Created in COMSOL Multiphysics 6.0

Gaussian Beam Incident at the Brewster Angle

Introduction

For a plane wave incident at an interface between two different media, there exists an angle of incidence for which there is no reflectance if the incident wave is polarized in the plane of incidence. The angle, for which the reflectance is zero, is called *the Brewster angle*.

Figure 1 shows an incident wave being reflected and refracted at the interface between the two media. The polarization component polarized in the plane of incident (the plane spanned by the wave vector of the incident wave and the normal to the interface) is not reflected. This polarization component is called the p-polarization.

The polarization component orthogonal to the plane of incidence (the s-polarization) is both reflected and refracted.

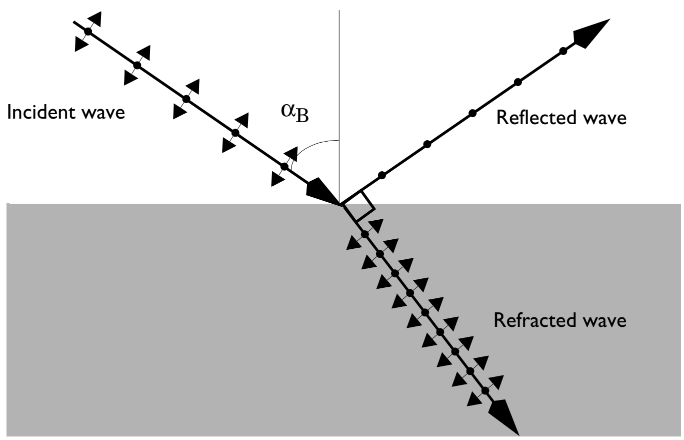


Figure 1: The figure shows the incident, reflected and refracted waves. At the Brewster angle α_B *the wave polarized in the plane of incidence is only refracted and not reflected.*

At the Brewster angle, the incident p-polarized wave creates a polarization in the second medium (where the refracted wave is propagating) with the components in the propagation direction of the reflected wave. Because this is a longitudinal polarization for the reflected wave, and not a transverse polarization, it is clear that this polarization cannot excite a reflected wave.

Referring to the angles defined in Figure 1, write Snell's law as

$$n_1 \sin\alpha_B = n_2 \sin\left(\Pi - \alpha_B - \frac{\Pi}{2}\right) = n_2 \cos\alpha_B \tag{1}$$

where n_1 and n_2 are the refractive indices above and below the interface, respectively.

Equation 1 results in the Brewster angle definition

$$\tan\alpha_B = \frac{n_2}{n_1}$$

From the Fresnel Equations Application Libraries example, the reflectance for the s-polarization at the Brewster angle is given by

$$R_s = \left|\frac{n_1 \cos\theta_\mathrm{i} - n_2 \cos\theta_\mathrm{t}}{n_1 \cos\theta_\mathrm{i} + n_2 \cos\theta_\mathrm{t}}\right|^2 = \left|\frac{n_1 \cos\alpha_B - n_2 \sin\alpha_B}{n_1 \cos\alpha_B + n_2 \sin\alpha_B}\right|^2 = \left|\frac{n_1^2 - n_2^2}{n_1^2 + n_2^2}\right|^2 \tag{2}$$

This model does not use plane waves, but Gaussian beams (see for instance the Second Harmonic Generation of a Gaussian Beam Application Libraries model for a discussion about Gaussian beams). However, because the spot size for the beam is much larger than the wavelength, the plane wave relations above are good approximations also for the Gaussian beams.

Model Definition

This model demonstrates the use of the User defined phase specification, when using the Electromagnetic Waves, Beam Envelopes interface. Secondly, the model shows how the Matched Boundary Condition feature can be used to absorb waves that propagate toward a boundary in a direction different from the boundary's normal direction. Here, a Scattering Boundary Condition feature is not an option, as that feature only absorbs waves propagating at or close to the normal direction to the boundary normal. A second alternative would be to use a Perfectly Matched Layer (PML) domain. However, in that case, extra degrees of freedom would have to be included for the PML domain. Thus, the Matched Boundary Condition feature is the best feature to use for absorbing beams propagating in directions that are not in the normal direction to the boundary.

The User defined phase is specified by defining parameters for the wave vectors of the forward- and backward-propagating waves in the two different media and then defining phase variables, phi1 and phi2, for the two waves in the two different media, as shown in Table 1.

