

Frozen Inclusion

Introduction

This benchmark example simulates phase change in porous media. The model studies the melting process of an ice inclusion within a porous medium, and it demonstrates how to couple Darcy's Law and the Heat Transfer in Porous Media interfaces including phase change. It is one of the test cases from the INTERFROST project — a comparison project for coupled thermo-hydraulic systems with respect to climate change in permafrost regions (see Ref. 2).

Model Definition

The model setup is shown in Figure 1. A flow channel 3 meters in length and 1 meter in width contains a squared ice inclusion of 33.3 cm in length, centered at the coordinates x = 1 m and y = 0.5 m.

Zero conductive flux, zero flux



Zero conductive flux, zero flux

Figure 1: Model geometry, initial temperature distribution, and boundary conditions.

The initial temperature of the ice inclusion is $T_{ini} = -5$ °C. The temperature in the porous channel is $T_{inw} = +5$ °C. The fluid flow is driven by a hydraulic head gradient of $\Delta H = 3\%$ over the length of the channel.

The initial pressure and velocity field are computed from a steady state flow simulation for a uniformly distributed temperature and permeability (as if the ice inclusion was not present). Considering the symmetry of the benchmark test model, only the lower half of the domain is modeled.

The following simplifications have been made in the INTERFROST benchmark and therefore are also assumed in this example: No thermal dispersion is included in the heat transfer equation, and water density and dynamic viscosity are considered constant with respect to the temperature (see Table 1).

MODEL EQUATIONS

The Porous Medium feature in Darcy's Law interface includes an option to define a user-defined storage $S_{\rm p}$ (SI unit: 1/Pa) which in this case is defined as

$$S_{p} = S_{w} \beta \varepsilon_{p} \tag{1}$$

where $S_{\rm w}$ is the water saturation, $\varepsilon_{\rm p}$ is the porosity, and β the effective compressibility; which is a combined value of water, ice, and solid matrix compressibility. For simplicity, gravity is neglected. The variable $Q_{\rm m}$ is a mass source which represents the additional liquid water due to the melting of the ice inclusion:

$$Q_{\rm m} = S_{\rm w} \varepsilon_{\rm p} (\rho_{\rm i} - \rho_{\rm w}) \frac{\partial S_{\rm w}}{\partial t}. \tag{2}$$

here ρ_i and ρ_w are the ice and liquid water densities. The liquid water saturation S_w depends on the phase change as:

$$S_{w} = S_{w, res} + (1 - S_{w, res})\theta_{\varsigma}$$
(3)

here, $S_{\mathrm{w, res}}$ it the residual liquid water saturation and θ_2 is a smooth step function defined in the **Phase Change Material** node. The step function $\theta_2(T)$ is zero for temperatures below the melting temperature T_{pc} , and it equals 1 for temperatures above T_{pc} .

In the INTERFROST benchmark it is assumed that the mushy ice zone extends from 0° C to -1° C. Therefore, T_{pc} is set to -0.5° C and the transition interval of θ_2 is defined as 1K to represent this.

The heat transfer in the porous medium is described by the following equation:

$$(\rho C)_{\text{eq}} \frac{\partial T}{\partial t} + \rho C_{\text{w}} \mathbf{u} \cdot \nabla T + \nabla \cdot (-k_{\text{eq}} \nabla T) = Q$$
(4)

here $(\rho C)_{\rm eq}$ is the equivalent value of density ρ (kg/m³) and heat capacity C at constant pressure (J/kg·K), $k_{\rm eq}$ is the effective thermal conductivity (W/m·K), T is the temperature (K), and Q is a heat source (W/m³). The variable $C_{\rm w}$ is the effective fluid's heat capacity at constant pressure.

MODEL DATA

The benchmark example describes the ice-to-water phase change of an ice inclusion within a porous channel. The values for thermal properties of water, ice, and solid matrix are presented in Table 1.

TABLE I: MATERIAL PROPERTIES OF ICE, WATER, AND SOLID MATRIX.

MATERIAL PROPERTY	ICE	WATER	SOLID MATRIX
Density	920 kg/m ³	1000 kg/m ³	2650 kg/m ³
Heat capacity at constant pressure	2060 J/(kg·K)	4182 J(kg·K)	835 J(kg·K)
Thermal conductivity	2.14 W/(m·K)	0.6 W/(m·K)	9 W/(m·K)

A latent heat of melting, L=334 kJ/kg is used. Additional physical parameters used in the example are summarized in Table 2.

TABLE 2: ADDITIONAL PHYSICAL PARAMETERS.

