

# Frequency Domain Study of Three-Phase Motor

## Introduction

This three-phase induction motor model is used to compare with TEAM benchmark problem 30, "Induction Motor Analyses" (Ref. 1). The Magnetic Fields physics is used to model the motor in the frequency domain at 60 Hz. The Velocity (Lorentz Term) feature is used to model the armature's rotation. Solving the model generates values for the motor's electromagnetic torque, induced voltage, and rotor losses at various rotor speeds. These computed results are then compared to the TEAM problem's analytical results.

## Model Definition

The model geometry is two-dimensional and is represented in Figure 1. This simplified geometry was chosen in Ref. 1 to allow computing the analytic solution to use as a reference for the numerical results.



Figure 1: Two-dimensional geometry of the three-phase induction motor.

The rotor consists of an iron core surrounded by an aluminum layer. The air gap and the region containing the six coils (windings), including the coil themselves, are nonmagnetic and nonconductive. The motor is surrounded by a layer of laminated iron and the outer region is air.

The rotating part is rotationally symmetric and it does not have magnetic sources (such as permanent magnets), so the rotation can be modeled by adding an artificial term (Lorentz's term) to the constitutive relation for the conduction current:

$$\mathbf{J} = \sigma(\mathbf{E} + \mathbf{v} \times \mathbf{B}) \tag{1}$$

where v is the local velocity of the material. The properties of the rotor and the constant rotational velocity ensure that all the fields and currents in the model are time-harmonic; so a frequency-domain analysis can be performed.

Using Lorentz's term and a frequency-domain analysis allows setting up a numerical problem without the need for moving meshes, as it is normally done with the **Rotating Machinery, Magnetic** interface. The **Magnetic Fields** interface can be used instead. The Lorentz's term is added by using a **Velocity (Lorentz's Term)** feature.

The three-phase coils (windings) are modeled with three **Coil** features using the **Homogenized multiturn** conductor model, which provide a homogenized model of bundles of tiny conducting wires. The three Coils use the **Coil group** functionality to model the two series-connected phase groups with current flowing in opposite direction.

The quantities of interests, for which analytic data is provided in Ref, are:

- the torque at different rotational speeds, computed using a Force Calculation feature,
- the coil voltage, computed automatically by the Coil features,
- the losses in the rotor, computed by integrating the power loss density.

# Results and Discussion

Figure 2 and Figure 3 show the axial current density and the magnetic flux density lines for two different angular velocities,  $\Omega = 200$  rad/s and  $\Omega = 800$  rad/s, respectively, below and above the rotational velocity of the stator field (about 377 rad/s or 3600 RPM).



Figure 2: Axial current density and magnetic flux density lines for  $\Omega = 200$  rad/s.



Figure 3: Axial induced current density and magnetic flux density lines for  $\Omega = 800$  rad/s.

Since the analysis performed is in the frequency domain, the solutions plotted in Figure 2 and Figure 3 are the real part of the current density and magnetic flux density phasors. It is possible to visualize the evolution of the fields during a cycle by changing the value of the phase in the dataset or by creating an animation as demonstrated in the application.

Figure 4, Figure 5, and Figure 6 show the computed torque, the coil voltage, and the rotor losses as a function of the rotational velocity. The plots show the expected results for an induction motor; in particular, the torque is zero, the voltage is maximum, and the losses are at a minimum at the synchronous speed ( $\Omega = 377$  rad/s).

The computed solution shows very good agreement with the analytic data (green markers).



Figure 4: Computed (blue line) and analytic (green markers) torque as a function of the angular velocity of the rotor.



Figure 5: Computed (blue line) and analytic (green markers) coil voltage per turn as a function of the angular velocity of the rotor.



Figure 6: Computed (blue line) and analytic (green markers) rotor losses as a function of the angular velocity of the rotor.

## Reference

1. http://www.compumag.org/jsite/team.html

## **Application Library path:** ACDC\_Module/Verifications/ three\_phase\_motor\_frequency

## Modeling Instructions

From the File menu, choose New.

#### NEW

In the New window, click 🔗 Model Wizard.

#### MODEL WIZARD

- I In the Model Wizard window, click **2D**.
- 2 In the Select Physics tree, select AC/DC>Electromagnetic Fields>Magnetic Fields (mf).
- 3 Click Add.
- 4 Click  $\bigcirc$  Study.
- 5 In the Select Study tree, select General Studies>Frequency Domain.
- 6 Click M Done.