TABLE 1: PHASE VARIABLE DEFINITION.

VARIABLE	EXPRESSION
phi1	k1x_NN*x+k1y_NN*y
phi2	k2x_NN*x+k2y_NN*y

Here NN should be replaced by air and glass, respectively, for the two media.

As the geometry is centered at the origin, the phase variables are continuous across the boundary between the two media. A continuous phase across boundaries is a requirement when specifying the phase.

For this simple geometry, it would also have been possible to specify the phase using wave vectors that are different for the two media. However, in more complex geometries, it is not possible to use this approach as the phase expressions in this case are not continuous across all boundaries.

Results and Discussion

First, the results are computed for s-polarization, where the polarization is orthogonal to the plane of incidence (out-of-plane polarization). As shown in Figure 2, there are both a refracted and a reflected beam. The incident beam and the reflected beam form an inference pattern. Thanks to the fine mesh used in the model, the interference pattern is resolved. Equation 2 is also used to verify that the reflectance is correct. It should be close to 14.8%.

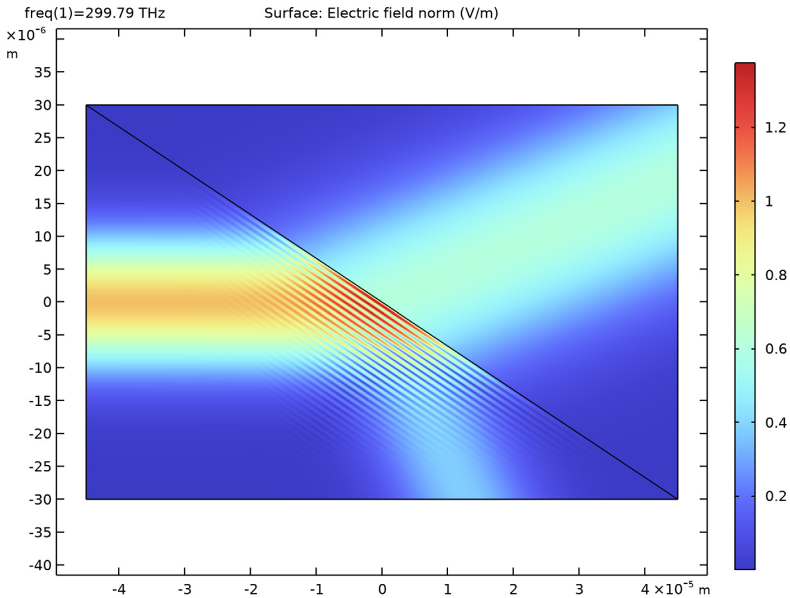


Figure 2: The incident, transmitted (refracted) and reflected Gaussian beams for s-polarization (out-of-plane polarization).

[Figure 3](#) shows the results for p-polarization (in-plane polarization). As expected, when the beam is incident at the Brewster angle, there is no reflected beam, but only a refracted (transmitted) beam.

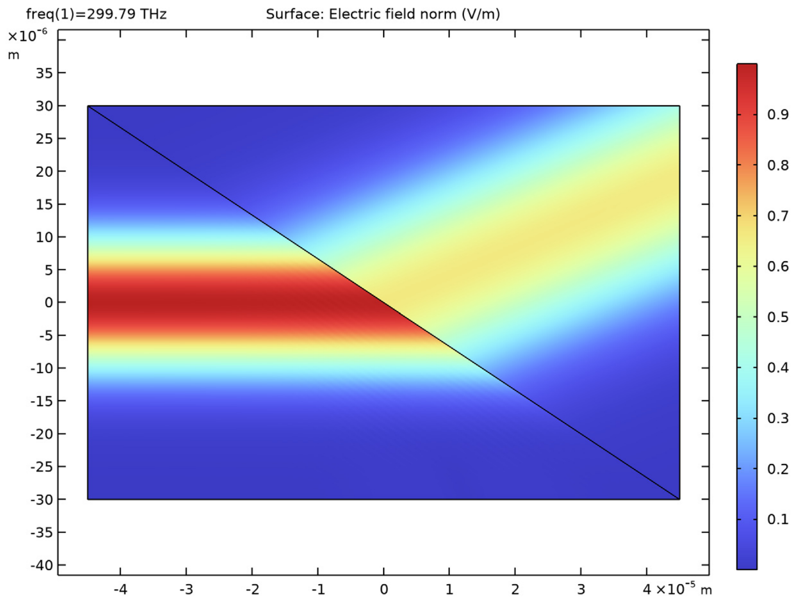


Figure 3: The incident and transmitted Gaussian beams for p-polarization (in-plane polarization). For this polarization, with Brewster angle incidence, the reflected beam is gone.

