Created in COMSOL Multiphysics 6.1

Self Inductance and Mutual Inductance of a Single Conductor and a Helical Coil

Introduction

The mutual inductance and induced currents between a single-turn primary and 20-turn secondary coil in a concentric coplanar arrangement is computed using a frequency domain model. Each turn of the secondary coil is modeled explicitly. Static and AC results are compared one against the other, and against analytic predictions.

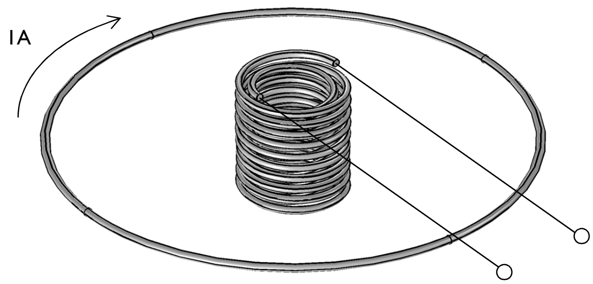


Figure 1: A 20-turn secondary coil inside of a single-turn primary coil (not to scale).

Model Definition

The physical situation being modeled is shown in Figure 1. A secondary coil composed of 20 turns, wound two deep, is concentric with the primary, and in the same plane. The radius of the centroid of the secondary coil is $R_2 = 10$ mm. The wire radii in both coils is $r_0 = 1$ mm. Although the coils are shown in 3D, they are modeled in the 2D axisymmetric space, assuming no physical variation around the centerline. First two DC analysis are solved in order to extract the inductance matrix of the system. Then a prescribed current of 1 A is flowing through a single-turn coil of radius $R_1 = 100$ mm, at a frequency of 1 kHz. The objective of this is computing the voltage difference at the secondary coil for the open-circuit case, and the induced currents for the closed-circuit case.

For the case of a secondary multi-turn coil with N turns, and in the limit as $R_1 >> R_2 >> r_0$, the analytic expression for the mutual inductance between the two coils is:

$$M = N\mu_0 \frac{\pi R_2^{\,2}}{2R_1}$$

where μ_0 is the permeability of free space.

The two concentric coils are modeled in a 2D axisymmetric sense, as shown schematically in Figure 2. The modeling domain is surrounded by a region of Infinite Elements, which are a way to truncate a domain which stretches to infinity. Although the thickness of the Infinite Element Domain is finite, it can be thought of as a domain of infinite extent.

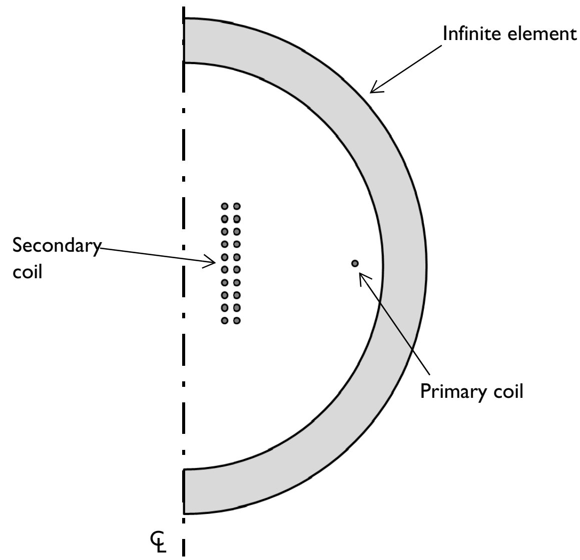


Figure 2: A schematic representation of the 2D axisymmetric model of the concentric coils.

The primary coil is modeled using the **Coil** feature, which can be thought of as introducing an infinitesimal slit in an otherwise continuous torus. Since the coil has a single turn and is made up of conductive material, the **Single conductor** model is used in the Coil feature. The feature is used to excite the coil by specifying a current of 1 A.

The secondary coil is modeled using the **Coil** feature with the **Coil group** setting that enforces that the same amount of current flow through each circular domain which represents one turn of the coil (the coil turns are connected in series). This feature can be used to model both the open circuit and the closed-circuit case. To model the open-circuit case, the current through the coil is specified to be 0 A, which specifies that there is no current flowing through the coil. To model a closed circuit it is enough to put 0 V. The Coil feature with the Coil group setting computes the total current and potential drop on the entire coil. Furthermore, if just one coil only is fed, also self and mutual inductance of the coil system is available in the output. For AC feeding, assuming the system to be purely reactive, mutual inductance can be computed via:

$$M = \frac{V_{coil}}{i\omega I_p} \tag{1}$$

Where ω is the angular frequency of excitation of the driving current in the primary, I_p. The inductance computed in this fashion will have a small imaginary component as, due to finite conductivity there are eddy current losses in the wires and the coil impedance, though mainly reactive, has a small resistive part. The deduced inductance is compared then also against the inductance predicted by taking the integral of the magnetic flux through the center of the coil.

