



Optical Ring Resonator Notch Filter

Introduction

The simplest optical ring resonator consists of a straight waveguide and a ring waveguide. The two waveguide cores are placed close to each other, so light couples from one waveguide to the other.

When the length of the ring waveguide is an integer number of wavelengths, the ring waveguide is resonant to the wavelength and the light power stored in the ring builds up.

The wave transmitted through the straight waveguide is the interference of the incident wave and the wave that couples over from the ring to the straight waveguide.

Schematically, you can think of the ring resonator as shown in [Figure 1](#) below. A part of the incident wave E_{i1} is transmitted in the straight waveguide, whereas a fraction of that field couples over to the ring. Similarly, some of the light in the ring couples over to the straight waveguide, whereas the rest of that wave continues around the ring waveguide.

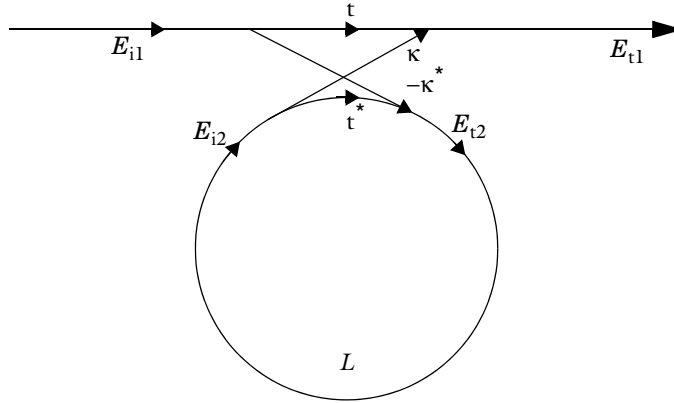


Figure 1: Schematic of an optical ring resonator, showing the incident fields E_{i1} and E_{i2} and the transmitted/coupled fields E_{t1} and E_{t2} . The transmission and coupling coefficients t and κ are also indicated, as well as the round-trip loss L .

The transmitted fields are related to the incident fields through the matrix-vector relation

$$\begin{bmatrix} E_{t1} \\ E_{t2} \end{bmatrix} = \begin{bmatrix} t & \kappa \\ -\kappa^* & t^* \end{bmatrix} \begin{bmatrix} E_{i1} \\ E_{i2} \end{bmatrix}. \quad (1)$$

The matrix elements defined above assure that the total input power equals the total output power,

$$|E_{t1}|^2 + |E_{t2}|^2 = |E_{i1}|^2 + |E_{i2}|^2, \quad (2)$$

by assuming that the transmission and coupling coefficients are related by

$$|t|^2 + |\kappa|^2 = 1. \quad (3)$$

Furthermore, as the wave propagates around the ring waveguide, one gets the relation

$$E_{i2} = E_{t2}L \exp(-j\phi), \quad (4)$$

where L is the loss coefficient for the propagation around the ring and ϕ is the accumulated phase.

Combining Equation 1, Equation 3 and Equation 4, the transmitted field can be written

$$E_{t1} = \frac{|t| - L \exp(-j(\phi - \phi_t))}{1 - |t|L \exp(-j(\phi - \phi_t))} E_{i1} e^{-j\phi_t}. \quad (5)$$

Here the transmission coefficient is separated into the transmission loss $|t|$ and the corresponding phase ϕ_t ,

$$t = |t|e^{-j\phi_t}. \quad (6)$$

Notice that on resonance, when $\phi - \phi_t$ is an integer multiple of 2π , and when $|t| = L$, the transmitted field is zero. The condition $|t| = L$ is called critical coupling. Thus, when the coupler transmission loss balances the loss for the wave propagating around the ring waveguide, one gets the optimum condition for a bandstop filter, a notch filter.

Model Definition

This application is set up using the Electromagnetic Waves, Beam Envelopes interface, to handle the propagation over distances that are many wavelengths long. Since the wave propagates in essentially one direction along the straight waveguide and along the waveguide ring, the unidirectional formulation is used. This assumes that the electric field for the wave can be written as

$$\mathbf{E} = \mathbf{E}_1 \exp(-j\phi), \quad (7)$$

where \mathbf{E}_1 is a slowly varying field envelope function and ϕ is an approximation of the propagation phase for the wave. The definitions used for the phase in the straight and ring waveguide are shown in [Table 1](#), [Table 2](#) and [Table 3](#).

TABLE 1: PHASE DEFINITION IN STRAIGHT WAVEGUIDE DOMAINS.

NAME	EXPRESSION	UNIT	DESCRIPTION
phi	$ewbe.beta_1*y$	rad	Phase

TABLE 2: PHASE DEFINITION IN RING WAVEGUIDE - LEFT DOMAIN.

NAME	EXPRESSION	UNIT	DESCRIPTION
phi	$ewbe.beta_1*r0*atan2(y, -(x-r0-dx))$	rad	Phase

TABLE 3: PHASE DEFINITION IN RING WAVEGUIDE - RIGHT DOMAIN.

NAME	EXPRESSION	UNIT	DESCRIPTION
phi	$ewbe.beta_1*r0*atan2(-y, (x-r0-dx))$	rad	Phase

The parameters $r0$ and dx correspond, respectively, to the curvature radius of the ring waveguide and to the separation between the straight and ring waveguide cores. The phase approximation defined in the tables above is discontinuous at the boundary between the straight waveguide and the ring waveguide as well as at the boundary between the left and the right ring waveguide domains. To handle this phase discontinuity and thereby the discontinuity in the field envelope, \mathbf{E}_1 , a Field Continuity boundary condition is used at the aforementioned boundaries. The Field Continuity boundary condition ensures that

the tangential components of the electric and the magnetic fields are continuous at the boundary, despite the phase jump.