Application Library path: `Wave_Optics_Module/Optical_Scattering/brewster_interface`

Modeling Instructions

From the **File** menu, choose **New**.

NEW

In the **New** window, click **Model Wizard**.

MODEL WIZARD

1 In the **Model Wizard** window, click **2D**.

2 In the **Select Physics** tree, select **Optics>Wave Optics>Electromagnetic Waves, Beam Envelopes (ewbe)**.

3 Click **Add**.

4 Click **Study**.

5 In the **Select Study** tree, select **General Studies>Frequency Domain**.

6 Click **Done**.

GLOBAL DEFINITIONS

Parameters 1

The parameters for the model will be read from a file.

1 In the **Model Builder** window, under **Global Definitions** click **Parameters 1**.

2 In the **Settings** window for **Parameters**, locate the **Parameters** section.

3 Click **Load from File**.

4 Browse to the model's Application Libraries folder and double-click the file `brewster_interface.txt`.

GEOMETRY 1

Define the geometry as a rectangle with a diagonal boundary.

Rectangle 1 (r1)

1 In the **Geometry** toolbar, click **Rectangle**.

2 In the **Settings** window for **Rectangle**, locate the **Size and Shape** section.

3 In the **Width** text field, type `b`.

4 In the **Height** text field, type `a`.

5 Locate the **Position** section. From the **Base** list, choose **Center**.

Polygon 1 (pol1)

1 In the **Geometry** toolbar, click **Polygon**.

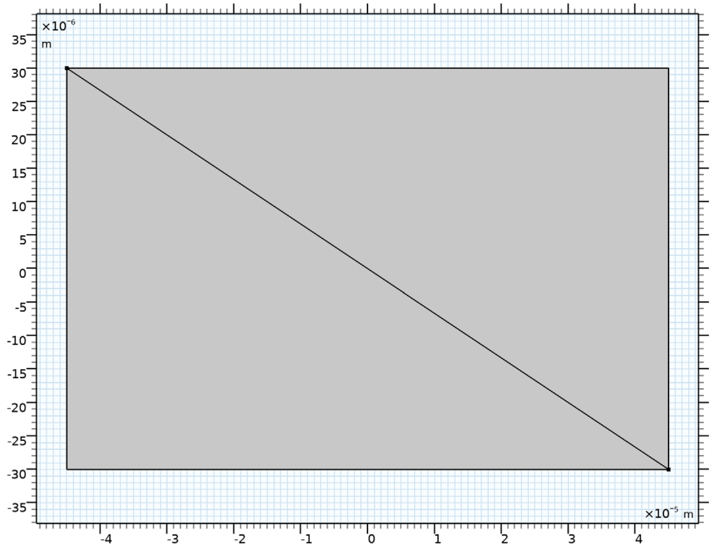
2 In the **Settings** window for **Polygon**, locate the **Coordinates** section.

3 In the table, enter the following settings:

x (m)	**y (m)**
-b/2	a/2
b/2	-a/2

4 Click **Build All Objects**.

5 Click the **Zoom Extents** button in the **Graphics** toolbar.



ADD MATERIAL

1 In the **Home** toolbar, click **Add Material** to open the **Add Material** window.

The leftmost part consists of air and the rightmost part will be glass.