For the closed-circuit case, the voltage drop across the coil is fixed at 0 V. Although this seems to imply a short circuit, the reactance of the copper coil is inherently included, so the case being modeled is analogous to a closed continuous loop of wire. The Coil feature with the Coil group setting enforces that the same current flows through each turn.

Via a simple circuital analogy, and for frequencies such that conductor size is significantly larger than skin depth, this current can be estimated from DC values via:

$$I_2 = -iw(L_{21}/Z_2) \tag{2}$$

where L_{21} is mutual inductance and $Z_2 = R_2 + iwL_2$ is the impedance of the inner coil. At the simulated 1 kHz deviations are really small. Increasing the frequency, this estimate will fail as the model is able to capture the autoinductance effects on the coil that are not included in the simplest circuital analogy.

Results and Discussion

From two initial static analyses — one feeding the single coil and the other feeding the 20-turn coil group — it is possible to extract the following inductance matrix from built-in variables:

$$\begin{bmatrix} 619\ \text{nH} & 40\ \text{nH} \\ 40\ \text{nH} & 3300\ \text{nH} \end{bmatrix}$$

A similar estimate (which is however less accurate) is extracted computing the integral of the normal magnetic flux. In case of an AC feeding the magnetic flux lines are plotted in Figure 3 for the open-circuit case. The Coil feature with the Coil group setting computes the voltage across the secondary, which can be used to evaluate the mutual inductance, 39.9 – 0.6*i* nH. This agrees well with the mutual inductance predicted by static calculations and with the analytic mutual inductance estimate of 39.478 nH.

The induced currents of the secondary coil is plotted in Figure 4 for the closed-circuit case. The skin effect is apparent, the current is being driven to the boundaries of the domains. The induced current through the secondary coil is $-10.94 - 3.5i$ mA, the imaginary component implies a reactive current. Both real and imaginary parts are correctly accounted for by Equation 2.

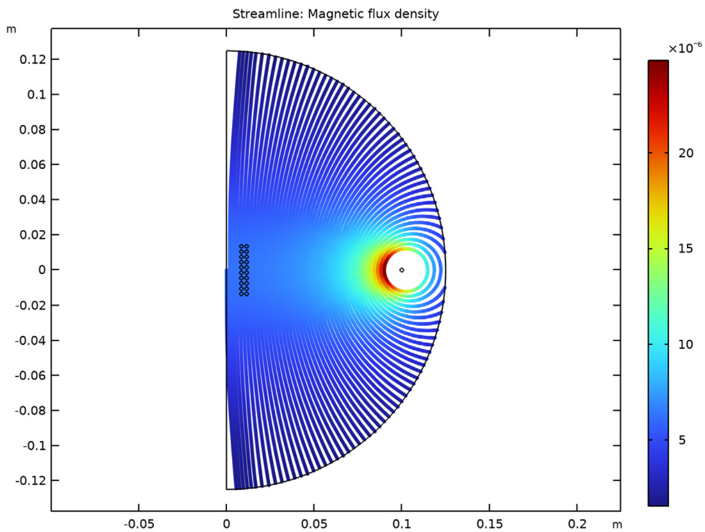


Figure 3: Magnetic flux lines for the open-circuit case.

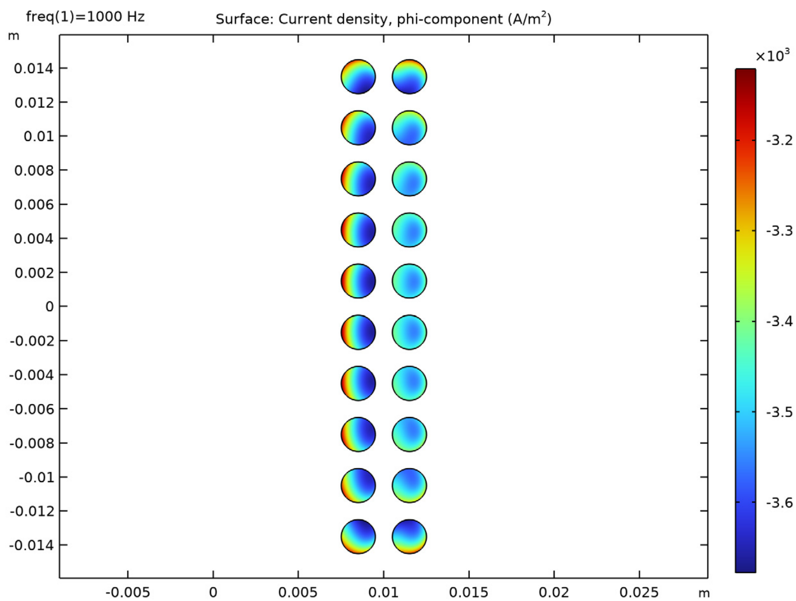


Figure 4: Induced currents in the coil for the closed-circuit case.