#### GLOBAL DEFINITIONS

Parameters 1

- I In the Model Builder window, under Global Definitions click Parameters I.
- 2 In the Settings window for Parameters, locate the Parameters section.
- **3** In the table, enter the following settings:

Name	Expression	Value	Description
fO	60[Hz]	60 Hz	Supply frequency
wO	2*pi*f0	376.99 Hz	Supply angular frequency
n0	1000	1000	Number of turns
L	1[m]	l m	Length of the motor
Omega	200[rad/s]	200 rad/s	Angular speed of the rotor
coil_wire_curre nt	2045.175[A]* sqrt(2)/n0	2.8923 A	Current amplitude in coil wire

The geometry sequence is available in a separate file.

#### GEOMETRY I

- I In the Geometry toolbar, click Insert Sequence and choose Insert Sequence.
- 2 Browse to the model's Application Libraries folder and double-click the file three\_phase\_motor\_frequency\_geom\_sequence.mph.
- **3** In the **Geometry** toolbar, click 🛄 **Build All**.



#### DEFINITIONS

Integration, Steel

- I In the Definitions toolbar, click / Nonlocal Couplings and choose Integration.
- **2** Select Domain 19 only.
- 3 In the Settings window for Integration, type Integration, Steel in the Label text field.
- 4 In the **Operator name** text field, type int\_steel.

Integration, Aluminum

- I In the Definitions toolbar, click 🖉 Nonlocal Couplings and choose Integration.
- 2 Select Domain 18 only.
- **3** In the **Settings** window for **Integration**, type **Integration**, Aluminum in the **Label** text field.
- 4 In the **Operator name** text field, type int\_al.

#### Cylindrical System 2 (sys2)

Add a cylindrical coordinate system to simplify the definition of the rotational velocity.

In the **Definitions** toolbar, click  $\sum_{x}^{z}$  **Coordinate Systems** and choose **Cylindrical System**.

#### Infinite Element Domain 1 (ie1)

In the original specification, the exterior region extends to infinity. Apply an Infinite Element Domain scaling system on the outer layer for this purpose.

- I In the Definitions toolbar, click  $i \sim$  Infinite Element Domain.
- **2** Select Domains 1 and 2 only.
- 3 In the Settings window for Infinite Element Domain, locate the Geometry section.
- 4 From the Type list, choose Cylindrical.

#### ADD MATERIAL

- I In the Home toolbar, click 🙀 Add Material to open the Add Material window.
- 2 Go to the Add Material window.
- 3 In the tree, select Built-in>Air.
- 4 Click Add to Component in the window toolbar.
- 5 In the Home toolbar, click 🙀 Add Material to close the Add Material window.

#### MATERIALS

Aluminum

- I In the Model Builder window, under Component I (comp1) right-click Materials and choose Blank Material.
- 2 In the Settings window for Material, type Aluminum in the Label text field.
- **3** Select Domain 18 only(the outer layer of the rotor).
- 4 Locate the Material Contents section. In the table, enter the following settings:

Property	Variable	Value	Unit	Property group
Relative permeability	mur_iso ; murii = mur_iso, murij = 0	1	I	Basic
Electrical conductivity	sigma_iso ; sigmaii = sigma_iso, sigmaij = 0	3.72e7[S /m]	S/m	Basic
Relative permittivity	epsilonr_iso ; epsilonrii = epsilonr_iso, epsilonrij = 0	1	I	Basic

Rotor Steel

I Right-click Materials and choose Blank Material.

2 In the Settings window for Material, type Rotor Steel in the Label text field.

**3** Select Domain 19 only(the inner core of the rotor).

**4** Locate the **Material Contents** section. In the table, enter the following settings:

Property	Variable	Value	Unit	Property group
Relative permeability	mur_iso ; murii = mur_iso, murij = 0	30	I	Basic
Electrical conductivity	sigma_iso ; sigmaii = sigma_iso, sigmaij = 0	1.6e6[S /m]	S/m	Basic
Relative permittivity	epsilonr_iso ; epsilonrii = epsilonr_iso, epsilonrij = 0	1	I	Basic

Stator Steel

I Right-click Materials and choose Blank Material.