2 Go to the **Add Material** window.

3 In the tree, select **Built-in>Air**.

4 Click **Add to Component** in the window toolbar.

5 In the **Home** toolbar, click **Add Material** to close the **Add Material** window.

MATERIALS

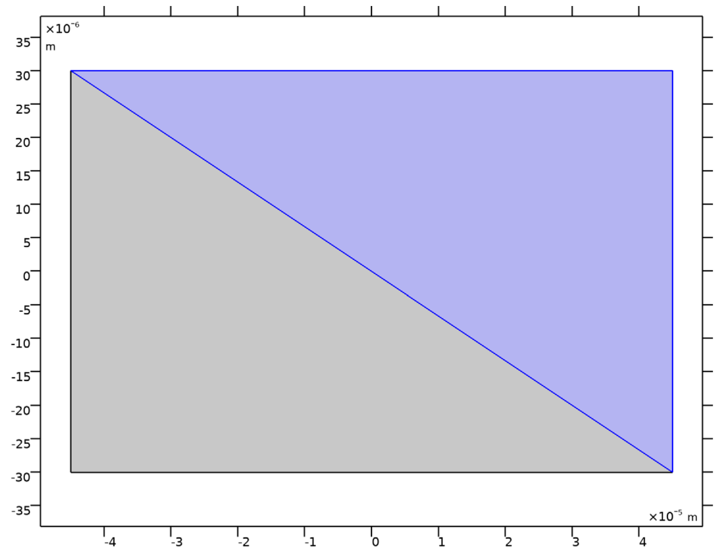
Glass

1 In the **Model Builder** window, under **Component 1 (comp1)** right-click **Materials** and choose **Blank Material**.

2 In the **Settings** window for **Material**, type `Glass` in the **Label** text field.

3 Select Domain 2 only.

4 Click the **Zoom Extents** button in the **Graphics** toolbar.



Define the refractive index for glass, using the parameter n2.

5 Locate the **Material Contents** section. In the table, enter the following settings:

Property	**Variable**	**Value**	**Unit**	**Property group**
Refractive index, real part	n_iso ; nii = n_iso, nij = 0	n2	1	Refractive index

DEFINITIONS

Set up expressions for the user-defined phases for the two waves, with different expressions in the two domains.

Variables 1

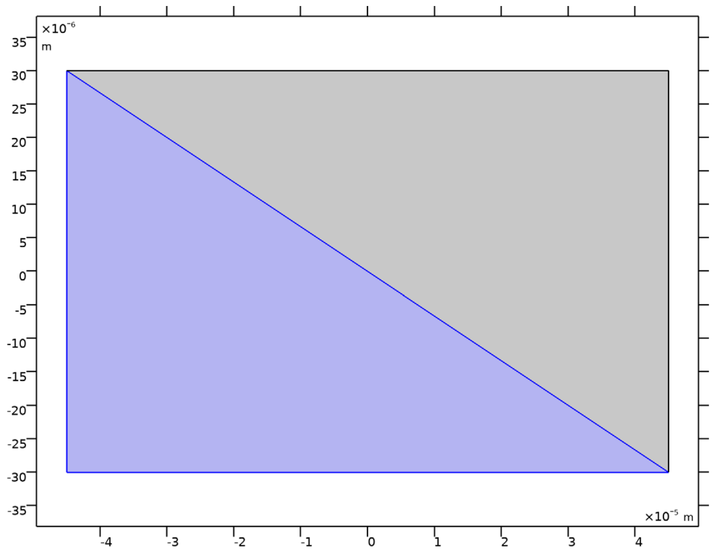
1 In the **Model Builder** window, under **Component 1 (comp1)** right-click **Definitions** and choose **Variables**.

2 In the **Settings** window for **Variables**, locate the **Geometric Entity Selection** section.

3 From the **Geometric entity level** list, choose **Domain**.

4 Select Domain 1 only.

5 Click the **Zoom Extents** button in the **Graphics** toolbar.



6 Locate the **Variables** section. In the table, enter the following settings:

Name	Expression	Unit	Description
`phi1`	`k1x_air*x+k1y_air*y`		`Phase in air, first wave`
`phi2`	`k2x_air*x+k2y_air*y`		`Phase in air, second wave`