Parameter	Description	Value
$\mu_{\mathbf{w}}$	Dynamic viscosity of water	1.793·10 ⁻³ Pa·s
ε	Porosity	0.37
β	Effective compressibility	10 ⁻⁸ Pa ⁻¹
S_{wres}	Residual water saturation	0.05
Ω	Impedance factor	50
k_{int}	Intrinsic permeability	1.3·10 ⁻¹⁰ m ²

Results and Discussion

The simulation is run for 56 hours which is the time until the frozen inclusion is completely melted and the cooler temperatures have been convected out of the channel. However, it is also interesting to display the effect of the molten inclusion on the pressure and temperature distribution; Figure 2 shows the pressure distribution after 9 hours of

simulated time when there is still some ice present. As gravity has been neglected, only the pressure effects of the ice inclusion are shown.]

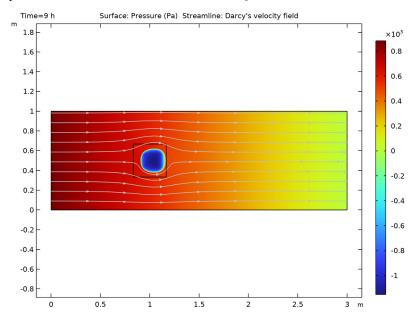


Figure 2: Pressure distribution after 9 hours when there is still some ice present in the model domain.

The temperature field is shown at the same time (after 9 hours) in Figure 3 and the liquid water saturation in Figure 4. Both figures indicate the ice inclusion and its influence on the temperature field. The low temperatures are convected in the downstream part of the domain.

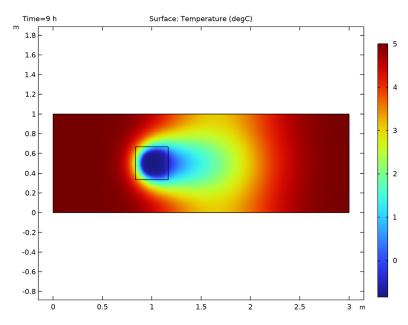


Figure 3: Surface plot showing the temperature distribution after 9 hours.

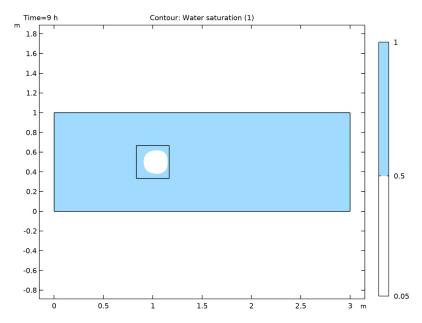


Figure 4: Surface plot showing the liquid water saturation after 9 hours. Effectively the areas with liquid water appear in blue whereas the areas of ice appear in white.

The plots below have been used as performance measures in the INTERFROST benchmark, and therefore are also included here:

The evolution of the minimum of the simulated temperature field as a function of time is shown in Figure 5..

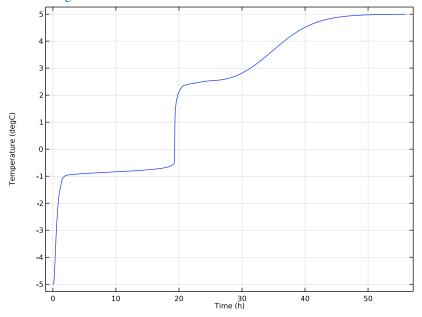


Figure 5: The minimum temperature in the model domain as a function of time.

Figure 6 shows the time evolution of the total heat flux exiting the system at the downstream boundary.

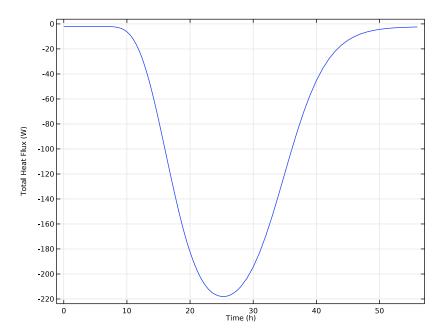


Figure 6: Total heat flux exiting the system via the outflow boundary as a function of time.

Figure 7 displays the evolution of the total liquid water volume in the domain as a function of time.

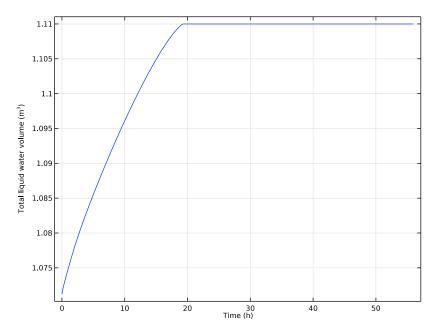


Figure 7: Total liquid water volume in the domain as a function of time.