Application Library path: `ACDC_Module/Tutorials,_Coils/mutual_inductance_coil_group`

Modeling Instructions

From the **File** menu, choose **New**.

NEW

In the **New** window, click **Model Wizard**.

MODEL WIZARD

1 In the **Model Wizard** window, click **2D Axisymmetric**.

2 In the **Select Physics** tree, select **AC/DC>Electromagnetic Fields>Magnetic Fields (mf)**.

3 Click **Add**.

4 Click **Study**.

5 In the **Select Study** tree, select **General Studies>Stationary**.

6 Click **Done**.

GLOBAL DEFINITIONS

Parameters 1

1 In the **Model Builder** window, under **Global Definitions** click **Parameters 1**.

2 In the **Settings** window for **Parameters**, locate the **Parameters** section.

3 In the table, enter the following settings:

Name	Expression	Value	Description
r_wire	1[mm]	0.001 m	Radius, wire
R1	100[mm]	0.1 m	Radius, outer coil
R2	10[mm]	0.01 m	Radius, inner coil
M	20*(mu0_const*pi* R2^2)/(2*R1)	3.9478E-8 H	Analytic mutual inductance
I1	1[A]	1 A	Current, outer coil
I2	0[A]	0 A	Current, inner coil

Here, mu0_const is a predefined COMSOL constant for the permeability in vacuum.

GEOMETRY 1

Create a circle for the simulation domain. Define a layer in the circle where you will assign the Infinite Element Domain.

Circle 1 (c1)

1 In the **Geometry** toolbar, click **Circle**.

2 In the **Settings** window for **Circle**, locate the **Size and Shape** section.

3 In the **Sector angle** text field, type 180.

4 In the **Radius** text field, type 1.75*R1.

5 Locate the **Rotation Angle** section. In the **Rotation** text field, type -90.

6 Click to expand the **Layers** section. In the table, enter the following settings:

Layer name	Thickness (m)
Layer 1	50[mm]

Create a circle for the outer coil.

Circle 2 (c2)

1 In the **Geometry** toolbar, click **Circle**.

2 In the **Settings** window for **Circle**, locate the **Size and Shape** section.

3 In the **Radius** text field, type `r_wire`.

4 Locate the **Position** section. In the **r** text field, type `R1`.

Then, create a circle for the inner coil.

Circle 3 (c3)

1 In the **Geometry** toolbar, click **Circle**.

2 In the **Settings** window for **Circle**, locate the **Size and Shape** section.

3 In the **Radius** text field, type `r_wire`.

4 Locate the **Position** section. In the **r** text field, type `R2-1.5*r_wire`.

5 In the **z** text field, type `-13.5*r_wire`.

Array 1 (arr1)

1 In the **Geometry** toolbar, click **Transforms** and choose **Array**.

2 Select the object **c3** only.

3 In the **Settings** window for **Array**, locate the **Size** section.

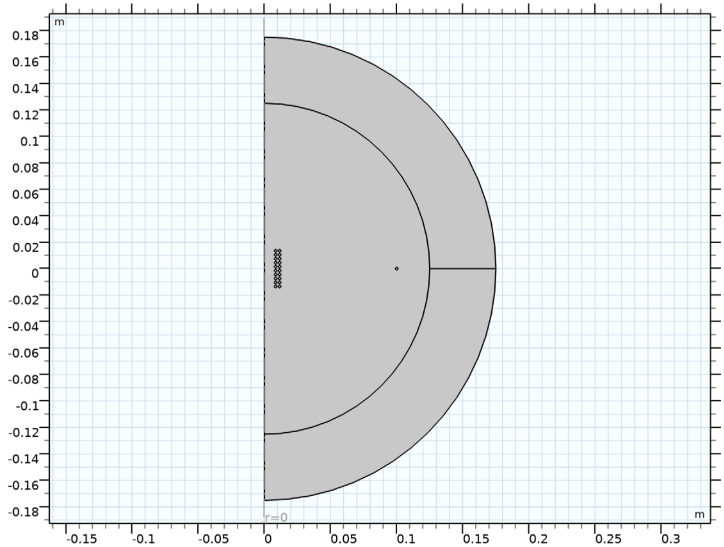
4 In the **r size** text field, type `2`.