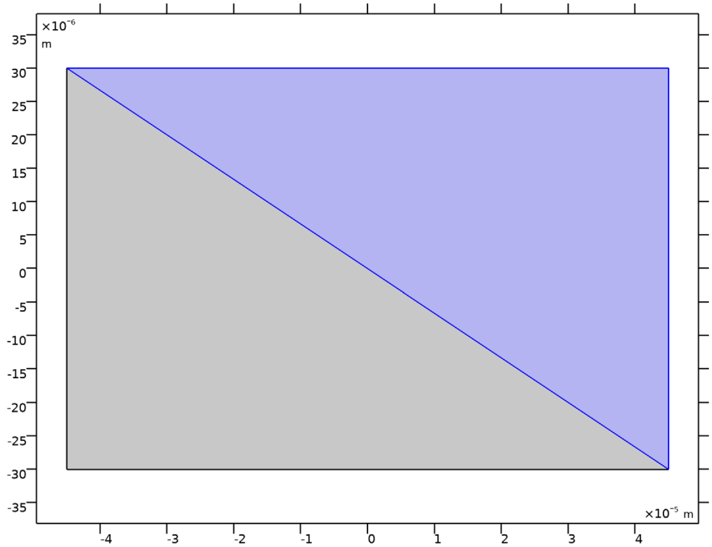
Variables 2

1 In the **Model Builder** window, right-click **Definitions** and choose **Variables**.

2 In the **Settings** window for **Variables**, locate the **Geometric Entity Selection** section.

3 From the **Geometric entity level** list, choose **Domain**.

4 Select Domain 2 only.



5 Locate the **Variables** section. In the table, enter the following settings:

Name	**Expression**	**Unit**	**Description**
`phi1`	`k1x_glass*x+k1y_glass*y`		`Phase in glass, first wave`
`phi2`	`k2x_glass*x+k2y_glass*y`		`Phase in glass, second wave`

ELECTROMAGNETIC WAVES, BEAM ENVELOPES (EWBE)

Now, use the phase variables to define the user-defined phases for the two waves.

1 In the **Model Builder** window, under **Component 1 (comp1)** click **Electromagnetic Waves, Beam Envelopes (ewbe)**.

2 In the **Settings** window for **Electromagnetic Waves, Beam Envelopes**, locate the **Wave Vectors** section.

3 From the **Type of phase specification** list, choose **User defined**.

4 In the ϕ_1 text field, type `phi1`.

5 In the ϕ_2 text field, type `phi2`.

First, simulate the case for s-polarization, where the polarization is orthogonal to the plane of incidence.

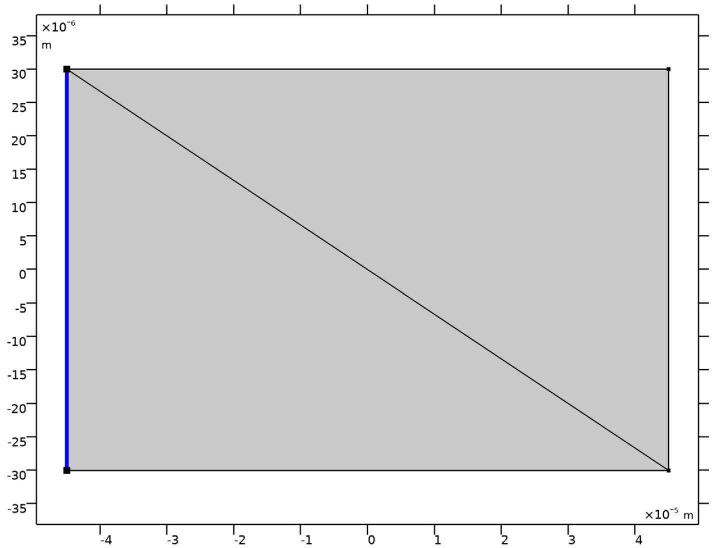
6 Locate the **Components** section. From the **Electric field components solved for** list, choose **Out-of-plane vector**.

On the leftmost boundary, a normally incident Gaussian beam is expected.

Matched Boundary Condition 1

1 In the **Physics** toolbar, click **Boundaries** and choose **Matched Boundary Condition**.

2 Select Boundary 1 only.



3 In the **Settings** window for **Matched Boundary Condition**, locate the **Matched Boundary Condition** section.

4 From the **Incident field** list, choose **Gaussian beam**.

5 In the w_0 text field, type `w0`.

6 Specify the $\mathbf{E}_{g0}$ vector as

0	x
0	y
1[V/m]	z

On this boundary, only an incident field is expected, but there should not be any scattered field. Thus, provide that information, with the following setting, to avoid any potential spurious solutions.