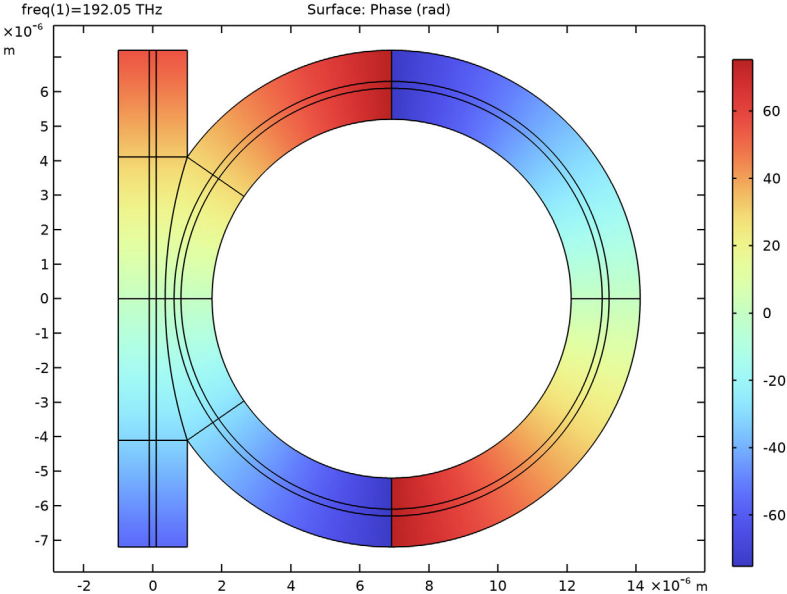


Figure 2: Plot of the predefined phase approximation. Notice the phase jump at the boundary between the left and right part of the ring waveguide. The discontinuity at the boundary between the straight and the ring waveguide is not visible at this scale.

Results and Discussion

Figure 3 below shows the transmittance spectrum for the optical ring resonator.

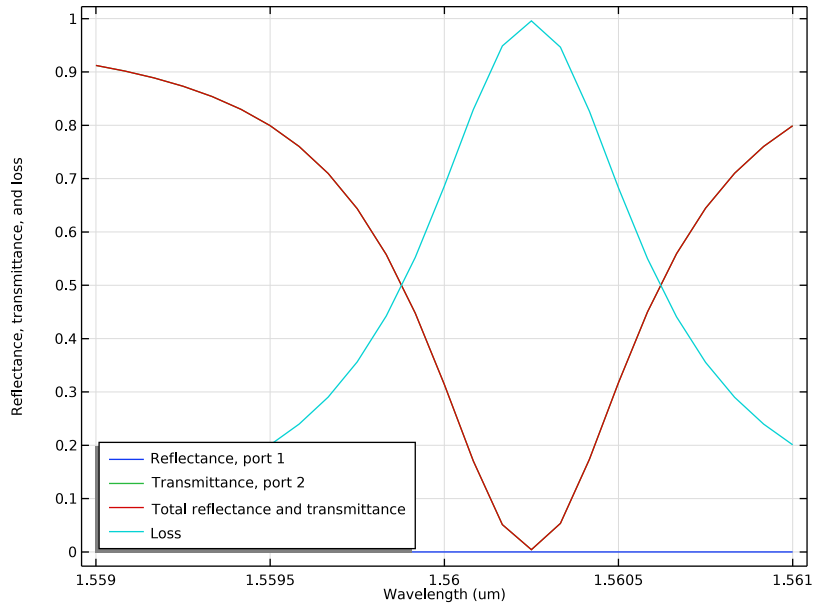


Figure 3: Transmittance spectrum for the optical ring resonator.

and Figure 4 shows a field plot for a resonant wavelength. Notice that the field in the straight waveguide and the field incoming from the ring are out of phase when they

interfere in the coupler. Thereby the outgoing field in the straight waveguide is almost zero.

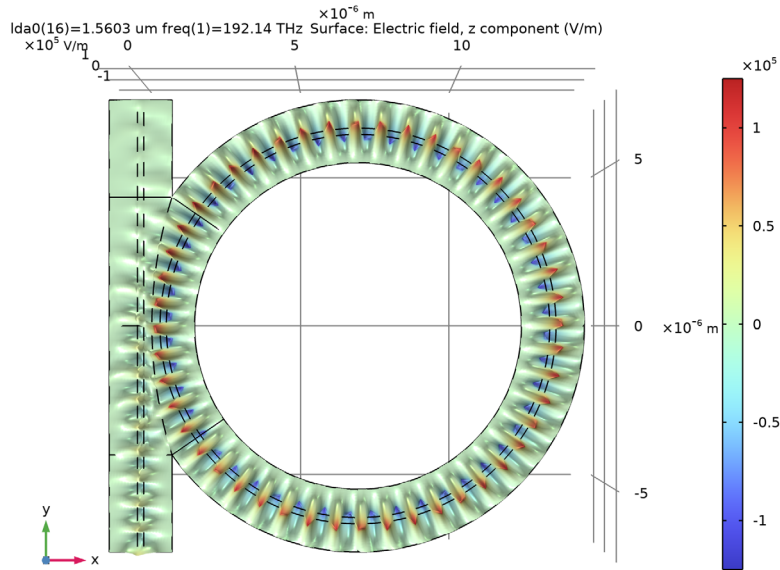


Figure 4: The out-of-plane component of the electric field for the resonant wavelength.