5 In the **z size** text field, type `10`.

6 Locate the **Displacement** section. In the **r** text field, type `3*r_wire`.

7 In the **z** text field, type `3*r_wire`.

8 Click **Build All Objects**.



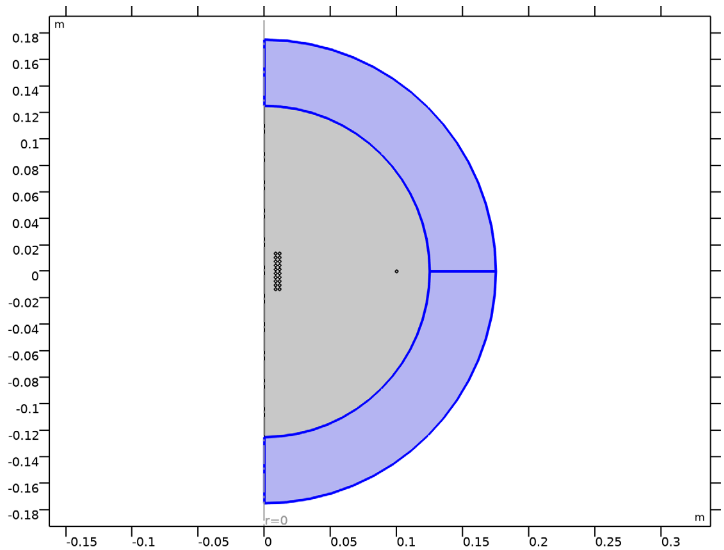
DEFINITIONS

Define the Infinite Element Domain to apply a coordinate transformation that mathematically stretches the layer to infinity. The **Physics-controlled mesh** creates a **Swept Mesh** inside the **Infinite Elements** domains.

Infinite Element Domain 1 (ie1)

1 In the **Definitions** toolbar, click **Infinite Element Domain**.

2 Select Domains 1 and 3 only.



MAGNETIC FIELDS (MF)

Now, set up the physics. Assign a **Coil** feature on the outer and the inner coil. The outer coil will be initially fed with a current of 1 A.

1 Click the **Zoom In** button in the **Graphics** toolbar.

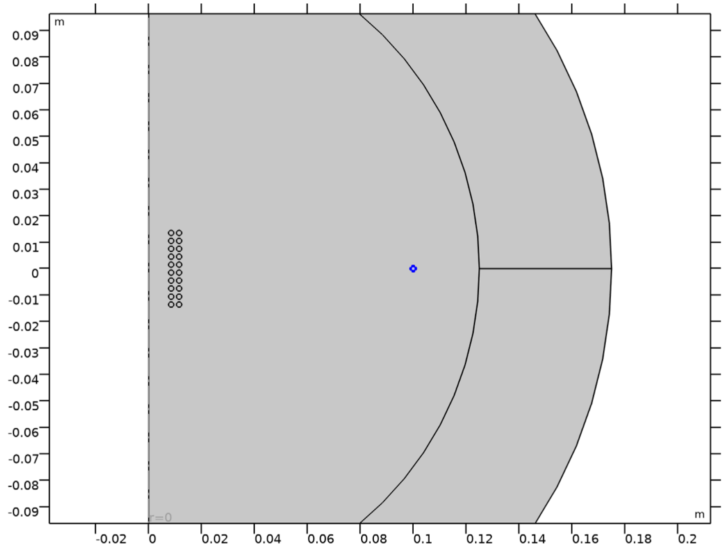
Coil 1

1 In the **Model Builder** window, under **Component 1 (comp1)** right-click **Magnetic Fields (mf)** and choose the domain setting **Coil**.

2 Select Domain 24 only.

3 In the **Settings** window for **Coil**, locate the **Coil** section.

4 In the I_{coil} text field, type `I1`.



Specify 0 A current for the **Coil** feature assigned to the inner coil to model the open circuit case.