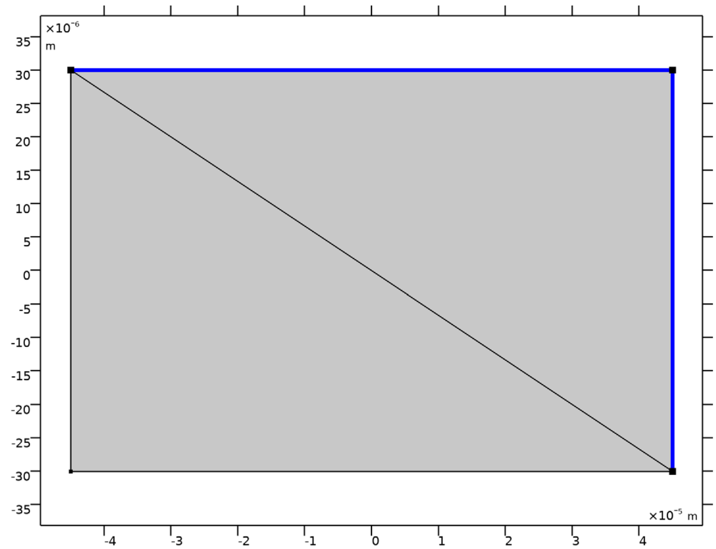
7 Find the **Scattered field** subsection. Select the **No scattered field** check box.

On the rightmost boundary, a transmitted Gaussian beam, propagating at an angle to the boundary normal, is expected. Thus, add a Matched Boundary Condition feature that will absorb this transmitted Gaussian beam.

Matched Boundary Condition 2

1 In the **Physics** toolbar, click **Boundaries** and choose **Matched Boundary Condition**.

2 Select Boundaries 4 and 5 only.



3 In the **Settings** window for **Matched Boundary Condition**, locate the **Matched Boundary Condition** section.

4 From the **Input wave** list, choose **Second wave**.

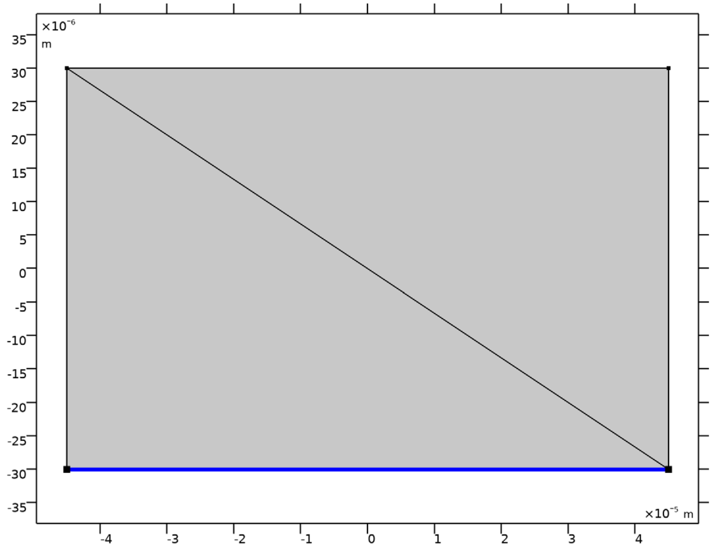
When specifying the user-defined phase functions, you defined wave 1 to correspond to the transmitted wave. Thus, here specify that there should be no second wave incident at this boundary.

The reflected wave, propagating toward the bottom boundary, will also propagate at an angle to the normal to the bottom boundary. Thus, add a Matched Boundary Condition feature here, too, to absorb the reflected beam.

Matched Boundary Condition 3

1 In the **Physics** toolbar, click **Boundaries** and choose **Matched Boundary Condition**.

2 Select Boundary 2 only.



Notice that when the user-defined phase functions were defined, the second wave was defined to correspond to the reflected wave. Thus, there should be no first wave incident at this boundary. Because this is the default setting, you do not need to make any manual settings.

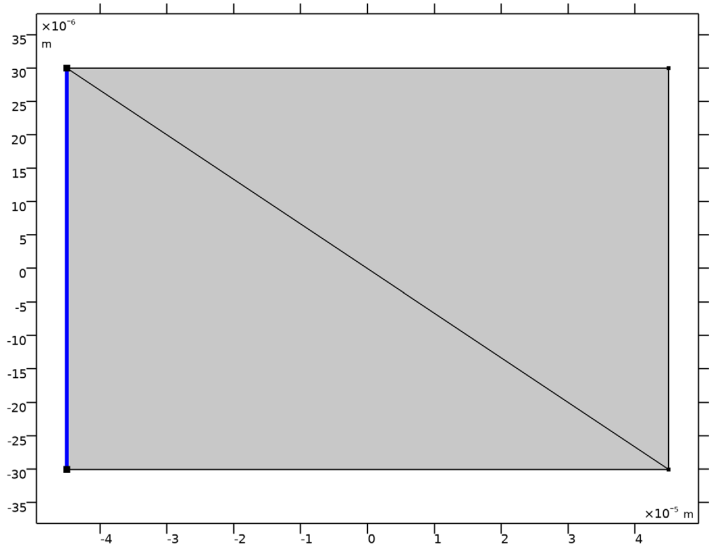
DEFINITIONS

Set up integration operators to calculate the power of the incident, reflected, and transmitted beams.