2 In the Settings window for Material, type Stator Steel in the Label text field.

**3** Select Domain 15 only(the outer layer of the stator, before the infinite element region).

**4** Locate the **Material Contents** section. In the table, enter the following settings:

Property	Variable	Value	Unit	<b>P</b> roperty group
Relative permeability	mur_iso ; murii = mur_iso, murij = 0	30	I	Basic
Electrical conductivity	sigma_iso ; sigmaii = sigma_iso, sigmaij = 0	0	S/m	Basic
Relative permittivity	epsilonr_iso ; epsilonrii = epsilonr_iso, epsilonrij = 0	1	I	Basic

#### MAGNETIC FIELDS (MF)

Create three **Coil** features using the **Coil group** functionality and apply each of them on pairs of windings on opposite sides of the rotor.

Coil, Phase A

- I In the Model Builder window, under Component I (compl) right-click Magnetic Fields (mf) and choose the domain setting Coil.
- 2 Select Domains 4 and 13 only.



- 3 In the Settings window for Coil, type Coil, Phase A in the Label text field.
- 4 Locate the Coil section. From the Conductor model list, choose Homogenized multiturn.
- 5 Select the Coil group check box.
- **6** In the  $I_{\text{coil}}$  text field, type coil\_wire\_current.
- 7 Locate the Homogenized Multiturn Conductor section. In the N text field, type n0.

The current flows in opposite directions in the two domains. Use a dedicated subfeature to specify this.

#### Reversed Current Direction 1

- I In the Physics toolbar, click Attributes and choose Reversed Current Direction.
- **2** Select Domain 13 only.

#### Coil, Phase B

I In the Model Builder window, right-click Coil, Phase A and choose Duplicate.

2 In the Settings window for Coil, type Coil, Phase B in the Label text field.



- **3** Locate the **Domain Selection** section. Click **Clear Selection**.
- **4** Select Domains 7 and 9 only.

#### Reversed Current Direction I

- I In the Model Builder window, expand the Coil, Phase B node, then click Reversed Current Direction I.
- **2** In the Settings window for Reversed Current Direction, locate the Domain Selection section.
- 3 Click Clear Selection.
- 4 Select Domain 7 only.

### Coil, Phase B

Specify the second coil current with a 120° phase shift with respect to the first. In frequency domain, this corresponds to a multiplication by a complex phase factor.

- I In the Model Builder window, click Coil, Phase B.
- 2 In the Settings window for Coil, locate the Coil section.
- **3** In the *I*<sub>coil</sub> text field, type coil\_wire\_current\*exp(j\*2\*pi/3).

#### Coil, Phase C

I Right-click Coil, Phase B and choose Duplicate.

2 In the Settings window for Coil, type Coil, Phase C in the Label text field.



- **3** Locate the **Domain Selection** section. Click **Clear Selection**.
- 4 Select Domains 5 and 11 only.

Specify the third coil current with a -120° phase shift.

5 Locate the Coil section. In the I<sub>coil</sub> text field, type coil\_wire\_current\*exp(-j\*2\*pi/ 3).

#### Reversed Current Direction I

- I In the Model Builder window, expand the Coil, Phase C node, then click Reversed Current Direction 1.
- **2** In the **Settings** window for **Reversed Current Direction**, locate the **Domain Selection** section.
- 3 Click Clear Selection.
- 4 Select Domain 5 only.

#### Velocity (Lorentz Term) I

- I In the Physics toolbar, click 🔵 Domains and choose Velocity (Lorentz Term).
- 2 Select Domains 18 and 19 only.
- **3** In the Settings window for Velocity (Lorentz Term), locate the Coordinate System Selection section.
- 4 From the Coordinate system list, choose Cylindrical System 2 (sys2).

5 Locate the Velocity (Lorentz Term) section. Specify the v vector as

0 r Omega\*sys2.r phi

Here, Omega is the model parameter corresponding to the angular velocity, while sys2.r is the radial coordinate in the cylindrical coordinate system (with tag sys2).

Force Calculation 1

- I In the Physics toolbar, click 🔵 Domains and choose Force Calculation.
- **2** Select Domains 18 and 19 only.