Coil 2

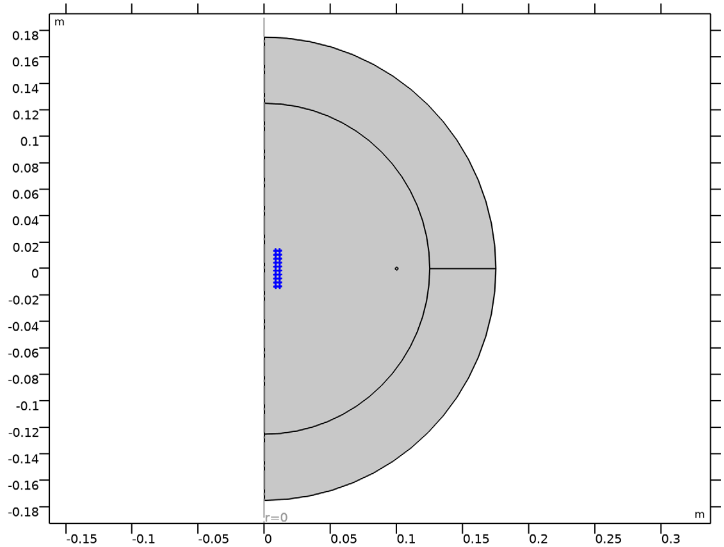
1 In the **Physics** toolbar, click **Domains** and choose **Coil**.

2 In the **Settings** window for **Coil**, locate the **Coil** section.

3 Select the **Coil group** check box.

4 Select Domains 4–23 only.

5 In the I_{coil} text field, type `I2`.



MATERIALS

Next, assign material properties. Use Air for all domains.

ADD MATERIAL

1. In the **Home** toolbar, click **Add Material** to open the **Add Material** window.
2. Go to the **Add Material** window.
3. In the tree, select **Built-in>Air**.
4. Click **Add to Component** in the window toolbar.

MATERIALS

Air (mat1)

Then, override the coil domains with copper.

ADD MATERIAL

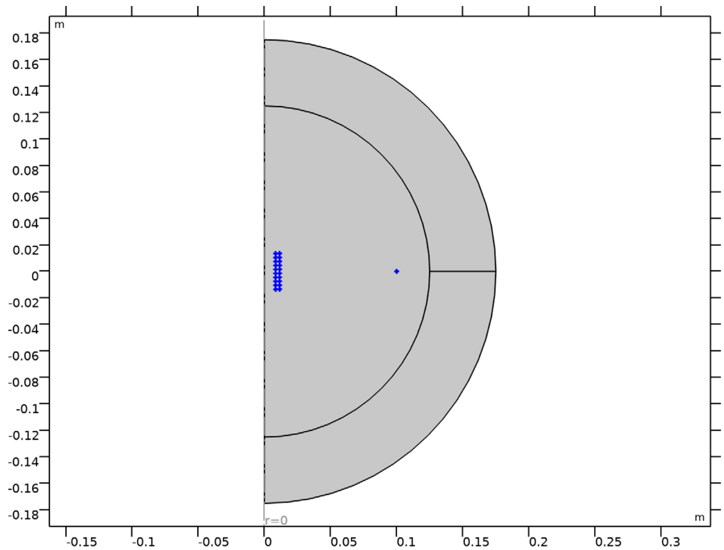
1. Go to the **Add Material** window.
2. In the tree, select **AC/DC>Copper**.
3. Click **Add to Component** in the window toolbar.
4. In the **Home** toolbar, click **Add Material** to close the **Add Material** window.

MATERIALS

Copper (mat2)

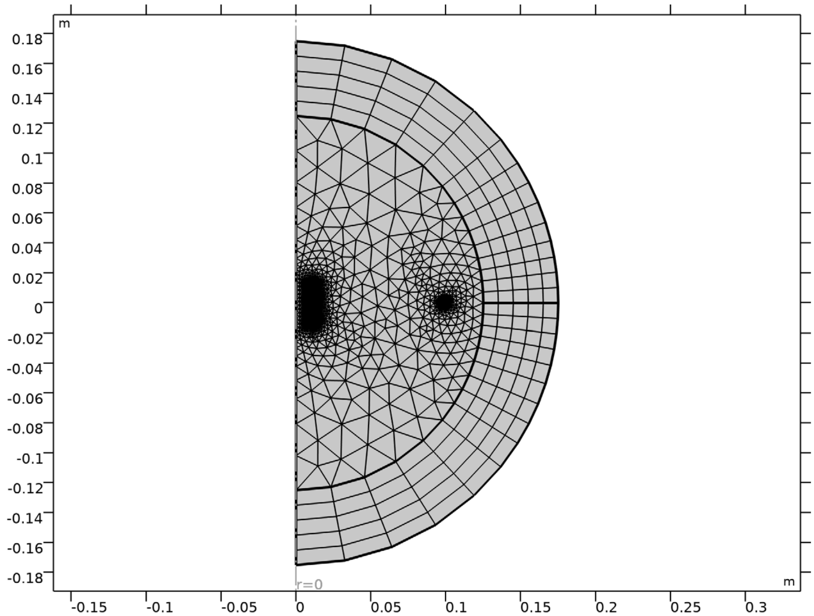
1 Select Domains 4–24 only.

