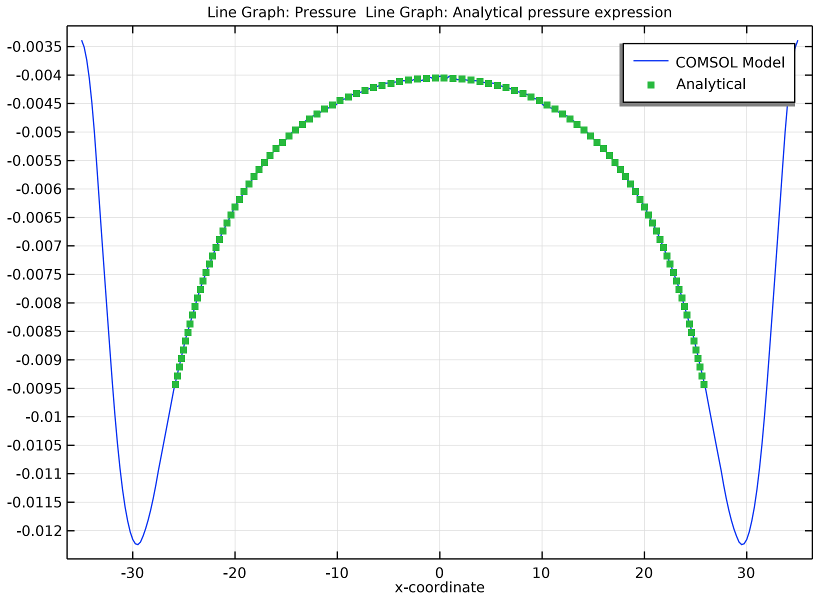

Figure 5: Pressure profile along the x-axis at t = 40 comparing the analytical solution with the COMSOL solution.

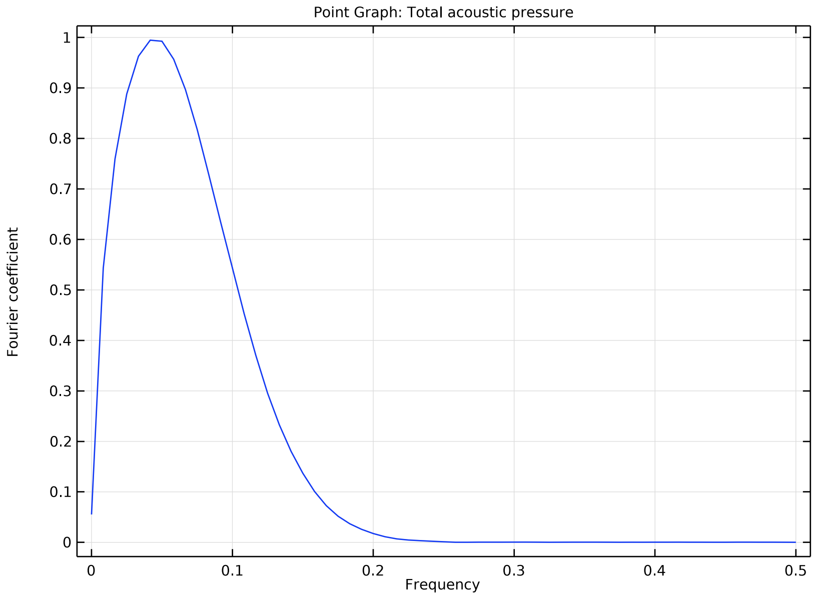

Figure 6: Frequency spectrum of the signal at point (x, y) = (20, 10).

Reference

1. H.L. Atkins and C.W. Shu, “Quadrature-Free Implementation of Discontinuous Galerkin Method for Hyperbolic Equations,” *AIAA Journal*, vol. 36, pp. 775–782, 1998.

Application Library path: `Acoustics_Module/Tutorials,_Pressure_Acoustics/gaussian_pulse_perfectly_matched_layers`

Modeling Instructions

From the **File** menu, choose **New**.

NEW

In the **New** window, click **Model Wizard**.