Notes About the COMSOL Implementation

This model geometry is easily set up by importing a geometry part from the COMSOL Part Libraries. The slab waveguide coupling between a straight and a ring waveguide section, with core embedded in a cladding domain, is available in the Wave Optics Module under Slab Waveguides.

Predefined geometry parts can be quickly modified by changing the default input parameters. Moreover, geometry parts provide targeted selections of domains and boundaries that greatly simplify the model building. As demonstrated in this model, these built-in selections are useful when adding materials, physics features and mesh sequences.


Application Library path: Wave_Optics_Module/Waveguides_and_Couplers/optical_ring_resonator

Modeling Instructions




First add the physics interface and the study sequence.

From the **File** menu, choose **New**.

NEW



In the **New** window, click  **Model Wizard**.

MODEL WIZARD

- 1 In the **Model Wizard** window, click  **2D**.
- 2 In the **Select Physics** tree, select **Optics>Wave Optics>Electromagnetic Waves, Beam Envelopes (ewbe)**.
- 3 Click **Add**.
- 4 Click  **Study**.
- 5 In the **Select Study** tree, select **Preset Studies for Selected Physics Interfaces>Boundary Mode Analysis**.
- 6 Click  **Done**.


The geometry for the optical ring resonator is straightforward to set up. Load the Slab Waveguide Straight-to-Ring Coupler geometry part from the COMSOL Part Libraries and then modify the input parameters in order to build the desired geometry.

PART LIBRARIES

- 1 In the **Home** toolbar, click  **Windows** and choose **Part Libraries**.
- 2 In the **Part Libraries** window, select **Wave Optics Module>Slab Waveguides>slab_waveguide_straight_to_ring_coupler** in the tree.
- 3 Click  **Add to Geometry**.

GEOMETRY I


Slab Waveguide Straight-to-Ring Coupler I (pi1)

- 1 In the **Model Builder** window, under **Component 1 (comp1)>Geometry 1** click **Slab Waveguide Straight-to-Ring Coupler I (pi1)**.
- 2 In the **Settings** window for **Part Instance**, click  **Build All Objects**.

GLOBAL DEFINITIONS

Start by loading a few more parameters required for building the physics and defining the materials.

Parameters 1

- 1 In the **Model Builder** window, under **Global Definitions** click **Parameters 1**.
- 2 In the **Settings** window for **Parameters**, locate the **Parameters** section.
- 3 Click  **Load from File**.
- 4 Browse to the model's Application Libraries folder and double-click the file `optical_ring_resonator_parameters.txt`.



GEOMETRY 1

Slab Waveguide Straight-to-Ring Coupler 1 (pi1)

- 1 In the **Model Builder** window, under **Component 1 (comp1)>Geometry 1** click **Slab Waveguide Straight-to-Ring Coupler 1 (pi1)**.
- 2 In the **Settings** window for **Part Instance**, locate the **Input Parameters** section.
- 3 In the table, enter the following settings:

Name	Expression	Value	Description
core_width	w_core	2E-7 m	Core width
cladding_width	w_clad	2E-6 m	Cladding width
element_length	2*r0+w_clad	1.44E-5 m	Element length
coupler_core_separation	dx	7.1666E-7 m	Core separation in coupler region
ring_radius	r0	6.2E-6 m	Ring radius

and leave the rest of the input parameters unchanged.

- 4 Locate the **Position and Orientation of Output** section. In the **y-displacement** text field, type `-r0-w_clad/2`.
- 5 Click  **Build All Objects**.
- 6 Click the  **Zoom Extents** button in the **Graphics** toolbar.
Choose to keep those domain and boundary selections that will be useful later when adding materials, boundary conditions and the mesh sequence.
- 7 Click to expand the **Domain Selections** section. In the table, enter the following settings:

Name	Keep	Physics	Contribute to
All		√	None
Core	√	√	None
Cladding	√	√	None

Name	Keep	Physics	Contribute to
Ring domain 1	√	√	None
Ring domain 2	√	√	None
Straight domain	√	√	None

8 Click to expand the **Boundary Selections** section. In the table, enter the following settings:

Name	Keep	Physics	Contribute to
Exterior		√	None
Port 1	√	√	None
Port 1 core		√	None
Port 1 cladding		√	None
Port 2	√	√	None
Port 2 core		√	None
Port 2 cladding		√	None
Transverse perimeter	√	√	None
Edge mesh	√	√	None
Field continuity	√	√	None

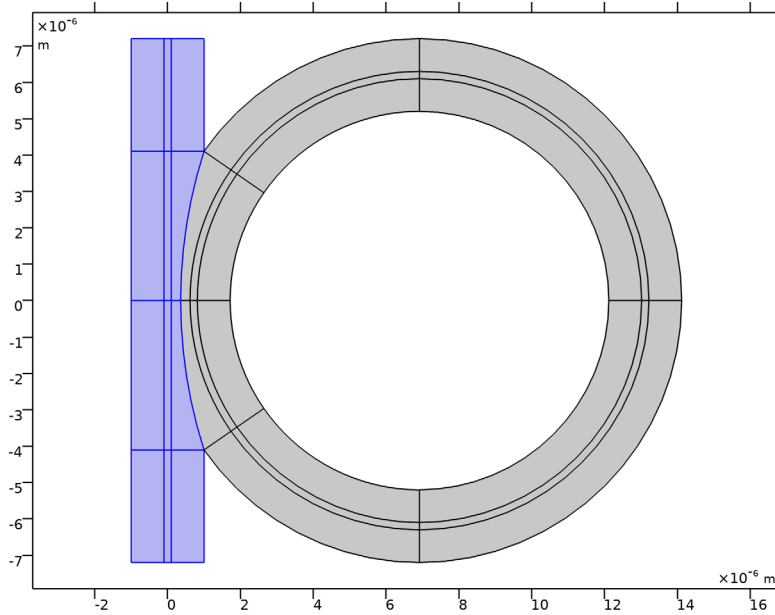
DEFINITIONS

Now add the definitions for the phase in the waveguide domains.