2 Click the  **Zoom Extents** button in the **Graphics** toolbar.



MESH 1

In the **Model Builder** window, under **Component 1 (comp1)** right-click **Mesh 1** and choose **Build All**.



Solve the first case in which the outer coil (named 1) is fed and the inner (named 2) is open.

STUDY 1

1 In the **Model Builder** window, click **Study 1**.

2 In the **Settings** window for **Study**, locate the **Study Settings** section.

3 Clear the **Generate default plots** check box.

4 In the **Home** toolbar, click **Compute**.

RESULTS

In the **Model Builder** window, expand the **Results** node.

Study 1/Solution 1 (sol1)

Select only the domains not part of the Infinite Element Domain to better visualize the magnetic flux density.

1 In the **Model Builder** window, expand the **Results>Datasets** node, then click **Study 1/ Solution 1 (sol1)**.

Selection

1 In the **Results** toolbar, click **Attributes** and choose **Selection**.

2 In the **Settings** window for **Selection**, locate the **Geometric Entity Selection** section.

3 From the **Geometric entity level** list, choose **Domain**.

4 Select Domains 2 and 4–24 only.

2D Plot Group 1

In the **Results** toolbar, click **2D Plot Group**.

Streamline 1

1 Right-click **2D Plot Group 1** and choose **Streamline**.

2 In the **Settings** window for **Streamline**, locate the **Streamline Positioning** section.

3 From the **Positioning** list, choose **Starting-point controlled**.

4 From the **Entry method** list, choose **Coordinates**.

5 In the **r** text field, type `range(0,0.9*R1/49,0.9*R1)`.

6 In the **z** text field, type `0`.

7 Locate the **Coloring and Style** section. Find the **Line style** subsection. From the **Type** list, choose **Tube**.

Color Expression 1

1 Right-click **Streamline 1** and choose **Color Expression**.

2 Click the **Zoom Extents** button in the **Graphics** toolbar.

This reproduces Figure 2.

Evaluate the self inductance of the external coil and the mutual inductance of the outer coil with respect to the inner. Some additional quantities are also computed to verify the results.

Global Evaluation 1

1 In the **Results** toolbar, click **Global Evaluation**.

2 In the **Settings** window for **Global Evaluation**, locate the **Expressions** section.

3 In the table, enter the following settings:

Expression	**Unit**	**Description**
mf.LCoil_1	nH	External coil inductance
2*mf.intWm/1[A^2]	nH	Energy estimate for external coil inductance
mf.L_2_1	nH	Computed mutual inductance
M	nH	Analytical mutual inductance

4 Click **Evaluate**.

Next, compute the self inductance of the inner coil and the mutual inductance of the inner coil with respect to the outer. Start by switching the currents in the coils.

GLOBAL DEFINITIONS

Parameters 1

1 In the **Model Builder** window, under **Global Definitions** click **Parameters 1**.

2 In the **Settings** window for **Parameters**, locate the **Parameters** section.

3 In the table, enter the following settings:

Name	**Expression**	**Value**	**Description**
I1	0[A]	0 A	Current, outer coil
I2	1[A]	1 A	Current, inner coil

Then add and solve a second study for this case. The solution previously computed will still be available in Study 1.

ADD STUDY

1 In the **Home** toolbar, click **Add Study** to open the **Add Study** window.

2 Go to the **Add Study** window.

3 Find the **Studies** subsection. In the **Select Study** tree, select **General Studies>Stationary**.

4 Click **Add Study** in the window toolbar.

5 In the **Home** toolbar, click **Add Study** to close the **Add Study** window.

STUDY 2

1 In the **Model Builder** window, click **Study 2**.

2 In the **Settings** window for **Study**, locate the **Study Settings** section.

3 Clear the **Generate default plots** check box.

4 In the **Home** toolbar, click **Compute**.

The quantities of interest are evaluated in the following steps.

RESULTS

Global Evaluation 2

1 In the **Results** toolbar, click **Global Evaluation**.

2 In the **Settings** window for **Global Evaluation**, locate the **Data** section.

3 From the **Dataset** list, choose **Study 2/Solution 2 (sol2)**.

4 Locate the **Expressions** section. In the table, enter the following settings:

Expression	Unit	Description
`mf.LCoil_2`	`nH`	`Internal coil inductance`
`2*mf.intWm/1[A^2]`	`nH`	`Energy estimate for internal coil inductance`
`mf.L_1_2`	`nH`	`Computed mutual inductance`
`M`	`nH`	`Analytical mutual inductance`

5 Click next to **Evaluate**, then choose **New Table**.

TABLE

1 Go to the **Table** window.

Self and mutual inductance variables as computed above are derived via concatenated flux, which is defined as the line integral of the magnetic vector potential along the coil. This approach gives the best accuracy.