#### MESH I

Create an appropriate mesh for the model. The mesh must be fine enough at the surface of the rotating conductor to resolve the skin depth even with the highest slip. For this particular model, this is verified already with the default mesh.

I In the Model Builder window, under Component I (compl) right-click Mesh I and choose Build All.

Now, set up the study, using a parametric sweep for the rotational speed.

2 Right-click Component I (compl)>Mesh I and choose Build Selected.

#### STUDY I

Parametric Sweep

- I In the Study toolbar, click **Parametric Sweep**.
- 2 In the Settings window for Parametric Sweep, locate the Study Settings section.
- 3 Click + Add.
- **4** In the table, enter the following settings:

Parameter name	Parameter value list	Parameter unit
Omega (Angular speed of the	range(0,20,1200)	Hz
rotor)		

Step 1: Frequency Domain

- I In the Model Builder window, click Step I: Frequency Domain.
- 2 In the Settings window for Frequency Domain, locate the Study Settings section.
- 3 In the Frequencies text field, type f0.
- **4** In the **Study** toolbar, click **= Compute**.

#### RESULTS

#### Magnetic Flux Density Norm (mf)

After the solution has been computed, the default surface plot will be shown. Default plot visualize the magnetic flux density lines by means of a contour plot. This setting will be added explicitly in the after the second study.

#### I In the Magnetic Flux Density Norm (mf) toolbar, click 🗿 Plot.

Create an additional plot for the current density induced in the rotor.

#### Current Density

I In the Home toolbar, click 🚛 Add Plot Group and choose 2D Plot Group.

Specify a custom title for the plot. Using the **Allow evaluation of expressions** functionality, it is possible to include values (such as the rotational velocity **Omega**) in the title.

- 2 In the Settings window for 2D Plot Group, type Current Density in the Label text field.
- 3 Click to expand the Title section. From the Title type list, choose Manual.
- 4 In the Title text area, type \OMEGA: eval(Omega) rad/s Surface: Current density (A/m<sup>2</sup>) Lines: Magnetic flux density.

#### Surface 1

- I Right-click Current Density and choose Surface.
- 2 In the Settings window for Surface, click Replace Expression in the upper-right corner of the Expression section. From the menu, choose Component I (compl)>Magnetic Fields> Currents and charge>Current density - A/m<sup>2</sup>>mf.Jz - Current density, z-component.

#### Contour I

- I In the Model Builder window, right-click Current Density and choose Contour.
- 2 In the Settings window for Contour, click Replace Expression in the upper-right corner of the Expression section. From the menu, choose Component I (compl)>Magnetic Fields> Magnetic>Magnetic vector potential - Wb/m>mf.Az - Magnetic vector potential, zcomponent.
- 3 Locate the Coloring and Style section. From the Coloring list, choose Uniform.
- 4 From the Color list, choose Gray.
- 5 Clear the Color legend check box.

#### Current Density

- I In the Model Builder window, click Current Density.
- 2 In the Settings window for 2D Plot Group, locate the Data section.

- 3 From the Parameter value (Omega (Hz)) list, choose 200.
- **4** In the **Current Density** toolbar, click **O Plot**.



The plot shows how the field is affected by the rotor rotating more slowly than the field.

5 From the Parameter value (Omega (Hz)) list, choose 380.

6 In the Current Density toolbar, click 💽 Plot.



At a speed close to the synchronous speed (377 rad/s) the field lines are not deformed and the induced current densities are at a minimum.

7 From the Parameter value (Omega (Hz)) list, choose 800.

8 In the Current Density toolbar, click **O** Plot.



The field applies a braking effect on the fast-spinning rotor.

Import the reference analytic data and compare it to the computed solution.

#### Reference Data

- I In the **Results** toolbar, click **Table**.
- 2 In the Settings window for Table, type Reference Data in the Label text field.
- 3 Locate the Data section. Click Import.
- **4** Browse to the model's Application Libraries folder and double-click the file three\_phase\_motor\_frequency\_data.txt.

#### Torque

- I In the **Results** toolbar, click  $\sim$  **ID Plot Group**.
- 2 In the Settings window for ID Plot Group, type Torque in the Label text field.
- 3 Click to expand the Title section. From the Title type list, choose Manual.
- 4 In the **Title** text area, type Torque (N\*m).
- 5 Locate the Plot Settings section.
- 6 Select the x-axis label check box. In the associated text field, type Angular Velocity (rad/s).