Phase, straight waveguide

- 1 In the **Model Builder** window, under **Component 1 (comp1)** right-click **Definitions** and choose **Variables**.
- 2 In the **Settings** window for **Variables**, type Phase, straight waveguide in the **Label** text field.
- 3 Locate the **Geometric Entity Selection** section. From the **Geometric entity level** list, choose **Domain**.

- 4 From the **Selection** list, choose **Straight domain (Slab Waveguide Straight-to-Ring Coupler 1)**.



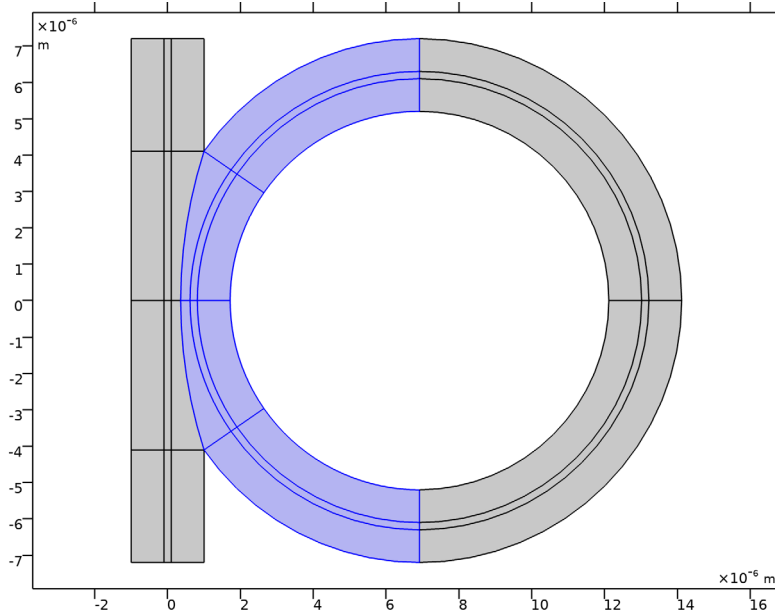
- 5 Locate the **Variables** section. In the table, enter the following settings:

Name	Expression	Unit	Description
phi	ewbe.beta_1*y		

Phase, ring waveguide 1

- 1 In the **Model Builder** window, right-click **Definitions** and choose **Variables**.
- 2 In the **Settings** window for **Variables**, type Phase, ring waveguide 1 in the **Label** text field.
- 3 Locate the **Geometric Entity Selection** section. From the **Geometric entity level** list, choose **Domain**.

- 4 From the **Selection** list, choose **Ring domain I (Slab Waveguide Straight-to-Ring Coupler I)**.



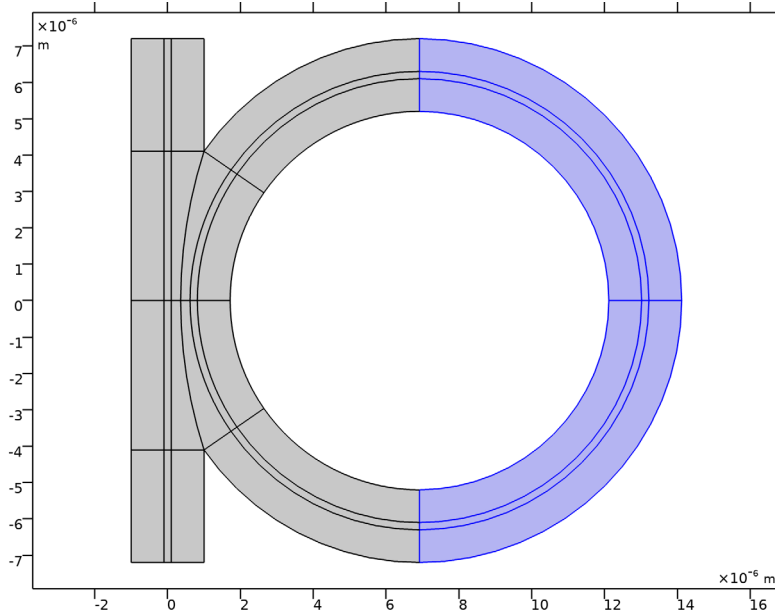
- 5 Locate the **Variables** section. In the table, enter the following settings:

Name	Expression	Unit	Description
phi	$\text{ewbe}.\text{beta}_1 * r_0 * \text{atan2}(y, -(x - r_0 - dx))$		

Phase, ring waveguide 2

- 1 Right-click **Definitions** and choose **Variables**.
- 2 In the **Settings** window for **Variables**, type Phase, ring waveguide 2 in the **Label** text field.
- 3 Locate the **Geometric Entity Selection** section. From the **Geometric entity level** list, choose **Domain**.

- 4 From the **Selection** list, choose **Ring domain 2 (Slab Waveguide Straight-to-Ring Coupler 1)**.