MODEL WIZARD

1 In the **Model Wizard** window, click **2D**.
2 In the **Select Physics** tree, select **Acoustics>Pressure Acoustics>Pressure Acoustics, Transient (actd)**.
3 Click **Add**.
4 Click **Study**.
5 In the **Select Study** tree, select **General Studies>Time Dependent**.
6 Click **Done**.

GEOMETRY 1

Before setting up the physics, change the unit system to be dimensionless.

ROOT

1 In the **Model Builder** window, click the root node.
2 In the root node's **Settings** window, locate the **Unit System** section.
3 From the **Unit system** list, choose **None**.

Load the parameters that define the geometry and the Gaussian pulse properties.

GLOBAL DEFINITIONS

Parameters 1

1 In the **Model Builder** window, under **Global Definitions** click **Parameters 1**.
2 In the **Settings** window for **Parameters**, locate the **Parameters** section.
3 Click **Load from File**.
4 Browse to the model's Application Libraries folder and double-click the file `gaussian_pulse_perfectly_matched_layers_parameters.txt`.

DEFINITIONS

Variables 1

1 In the **Model Builder** window, under **Component 1 (comp1)** right-click **Definitions** and choose **Variables**.

   Next, load the variables that define the initial Gaussian shape. These variables will be used as initial conditions for the acoustic pressure and its first time derivative.

2 In the **Settings** window for **Variables**, locate the **Variables** section.
3 Click **Load from File**.

4 Browse to the model's Application Libraries folder and double-click the file `gaussian_pulse_perfectly_matched_layers_variables.txt`.

The geometry contains a physical domain surrounded by perfectly matched layers (PML), see Figure 1. The latter are used to truncate the physical domain without introducing spurious numerical reflections from the outer boundary.

GEOMETRY 1

Square 1 (sq1)

1 In the **Geometry** toolbar, click **Square**.

2 In the **Settings** window for **Square**, locate the **Position** section.

3 From the **Base** list, choose **Center**.

4 Locate the **Size** section. In the **Side length** text field, type `W`.

5 Click to expand the **Layers** section. Select the **Layers to the left** check box.

6 Select the **Layers to the right** check box.

7 Select the **Layers on top** check box.

Specify the width of the PMLs.

8 In the table, enter the following settings:

Layer name	Thickness
Layer 1	dW

9 In the **Geometry** toolbar, click **Build All**.

DEFINITIONS

Set up a PML surrounding the physical domain.

Perfectly Matched Layer 1 (pml1)

1 In the **Definitions** toolbar, click **Perfectly Matched Layer**.

2 Select Domains 1–4 and 6–9 only.

Use cubic polynomial coordinate stretching in the PML domain.

3 In the **Settings** window for **Perfectly Matched Layer**, locate the **Scaling** section.

4 In the **PML scaling curvature parameter** text field, type `3`.

Modify the **Typical Wave Speed for Perfectly Matched Layers** and **Transient Solver Settings** according to the problem formulation.

PRESSURE ACOUSTICS, TRANSIENT (ACTD)

1 In the **Model Builder** window, under **Component 1 (comp1)** click **Pressure Acoustics, Transient (actd)**.

2 In the **Settings** window for **Pressure Acoustics, Transient**, locate the **Transient Solver and Mesh Settings** section.

3 In the f_{max} text field, type `0.3`.

The frequency content of sources is often known or can be analyzed with an FFT. In this model, the FFT of the source signal is depicted in Figure 6 created when analyzing the results. A maximal frequency of 0.3 (dimensionless) will capture the full signal information.

Transient Pressure Acoustics Model 1

1 In the **Model Builder** window, under **Component 1 (comp1)>Pressure Acoustics, Transient (actd)** click **Transient Pressure Acoustics Model 1**.

2 In the **Settings** window for **Transient Pressure Acoustics Model**, locate the **Transient Pressure Acoustics Model** section.