For simple geometries like the present one, concatenated flux can be also computed explicitly using its definition as the integral of magnetic flux through a surface, although this approach usually gives less accurate results.

RESULTS

Cut Line 2D 1

1 In the **Results** toolbar, click **Cut Line 2D**.

2 In the **Settings** window for **Cut Line 2D**, locate the **Line Data** section.

3 In row **Point 2**, set **r** to `R2`.

Cut Line 2D 2

1 In the **Results** toolbar, click **Cut Line 2D**.

2 In the **Settings** window for **Cut Line 2D**, locate the **Line Data** section.

3 In row **Point 2**, set **r** to `R1`.

Line Integration 1

1 In the **Results** toolbar, click 8.85 e-12 **More Derived Values** and choose **Integration>Line Integration**.

2 In the **Settings** window for **Line Integration**, locate the **Data** section.

3 From the **Dataset** list, choose **Cut Line 2D 1**.

4 Locate the **Expressions** section. In the table, enter the following settings:

Expression	Unit	Description
`20*mf.Bz/I1`	nH	

5 Click next to **Evaluate**, then choose **New Table**.

TABLE

Go to the **Table** window.

Line Integration 2

1 Right-click **Line Integration 1** and choose **Duplicate**.

2 In the **Settings** window for **Line Integration**, locate the **Data** section.

3 From the **Dataset** list, choose **Cut Line 2D 2**.

4 Locate the **Expressions** section. In the table, enter the following settings:

Expression	Unit	Description
`mf.Bz/I1`	nH	

5 Click next to **Evaluate**, then choose **Table 3 - Line Integration 1**.

Cut Line 2D 1

1 In the **Model Builder** window, under **Results>Datasets** click **Cut Line 2D 1**.

2 In the **Settings** window for **Cut Line 2D**, locate the **Data** section.

3 From the **Dataset** list, choose **Study 2/Solution 2 (sol2)**.

Cut Line 2D 2

1 In the **Model Builder** window, click **Cut Line 2D 2**.

2 In the **Settings** window for **Cut Line 2D**, locate the **Data** section.

3 From the **Dataset** list, choose **Study 2/Solution 2 (sol2)**.

Line Integration 1

1 In the **Model Builder** window, under **Results>Derived Values** click **Line Integration 1**.

2 In the **Settings** window for **Line Integration**, locate the **Expressions** section.

3 In the table, enter the following settings:

Expression	Unit	Description
20*mf.Bz/I2	nH	

4 Click ▾ next to **Evaluate**, then choose **Table 3 - Line Integration 1**.

Line Integration 2

1 In the **Model Builder** window, click **Line Integration 2**.

2 In the **Settings** window for **Line Integration**, locate the **Expressions** section.

3 In the table, enter the following settings:

Expression	Unit	Description
mf.Bz/I2	nH	

4 Click ▾ next to **Evaluate**, then choose **Table 3 - Line Integration 1**.

Experimentally, mutual inductance is measured by feeding an AC signal in the primary coil and measuring the voltage induced in the open-circuit secondary coil. This procedure can be simulated by using a Frequency Domain study step. Start by setting the AC feed on Coil 1 and the open circuit (zero current) condition on Coil 2.

GLOBAL DEFINITIONS

Parameters 1

1 In the **Model Builder** window, under **Global Definitions** click **Parameters 1**.

2 In the **Settings** window for **Parameters**, locate the **Parameters** section.

3 In the table, enter the following settings:

Name	Expression	Value	Description
I1	1[A]	1 A	Current, outer coil
I2	0[A]	0 A	Current, inner coil

ADD STUDY

1 In the **Home** toolbar, click **Add Study** to open the **Add Study** window.

2 Go to the **Add Study** window.

3 Find the **Studies** subsection. In the **Select Study** tree, select **General Studies> Frequency Domain**.

4 Click **Add Study** in the window toolbar.

5 In the **Home** toolbar, click **Add Study** to close the **Add Study** window.

STUDY 3

Step 1: Frequency Domain

1 In the **Settings** window for **Frequency Domain**, locate the **Study Settings** section.

2 In the **Frequencies** text field, type `1[kHz]`.

3 In the **Model Builder** window, click **Study 3**.

4 In the **Settings** window for **Study**, locate the **Study Settings** section.

5 Clear the **Generate default plots** check box.

6 In the **Home** toolbar, click **Compute**.

RESULTS

Study 3/Solution 3 (sol3)

Select the inner coil domains.