- 5 Locate the **Variables** section. In the table, enter the following settings:

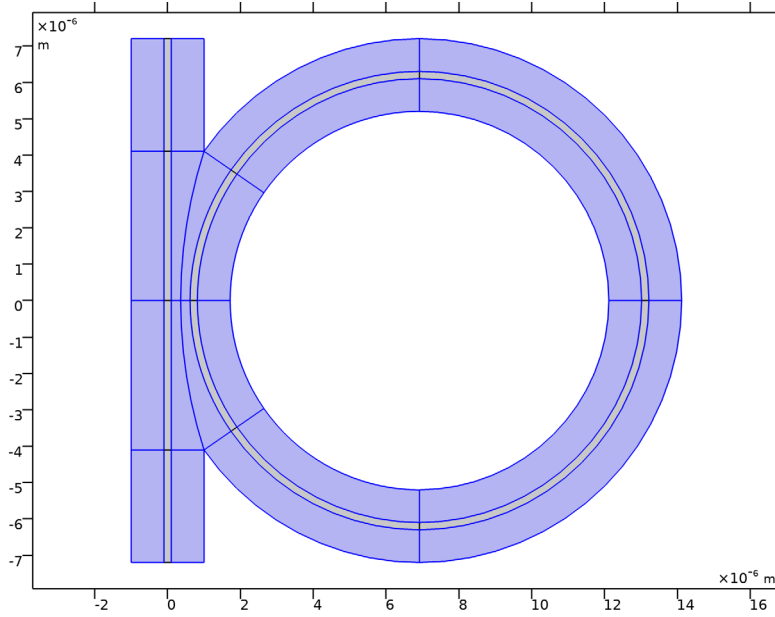
Name	Expression	Unit	Description
phi	$\text{ewbe}.\text{beta}_1 * r_0 * \text{atan2}(-y, (x - r_0 - dx))$		

MATERIALS

Cladding

- 1 In the **Model Builder** window, under **Component 1 (comp1)** right-click **Materials** and choose **Blank Material**.
- 2 In the **Settings** window for **Material**, type Cladding in the **Label** text field.

- 3 Locate the **Geometric Entity Selection** section. From the **Selection** list, choose **Cladding (Slab Waveguide Straight-to-Ring Coupler 1)**.



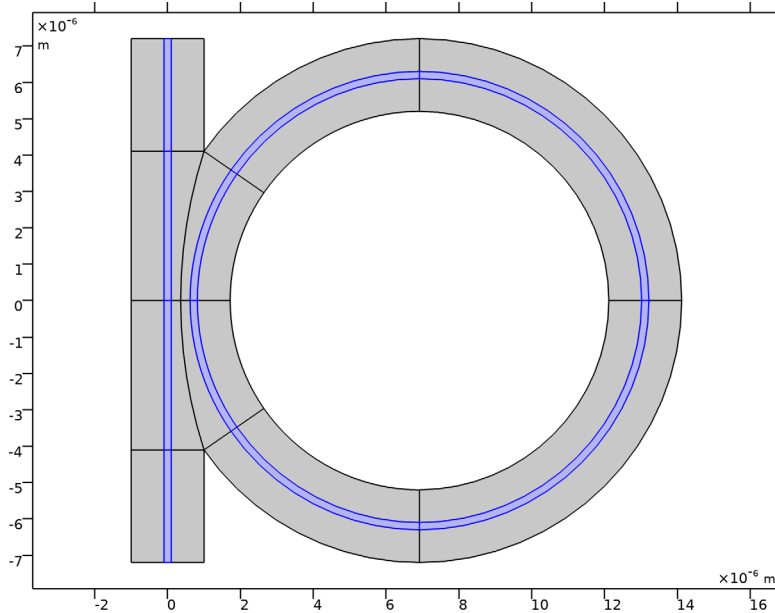
- 4 Locate the **Material Contents** section. In the table, enter the following settings:

Property	Variable	Value	Unit	Property group
Refractive index, real part	n_{iso} ; $n_{ii} = n_{iso}$, $n_{ij} = 0$	n_{clad}		Refractive index

Core

- 1 Right-click **Materials** and choose **Blank Material**.
- 2 In the **Settings** window for **Material**, type Core in the **Label** text field.

- 3 Locate the **Geometric Entity Selection** section. From the **Selection** list, choose **Core (Slab Waveguide Straight-to-Ring Coupler 1)**.




- 4 Locate the **Material Contents** section. In the table, enter the following settings:

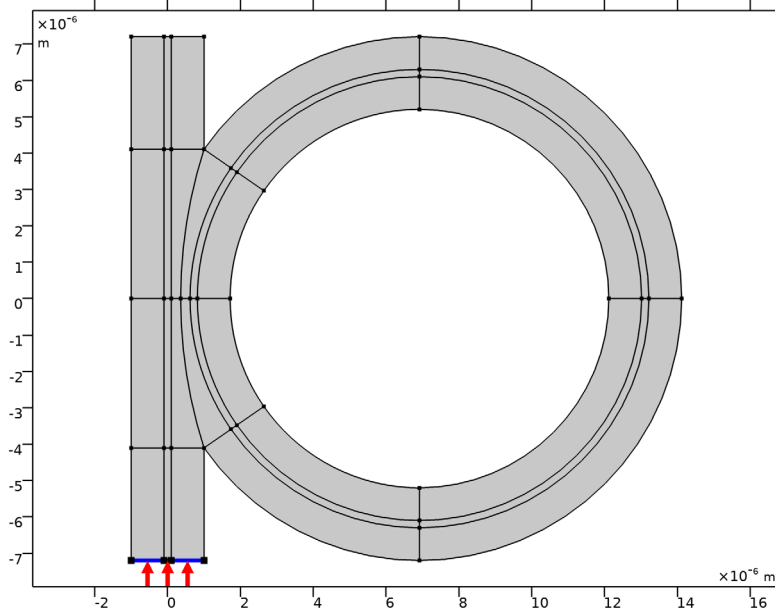
Property	Variable	Value	Unit	Property group
Refractive index, real part	n_{iso} ; $n_{ii} = n_{iso}$, $n_{ij} = 0$	n_{core}	1	Refractive index

ELECTROMAGNETIC WAVES, BEAM ENVELOPES (EWBE)

- 1 In the **Model Builder** window, under **Component 1 (comp1)** click **Electromagnetic Waves, Beam Envelopes (ewbe)**.
- 2 In the **Settings** window for **Electromagnetic Waves, Beam Envelopes**, locate the **Components** section.
- 3 From the **Electric field components solved for** list, choose **Out-of-plane vector**.
- 4 Locate the **Wave Vectors** section. From the **Number of directions** list, choose **Unidirectional**.
- 5 From the **Type of phase specification** list, choose **User defined**.
- 6 In the ϕ_1 text field, type ϕ_1 .