Integration 1 (intop1)

1. In the **Definitions** toolbar, click **Nonlocal Couplings** and choose **Integration**.
2. In the **Settings** window for **Integration**, locate the **Source Selection** section.
3. From the **Geometric entity level** list, choose **Boundary**.

4 Select Boundary 1 only.



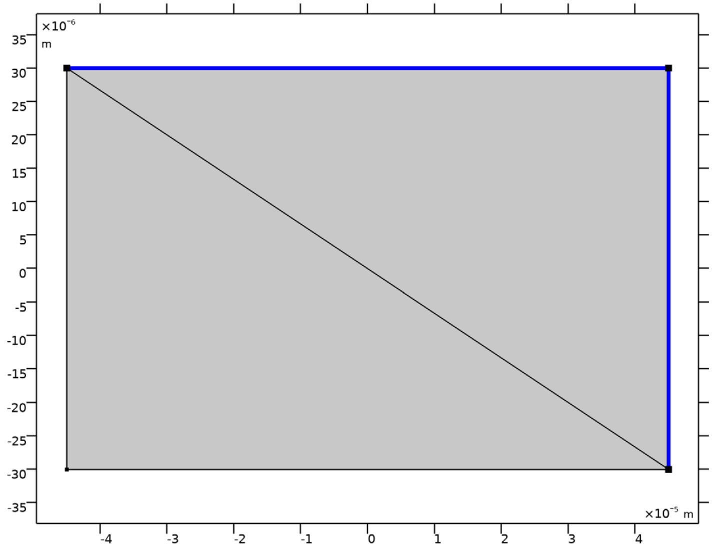
Integration 2 (intop2)

1 In the **Definitions** toolbar, click **Nonlocal Couplings** and choose **Integration**.

2 In the **Settings** window for **Integration**, locate the **Source Selection** section.

3 From the **Geometric entity level** list, choose **Boundary**.

4 Select Boundaries 4 and 5 only.



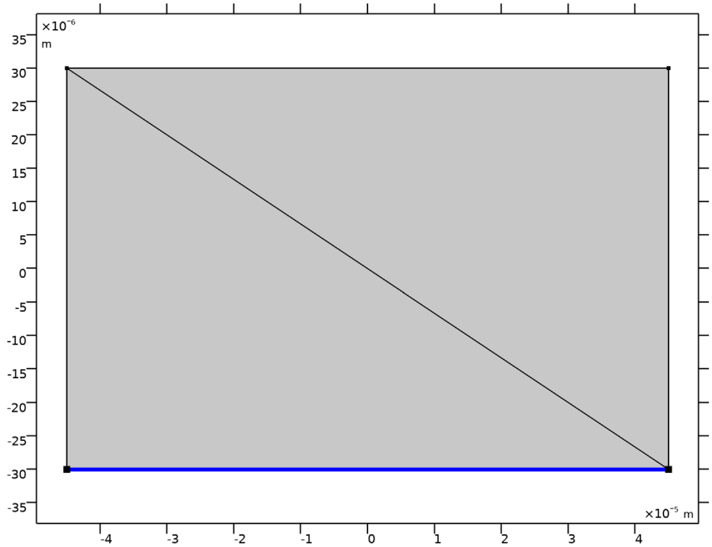
Integration 3 (intop3)

1 In the **Definitions** toolbar, click **Nonlocal Couplings** and choose **Integration**.

2 In the **Settings** window for **Integration**, locate the **Source Selection** section.

3 From the **Geometric entity level** list, choose **Boundary**.

4 Select Boundary 2 only.



Variables 3

Now, define the power variables for the beams, using the previously defined integration operators.