1 In the **Model Builder** window, under **Results>Datasets** click **Study 3/Solution 3 (sol3)**.

Selection

1 In the **Results** toolbar, click **Attributes** and choose **Selection**.

2 In the **Settings** window for **Selection**, locate the **Geometric Entity Selection** section.

3 From the **Geometric entity level** list, choose **Domain**.

4 Select Domains 4–23 only.

2D Plot Group 2

1 In the **Results** toolbar, click **2D Plot Group**.

2 In the **Settings** window for **2D Plot Group**, locate the **Data** section.

3 From the **Dataset** list, choose **Study 3/Solution 3 (sol3)**.

Surface 1

Right-click **2D Plot Group 2** and choose **Surface**.

Evaluate the mutual inductance using Equation 1.

Global Evaluation 3

1 In the **Results** toolbar, click **Global Evaluation**.

2 In the **Settings** window for **Global Evaluation**, locate the **Data** section.

3 From the **Dataset** list, choose **Study 3/Solution 3 (sol3)**.

4 Locate the **Expressions** section. In the table, enter the following settings:

Expression	Unit	Description
mf.VCoil_2/1[A]/mf.iomega	nH	

5 Click **Evaluate**.

Finally, simulate the system as if it were a transformer with a short-circuited secondary winding. Specify a voltage of 0 V for the **Coil** feature assigned to the inner coil to model the short-circuit condition.

MAGNETIC FIELDS (MF)

Coil 2

1 In the **Model Builder** window, under **Component 1 (comp1)>Magnetic Fields (mf)** click **Coil 2**.

2 In the **Settings** window for **Coil**, locate the **Coil** section.

3 From the **Coil excitation** list, choose **Voltage**.

4 In the V_{coil} text field, type 0.

ADD STUDY

1 In the **Home** toolbar, click **Add Study** to open the **Add Study** window.

2 Go to the **Add Study** window.

3 Find the **Studies** subsection. In the **Select Study** tree, select **General Studies> Frequency Domain**.

4 Click **Add Study** in the window toolbar.

5 In the **Home** toolbar, click **Add Study** to close the **Add Study** window.

STUDY 4

Step 1: Frequency Domain

1 In the **Settings** window for **Frequency Domain**, locate the **Study Settings** section.

2 In the **Frequencies** text field, type 1[kHz].

3 In the **Model Builder** window, click **Study 4**.

4 In the **Settings** window for **Study**, locate the **Study Settings** section.

5 Clear the **Generate default plots** check box.

6 In the **Home** toolbar, click **Compute**.

RESULTS

Study 4/Solution 4 (sol4)

Select the inner coil domains.

1 In the **Model Builder** window, under **Results>Datasets** click **Study 4/Solution 4 (sol4)**.

Selection

1 In the **Results** toolbar, click **Attributes** and choose **Selection**.

2 In the **Settings** window for **Selection**, locate the **Geometric Entity Selection** section.

3 From the **Geometric entity level** list, choose **Domain**.

4 Select Domains 4–23 only.

2D Plot Group 3

1 In the **Results** toolbar, click **2D Plot Group**.

2 In the **Settings** window for **2D Plot Group**, locate the **Data** section.

3 From the **Dataset** list, choose **Study 4/Solution 4 (sol4)**.

Surface 1

1 Right-click **2D Plot Group 3** and choose **Surface**.

2 In the **Settings** window for **Surface**, click **Replace Expression** in the upper-right corner of the **Expression** section. From the menu, choose **Component 1 (comp1)>Magnetic Fields> Currents and charge>Current density - A/m²>mf.Jphi - Current density, phi-component**.

3 In the **2D Plot Group 3** toolbar, click **Plot**.

4 Click the **Zoom Extents** button in the **Graphics** toolbar.

Compare the plot with Figure 4 describing the induced currents in the coil for the closed circuit case. Skin effect and proximity effects are clearly visible.

Evaluate the total induced current on the inner (secondary) coil. This quantity is related to static quantities, being in the simplest approximation $i\omega M/(R_2+i\omega L_2)$ times `1[A]`.

Global Evaluation 4

1 In the **Results** toolbar, click **Global Evaluation**.

2 In the **Settings** window for **Global Evaluation**, locate the **Data** section.

3 From the **Dataset** list, choose **Study 4/Solution 4 (sol4)**.

4 Locate the **Expressions** section. In the table, enter the following settings:

Expression	Unit	Description
`mf.ICoil_2`	`A`	`Coil current`
`-mf.iomega*withsol('sol1',mf.L_2_1)/ (withsol('sol2',mf.RCoil_2)+withsol('sol2', mf.LCoil_2)*mf.iomega)*mf.ICoil_1`	`A`	

5 Click **Evaluate**.