Port 1

- 1 In the **Physics** toolbar, click  **Boundaries** and choose **Port**.
- 2 In the **Settings** window for **Port**, locate the **Boundary Selection** section.
- 3 From the **Selection** list, choose **Port 1 (Slab Waveguide Straight-to-Ring Coupler I)**.

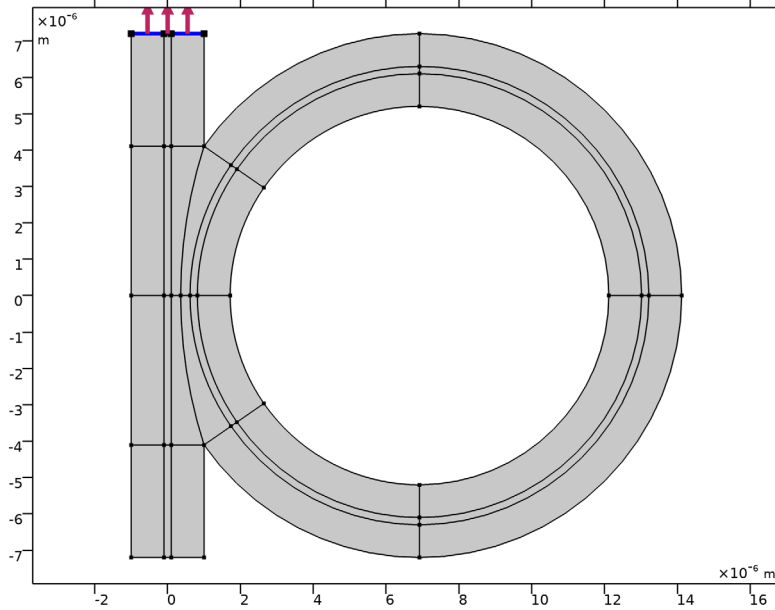


- 4 Locate the **Port Properties** section. From the **Type of port** list, choose **Numeric**.
For the first port, wave excitation is **on** by default.


Port 2

- 1 Right-click **Port 1** and choose **Duplicate**.
- 2 In the **Settings** window for **Port**, locate the **Boundary Selection** section.
- 3 From the **Selection** list, choose **Port 2 (Slab Waveguide Straight-to-Ring Coupler I)**.

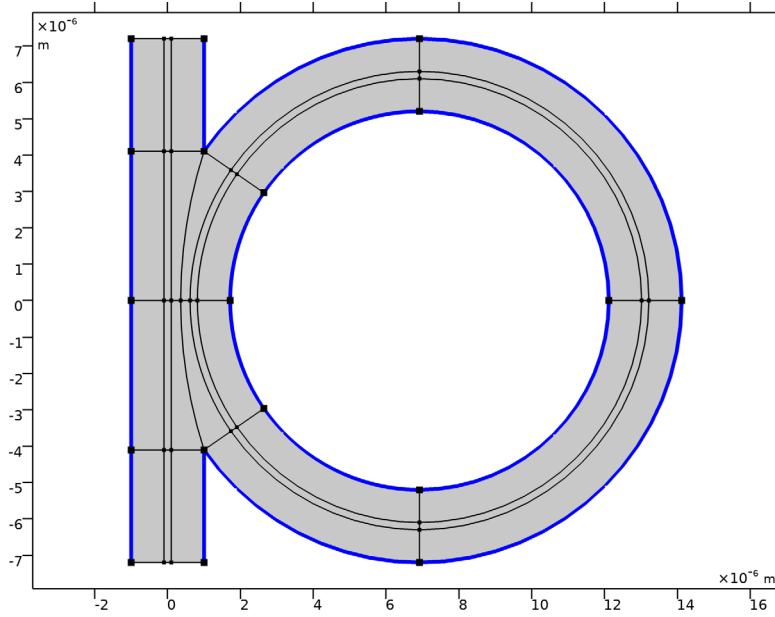
4 Locate the **Port Properties** section. From the **Wave excitation at this port** list, choose **Off**.




Scattering Boundary Condition 1


- 1 In the **Physics** toolbar, click  **Boundaries** and choose **Scattering Boundary Condition**.
- 2 In the **Settings** window for **Scattering Boundary Condition**, locate the **Boundary Selection** section.

- 3 From the **Selection** list, choose **Transverse perimeter (Slab Waveguide Straight-to-Ring Coupler 1)**.