1 In the **Model Builder** window, right-click **Definitions** and choose **Variables**.

2 In the **Settings** window for **Variables**, locate the **Variables** section.

3 In the table, enter the following settings:

Name	Expression	Unit	Description
`Pin`	`-intop1(ewbe.nPoav)`	W/m	`Input power`
`Pt`	`intop2(ewbe.nPoav)`	W/m	`Transmitted power`
`Pr`	`intop3(ewbe.nPoav)`	W/m	`Reflected power`

The minus sign for the input power is used as the power flow and the boundary normal point in the opposite directions.

MESH 1

Let the physics define a triangular mesh where the maximum mesh element size is set to half a wavelength, to resolve the interference pattern created by the incident and the reflected beam.

1 In the **Model Builder** window, under **Component 1 (comp1)** click **Mesh 1**.

2 In the **Settings** window for **Mesh**, locate the **Electromagnetic Waves, Beam Envelopes (ewbe)** section.

3 From the **Mesh type** list, choose **Triangular mesh**.

4 In the h_{max} text field, type `lda0/2`.

STUDY 1

Step 1: Frequency Domain

Define the frequency and compute the solution for the model.

1 In the **Model Builder** window, under **Study 1** click **Step 1: Frequency Domain**.

2 In the **Settings** window for **Frequency Domain**, locate the **Study Settings** section.

3 In the **Frequencies** text field, type `f0`.

4 In the **Home** toolbar, click **Compute**.

RESULTS

Electric Field

To really resolve the inference pattern to the left of the air-glass interface, set the resolution to extra fine.

1 In the **Model Builder** window, expand the **Electric Field (ewbe)** node, then click **Electric Field**.

2 In the **Settings** window for **Surface**, click to expand the **Quality** section.

3 From the **Resolution** list, choose **Extra fine**.

4 In the **Electric Field (ewbe)** toolbar, click **Plot**.

5 Click the **Zoom Extents** button in the **Graphics** toolbar. Compare your result with Figure 2.

Using the defined variables, compute the reflectance.

Global Evaluation 1

1 In the **Results** toolbar, click 8.5 **Global Evaluation**.

2 In the **Settings** window for **Global Evaluation**, locate the **Expressions** section.

3 In the table, enter the following settings:

Expression	Unit	Description
Pr/Pin	1	

4 Click **Evaluate**.

TABLE

1 Go to the **Table** window. Compare the calculated reflectance with the theoretical value for s-polarized plane waves. Notice that the values are almost the same.
2 In the **Settings** window for **Global Evaluation**, locate the **Expressions** section.
3 In the table, enter the following settings:

Expression	Unit	Description
`((1-n2^2)/(1+n2^2))^2`		

4 Click **Evaluate**.

Now check that all incident power is either reflected or transmitted.

5 In the table, enter the following settings:

Expression	Unit	Description
`(Pr+Pt)/Pin`	1	

6 Click **Evaluate**.

ELECTROMAGNETIC WAVES, BEAM ENVELOPES (EWBE)

In this simulation, set the polarization to be in-plane, that is p-polarization. For this case, there should be no reflected beam.

1 In the **Model Builder** window, under **Component 1 (comp1)** click **Electromagnetic Waves, Beam Envelopes (ewbe)**.
2 In the **Settings** window for **Electromagnetic Waves, Beam Envelopes**, locate the **Components** section.
3 From the **Electric field components solved for** list, choose **In-plane vector**.

Matched Boundary Condition 1

1 In the **Model Builder** window, under **Component 1 (comp1)>Electromagnetic Waves, Beam Envelopes (ewbe)** click **Matched Boundary Condition 1**.
2 In the **Settings** window for **Matched Boundary Condition**, locate the **Matched Boundary Condition** section.
3 Specify the $\mathbf{E}_{g0}$ vector as

0	x
1[V/m]	y
0	z

STUDY 1

In the **Home** toolbar, click **Compute**.

RESULTS

Electric Field (ewbe)

Click the **Zoom Extents** button in the **Graphics** toolbar. Compare your result with Figure 3. Notice that there is no reflected beam in this case.

Global Evaluation 1

Also check numerically that the reflected wave is almost gone for p-polarization at the Brewster angle.

1 In the **Model Builder** window, under **Results>Derived Values** click **Global Evaluation 1**.

2 In the **Settings** window for **Global Evaluation**, locate the **Expressions** section.

3 In the table, enter the following settings:

Expression	Unit	Description
`Pr/Pin`	1	

4 Click **Evaluate**.