3 From the *c* list, choose **User defined**. In the associated text field, type `c0`.

4 From the ρ list, choose **User defined**. In the associated text field, type `rho0`.

Specify the initial values for the pressure and its first time derivative.

Initial Values 1

1 In the **Model Builder** window, click **Initial Values 1**.

2 In the **Settings** window for **Initial Values**, locate the **Initial Values** section.

3 In the *p* text field, type `p_i`.

4 In the *dp/dt* text field, type `dp_i`.

Note that it is not required to impose any special boundary conditions on the outer boundary.

MESH 1

Free Triangular 1

1 In the **Mesh** toolbar, click **Free Triangular**.

2 In the **Settings** window for **Free Triangular**, locate the **Domain Selection** section.

3 From the **Geometric entity level** list, choose **Domain**.

4 Select Domain 5 only.

Size 1

1 Right-click **Free Triangular 1** and choose **Size**.
2 In the **Settings** window for **Size**, locate the **Element Size** section.
3 Click the **Custom** button.
4 Locate the **Element Size Parameters** section.
5 Select the **Maximum element size** check box. In the associated text field, type `1.5`.

Create a mapped mesh in the PML domains.

Mapped 1

1 In the **Mesh** toolbar, click **Mapped**.

Choose the same mesh element size as in the physical domain. The generated mesh will contain 6 mesh layers.

Size 1

1 Right-click **Mapped 1** and choose **Size**.
2 In the **Settings** window for **Size**, locate the **Element Size** section.
3 Click the **Custom** button.
4 Locate the **Element Size Parameters** section.
5 Select the **Maximum element size** check box. In the associated text field, type `1.5`.

STUDY 1

Step 1: Time Dependent

Solve the model from time t = 0 to 120 in steps of 1 (dimensionless time units).

1 In the **Model Builder** window, under **Study 1** click **Step 1: Time Dependent**.
2 In the **Settings** window for **Time Dependent**, locate the **Study Settings** section.
3 In the **Output times** text field, type `range(0,1,120)`.
4 In the **Home** toolbar, click **Compute**.

RESULTS

Acoustic Pressure (actd)

The first default plot shows the acoustic pressure in the computational domain. You can see the pressure distribution at various times in Figure 2 and Figure 3.

1 In the **Settings** window for **2D Plot Group**, locate the **Data** section.
2 From the **Time (s)** list, choose **0**.

3 In the **Acoustic Pressure (actd)** toolbar, click **Plot**.

Create a **Cut Point 2D** dataset to plot the pressure as a function of time.

Cut Point 2D 1

1 In the **Results** toolbar, click **Cut Point 2D**.

2 In the **Settings** window for **Cut Point 2D**, locate the **Point Data** section.

3 In the **X** text field, type `20`.

4 In the **Y** text field, type `10`.

The plot in Figure 4 shows the comparison of the pressure and the analytical solution at the cut point.

Pressure at Point

1 In the **Results** toolbar, click **1D Plot Group**.

2 In the **Settings** window for **1D Plot Group**, type `Pressure at Point` in the **Label** text field.

3 Locate the **Data** section. From the **Dataset** list, choose **Cut Point 2D 1**.

Point Graph 1

1 Right-click **Pressure at Point** and choose **Point Graph**.

2 In the **Settings** window for **Point Graph**, locate the **y-Axis Data** section.

3 In the **Expression** text field, type `p`.

4 Click to expand the **Legends** section. Select the **Show legends** check box.

5 From the **Legends** list, choose **Manual**.

6 In the table, enter the following settings:

Legends
`COMSOL Model`

Point Graph 2

1 In the **Model Builder** window, right-click **Pressure at Point** and choose **Point Graph**.

2 In the **Settings** window for **Point Graph**, locate the **y-Axis Data** section.

3 In the **Expression** text field, type `p_a`.

4 Locate the **Legends** section. Select the **Show legends** check box.

5 From the **Legends** list, choose **Manual**.

6 In the table, enter the following settings:

Legends
`Analytical`

7 Click to expand the **Coloring and Style** section. Find the **Line style** subsection. From the **Line** list, choose **None**.