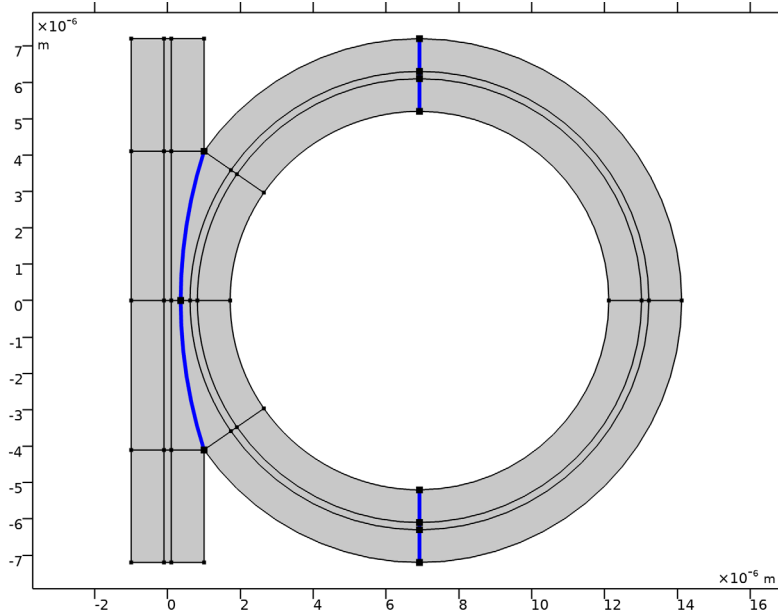


- 4 Click the  **Show More Options** button in the **Model Builder** toolbar.
- 5 In the **Show More Options** dialog box, in the tree, select the check box for the node **Physics>Advanced Physics Options**.
- 6 Click **OK**.

Field Continuity I

- 1 In the **Physics** toolbar, click  **Boundaries** and choose **Field Continuity**.
- 2 In the **Settings** window for **Field Continuity**, locate the **Boundary Selection** section.


- 3 From the **Selection** list, choose **Field continuity (Slab Waveguide Straight-to-Ring Coupler I)**.



MESH I

For this model a edge mesh and a mapped mesh will be used.


Edge I

- 1 In the **Mesh** toolbar, click  **Edge**.
- 2 In the **Settings** window for **Edge**, locate the **Boundary Selection** section.
- 3 From the **Selection** list, choose **Edge mesh (Slab Waveguide Straight-to-Ring Coupler I)**.

Distribution I


- 1 Right-click **Edge I** and choose **Distribution**.
- 2 In the **Settings** window for **Distribution**, locate the **Distribution** section.
- 3 In the **Number of elements** text field, type 3.

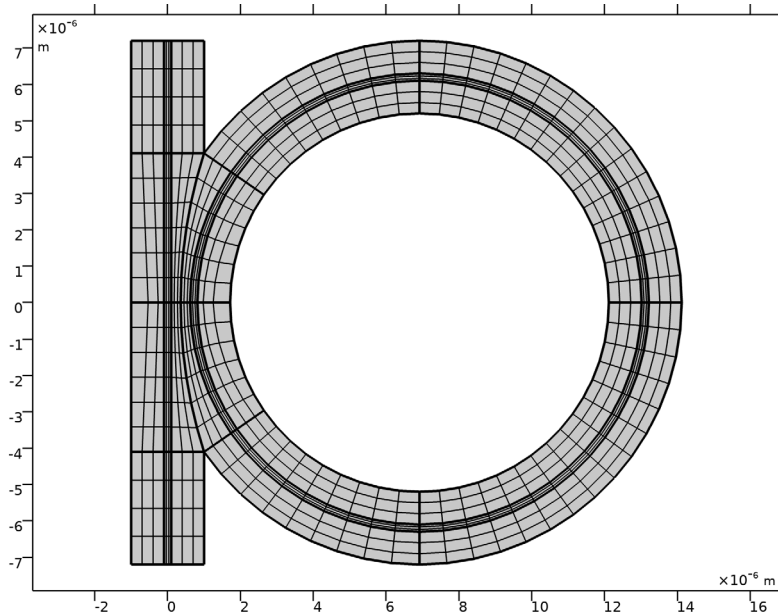
Mapped I

In the **Mesh** toolbar, click  **Mapped**.

Size

- 1 In the **Model Builder** window, click **Size**.

- 2 In the **Settings** window for **Size**, locate the **Element Size** section.
- 3 Click the **Custom** button.
- 4 Locate the **Element Size Parameters** section. In the **Maximum element size** text field, type $w10/2$.
- 5 Click  **Build All**.



STUDY 1

Step 1: Boundary Mode Analysis

- 1 In the **Model Builder** window, under **Study 1** click **Step 1: Boundary Mode Analysis**.
- 2 In the **Settings** window for **Boundary Mode Analysis**, locate the **Study Settings** section.
- 3 In the **Mode analysis frequency** text field, type $f0$.
- 4 Select the **Search for modes around** check box.
- 5 In the associated text field, type n_core .




Step 3: Boundary Mode Analysis 1

- 1 Right-click **Study 1 > Step 1: Boundary Mode Analysis** and choose **Duplicate**.
- 2 In the **Settings** window for **Boundary Mode Analysis**, locate the **Study Settings** section.
- 3 In the **Port name** text field, type 2.

Step 2: Frequency Domain

- 1 In the **Model Builder** window, click **Step 2: Frequency Domain**.
- 2 In the **Settings** window for **Frequency Domain**, locate the **Study Settings** section.
- 3 In the **Frequencies** text field, type f_0 .
- 4 Right-click **Study 1 > Step 2: Frequency Domain** and choose **Move Down**.