#### Global I

- I Right-click **Torque** and choose **Global**.
- 2 In the Settings window for Global, click Replace Expression in the upper-right corner of the y-Axis Data section. From the menu, choose Component I (compl)>Magnetic Fields> Mechanical>Torque - N·m>mf.Tz\_0 - Torque, z-component.
- 3 Click to expand the Legends section. From the Legends list, choose Manual.
- 4 In the table, type Computed in the first row.

- I In the Model Builder window, right-click Torque and choose Table Graph.
- 2 In the Settings window for Table Graph, locate the Data section.
- **3** From the **Plot columns** list, choose **Manual**.
- 4 In the Columns list, select Torque (N\*m).
- **5** Locate the **Coloring and Style** section. Find the **Line style** subsection. From the **Line** list, choose **None**.
- 6 Find the Line markers subsection. From the Marker list, choose Cycle.
- 7 Click to expand the Legends section. Select the Show legends check box.
- 8 From the Legends list, choose Manual.
- 9 In the table, type Reference in the first row.

**IO** In the **Torque** toolbar, click **ID Plot**.



#### Coil Voltage

- I Right-click Torque and choose Duplicate.
- 2 In the Settings window for ID Plot Group, type Coil Voltage in the Label text field.
- 3 Locate the Title section. In the Title text area, type Coil Voltage per Turn (V, RMS).

#### Global I

- I In the Model Builder window, expand the Coil Voltage node, then click Global I.
- 2 In the Settings window for Global, locate the y-Axis Data section.
- **3** In the table, enter the following settings:

Expression	Unit	Description
<pre>abs(mf.coil1.Vind)/(n0*sqrt(2))</pre>	V	

- I In the Model Builder window, click Table Graph I.
- 2 In the Settings window for Table Graph, locate the Data section.
- 3 In the Columns list, select Vin/turn (V).

**4** In the **Coil Voltage** toolbar, click **I Plot**.



#### Rotor Losses

- I In the Model Builder window, right-click Coil Voltage and choose Duplicate.
- 2 In the Settings window for ID Plot Group, type Rotor Losses in the Label text field.
- 3 Locate the Title section. In the Title text area, type Rotor Losses (W).

#### Global I

- I In the Model Builder window, expand the Rotor Losses node, then click Global I.
- 2 In the Settings window for Global, locate the y-Axis Data section.
- **3** In the table, enter the following settings:

Expression	Unit	Description
<pre>int_steel(mf.Qh*L)+int_al(mf.Qh*L)</pre>	W	

- I In the Model Builder window, click Table Graph I.
- 2 In the Settings window for Table Graph, locate the Data section.
- 3 In the Columns list, select Rotor\_Loss (W).



**4** In the **Rotor Losses** toolbar, click **I Plot**.

#### Steel Losses

- I In the Model Builder window, right-click Rotor Losses and choose Duplicate.
- 2 In the Settings window for ID Plot Group, type Steel Losses in the Label text field.
- 3 Locate the Title section. In the Title text area, type Steel Losses (W).

#### Global I

- I In the Model Builder window, expand the Steel Losses node, then click Global I.
- 2 In the Settings window for Global, locate the y-Axis Data section.
- **3** In the table, enter the following settings:

Expression	Unit	Description
<pre>int_steel(mf.Qh*L)</pre>	W	Integration, Steel

- I In the Model Builder window, click Table Graph I.
- 2 In the Settings window for Table Graph, locate the Data section.
- 3 In the Columns list, select Steel\_Loss (W).



**4** In the **Steel Losses** toolbar, click **I** Plot.

## Current Density

Click the  $\leftarrow$  **Zoom Extents** button in the **Graphics** toolbar.

Finally, to appreciate the dynamics of the rotating field, create an animation of the timeharmonic solution.

#### Animation I

- I In the **Results** toolbar, click **Animation** and choose **Player**.
- 2 In the Settings window for Animation, locate the Animation Editing section.
- **3** From the Sequence type list, choose Dynamic data extension.
- 4 Locate the Playing section. From the Repeat list, choose Forever.
- 5 In the Graphics toolbar, use the Play and Stop buttons to control the animation.

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