8 Find the **Line markers** subsection. From the **Marker** list, choose **Point**.

9 From the **Positioning** list, choose **Interpolated**.

10 In the **Number** text field, type `100`.

11 In the **Pressure at Point** toolbar, click **Plot**.

Create **Cut Line 2D** datasets to plot the pressure along the line at a given time.

Cut Line 2D 1

1 In the **Results** toolbar, click **Cut Line 2D**.

2 In the **Settings** window for **Cut Line 2D**, locate the **Line Data** section.

3 In row **Point 1**, set **X** to `-W/2`.

4 In row **Point 2**, set **X** to `W/2`.

Cut Line 2D 2

1 In the **Results** toolbar, click **Cut Line 2D**.

2 In the **Settings** window for **Cut Line 2D**, locate the **Line Data** section.

3 In row **Point 1**, set **X** to `-W/2+dW`.

4 In row **Point 2**, set **X** to `W/2-dW`.

The comparison of the pressure and the analytical solution along the x-axis for a given time is depicted in Figure 5.

Pressure along Cut Line

1 In the **Results** toolbar, click **1D Plot Group**.

2 In the **Settings** window for **1D Plot Group**, type `Pressure along Cut Line` in the **Label** text field.

3 Locate the **Data** section. From the **Dataset** list, choose **None**.

Line Graph 1

1 Right-click **Pressure along Cut Line** and choose **Line Graph**.

2 In the **Settings** window for **Line Graph**, locate the **Data** section.

3 From the **Dataset** list, choose **Cut Line 2D 1**.

4 From the **Time selection** list, choose **From list**.

5 In the **Times (s)** list, select **40**.

6 Locate the **y-Axis Data** section. In the **Expression** text field, type `p`.

7 Locate the **x-Axis Data** section. From the **Parameter** list, choose **Expression**.

8 In the **Expression** text field, type `x`.

9 Click to expand the **Legends** section. Select the **Show legends** check box.

10 From the **Legends** list, choose **Manual**.

11 In the table, enter the following settings:

Legends
COMSOL Model

Line Graph 2

1 In the **Model Builder** window, right-click **Pressure along Cut Line** and choose **Line Graph**.

2 In the **Settings** window for **Line Graph**, locate the **Data** section.

3 From the **Dataset** list, choose **Cut Line 2D 2**.

4 From the **Time selection** list, choose **From list**.

5 In the **Times (s)** list, select **40**.

6 Locate the **y-Axis Data** section. In the **Expression** text field, type `p_a`.

7 Locate the **x-Axis Data** section. From the **Parameter** list, choose **Expression**.

8 In the **Expression** text field, type `x`.

9 Locate the **Legends** section. Select the **Show legends** check box.

10 From the **Legends** list, choose **Manual**.

11 In the table, enter the following settings:

Legends
Analytical

12 Click to expand the **Coloring and Style** section. Find the **Line style** subsection. From the **Line** list, choose **None**.

13 Find the **Line markers** subsection. From the **Marker** list, choose **Point**.

14 From the **Positioning** list, choose **Interpolated**.

15 In the **Number** text field, type `100`.

16 In the **Pressure along Cut Line** toolbar, click **Plot**.

Figure 6 shows the frequency spectrum of the pressure wave at the cut point.

Pressure at Point, FFT

1 In the **Home** toolbar, click **Add Plot Group** and choose **1D Plot Group**.

2 In the **Settings** window for **1D Plot Group**, type `Pressure at Point, FFT` in the **Label** text field.

3 Locate the **Data** section. From the **Dataset** list, choose **Cut Point 2D 1**.

Point Graph 1

1 Right-click **Pressure at Point, FFT** and choose **Point Graph**.

2 In the **Settings** window for **Point Graph**, locate the **x-Axis Data** section.

3 From the **Parameter** list, choose **Discrete Fourier transform**.

4 From the **Show** list, choose **Frequency spectrum**.

5 In the **Pressure at Point, FFT** toolbar, click **Plot**.