Parametric Sweep

- 1 In the **Study** toolbar, click  **Parametric Sweep**.
- 2 In the **Settings** window for **Parametric Sweep**, locate the **Study Settings** section.
- 3 Click  **Add**.
- 4 From the list in the **Parameter name** column, choose **lda0 (Wavelength)**.
- 5 Click  **Range**.
- 6 In the **Range** dialog box, choose **Number of values** from the **Entry method** list.
- 7 In the **Start** text field, type $1.559[\mu\text{m}]$.
- 8 In the **Stop** text field, type $1.561[\mu\text{m}]$.
- 9 In the **Number of values** text field, type 25.
- 10 Click **Replace**.
- 11 In the **Settings** window for **Parametric Sweep**, locate the **Study Settings** section.
- 12 In the table, enter the following settings:

Parameter name	Parameter value list	Parameter unit
lda0 (Wavelength)	$\text{range}(1.559[\mu\text{m}], (1.561[\mu\text{m}] - 1.559[\mu\text{m}])) / 24, 1.561[\mu\text{m}]$	μm

In practice just replace the Parameter unit with μm .

- 13 In the **Study** toolbar, click  **Compute**.

RESULTS




Electric Field

- 1 In the **Model Builder** window, expand the **Electric Field (ewbe)** node, then click **Electric Field**.
- 2 In the **Settings** window for **Surface**, locate the **Expression** section.
- 3 In the **Expression** text field, type ewbe.Ez .

Height Expression 1

- Right-click **Electric Field** and choose **Height Expression**.

Electric Field (ewbe)

- 1 In the **Settings** window for **2D Plot Group**, locate the **Data** section.
- 2 From the **Parameter value (Ida0 (um))** list, choose **1.5603 (1)**.
- 3 In the **Electric Field (ewbe)** toolbar, click  **Plot**.
- 4 Click the  **Go to XY View** button in the **Graphics** toolbar.
- 5 Click the  **Zoom Extents** button in the **Graphics** toolbar. The plot should now look like [Figure 4](#).

Reflectance, Transmittance, and Loss (ewbe)

For the optical ring resonator, where there is loss due to the propagation in the ring and not due to material absorption, it is more appropriate to use the term loss than absorbance. Thus, replace absorbance with loss in the node label, y-axis label and the legend.


- 1 In the **Model Builder** window, under **Results** click **Reflectance, Transmittance, and Absorbance (ewbe)**.
- 2 In the **Settings** window for **ID Plot Group**, type Reflectance, Transmittance, and Loss (ewbe) in the **Label** text field.
- 3 Locate the **Plot Settings** section. In the **y-axis label** text field, type Reflectance, transmittance, and loss.
- 4 Locate the **Legend** section. From the **Position** list, choose **Lower left**.

Global I

- 1 In the **Model Builder** window, expand the **Reflectance, Transmittance, and Loss (ewbe)** node, then click **Global I**.
- 2 In the **Settings** window for **Global**, locate the **y-Axis Data** section.
- 3 In the table, enter the following settings:


Expression	Unit	Description
ewbe.Atotal	1	Loss

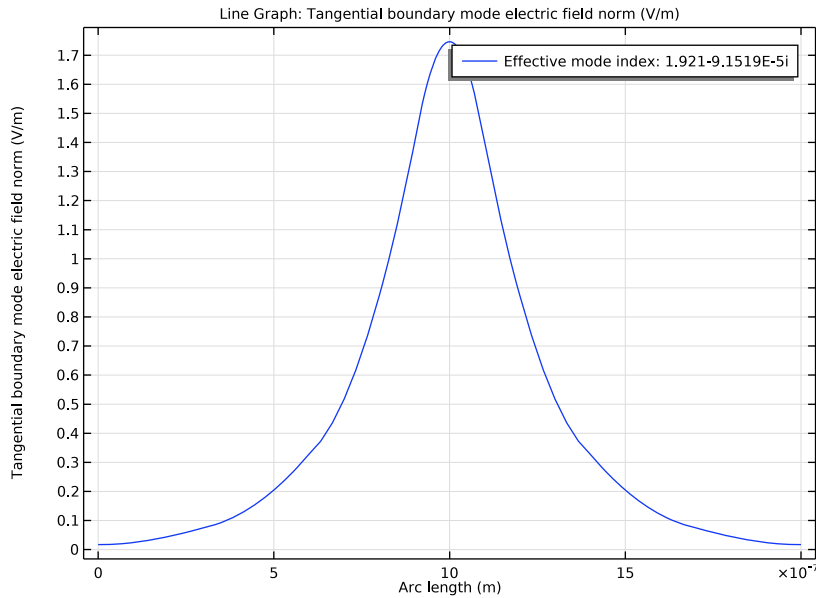
In practice just replace Absorbance with Loss in the Description field. Leave unmodified the other expressions in the same table.

- 4 In the **Reflectance, Transmittance, and Loss (ewbe)** toolbar, click  **Plot**. The plot should now look like [Figure 3](#).

Finally inspect the mode field plot resulting from the boundary mode analysis performed for each port. Since the resulting graphs overlap each other, it is enough to plot only one of them. For example, pick the first one.

Electric Mode Field, Port 1 (ewbe)

- 1 In the **Model Builder** window, under **Results** click **Electric Mode Field, Port 1 (ewbe)**.
- 2 In the **Settings** window for **ID Plot Group**, locate the **Data** section.
- 3 From the **Parameter selection (Ida0)** list, choose **First**.
- 4 In the **Electric Mode Field, Port 1 (ewbe)** toolbar, click  **Plot**.



Electric Mode Field, Port 2 (ewbe)

- 1 In the **Model Builder** window, click **Electric Mode Field, Port 2 (ewbe)**.
- 2 In the **Settings** window for **ID Plot Group**, locate the **Data** section.
- 3 From the **Parameter selection (Ida0)** list, choose **First**.

4 In the **Electric Mode Field, Port 2 (ewbe)** toolbar, click  **Plot**.