Notes About the COMSOL Implementation

A residual liquid phase is present in soils even for temperatures significantly below 0°C. To make sure that this residual water is not subject to phase change in the model and that it is not convected, it is considered as a second immobile solid (in addition to the solid matrix). According to Ref. 1, the residual saturation $S_{w.\, res}$ is set to 0.05.

References

- 1. C. Grenier and others, "Groundwater flow and heat transport for systems undergoing freeze-thaw: Intercomparison of numerical simulators for 2D test cases," Advances in Water Resources, vol. 114, pp. 196-218, 2018.
- 2. https://wiki.lsce.ipsl.fr/interfrost/doku.php?id=test_cases:five

Application Library path: Subsurface Flow Module/Heat Transfer/

frozen_inclusion

Modeling Instructions

From the File menu, choose New.

NEW

In the New window, click Model Wizard.

MODEL WIZARD

- I In the Model Wizard window, click **2** 2D.
- 2 In the Select Physics tree, select Fluid Flow>Porous Media and Subsurface Flow> Darcy's Law (dl).
- 3 Right-click and choose Add Physics.
- 4 In the Select Physics tree, select Heat Transfer>Porous Media> Heat Transfer in Porous Media (ht).
- 5 Right-click and choose Add Physics.
- 6 Click 🔁 Study.
- 7 In the Select Study tree, select General Studies>Stationary.
- 8 Click M Done.

GLOBAL DEFINITIONS

Parameters 1

- I In the Model Builder window, under Global Definitions click Parameters I. The parameters defining the material properties of the solid matrix, water, and ice have to be declared. They are available in a file and you can import them as follows:
- 2 In the Settings window for Parameters, locate the Parameters section.
- 3 Click Load from File.
- **4** Browse to the model's Application Libraries folder and double-click the file frozen inclusion parameters.txt.

GEOMETRY I

Now, the model domain is drawn as two rectangles, one representing the porous domain, the other the frozen inclusion within the porous domain. Create the geometry as follows and note that due to symmetry only half of the model domain is drawn.

Rectangle I (rI)

- I In the Geometry toolbar, click Rectangle.
- 2 In the Settings window for Rectangle, locate the Size and Shape section.
- 3 In the Width text field, type 3.
- 4 In the Height text field, type 0.5.

Rectangle 2 (r2)

- I In the Geometry toolbar, click Rectangle.
- 2 In the Settings window for Rectangle, locate the Size and Shape section.
- 3 In the Width text field, type 0.333.
- 4 In the Height text field, type 0.333/2.
- 5 Locate the Position section. From the Base list, choose Center.
- 6 In the x text field, type 1.
- 7 In the y text field, type 0.5-0.333/4.
- 8 In the Geometry toolbar, click **Build All**.

MATERIALS

Define the materials in the next step. Introduce them as empty material nodes, first. As soon as the physics has been defined, the material node menu will show you which properties are needed for the simulation. You can then just fill in the values defined in the Parameters list.

Solid Matrix

- I In the Model Builder window, under Component I (comp I) right-click Materials and choose Blank Material.
- 2 In the Settings window for Material, type Solid Matrix in the Label text field.

Water

- I Right-click Materials and choose Blank Material.
- 2 In the Settings window for Material, type Water in the Label text field.

Ice

I Right-click Materials and choose Blank Material.

2 In the Settings window for Material, type Ice in the Label text field.

Porous Material I (pmat I)

- I Right-click Materials and choose More Materials>Porous Material.
- 2 In the Settings window for Porous Material, locate the Geometric Entity Selection section.
- 3 From the Selection list, choose All domains.
- 4 Locate the Phase-Specific Properties section. Click 👯 Add Required Phase Nodes.

Fluid I (pmat1.fluid1)

- I In the Model Builder window, click Fluid I (pmat1.fluid1).
- 2 In the Settings window for Fluid, locate the Fluid Properties section.
- 3 From the Material list, choose Water (mat2).

Solid I (pmat1.solid1)

- I In the Model Builder window, click Solid I (pmat1.solid1).
- 2 In the Settings window for Solid, locate the Solid Properties section.
- 3 From the Material list, choose Solid Matrix (matl).
- **4** In the θ_s text field, type 1-epsilon p.

Immobile Fluid | (pmat1.imfluid1)

- I In the Model Builder window, right-click Porous Material I (pmatl) and choose Immobile Fluid.
- 2 In the Settings window for Immobile Fluid, locate the Immobile Fluid Properties section.
- 3 From the Material list, choose Water (mat2).
- **4** In the θ_{imf} text field, type epsilon_p*Sw_res.

HEAT TRANSFER IN POROUS MEDIA (HT)

Continue with setting up the physics. Since some heat transfer variables are needed to define the saturation variables as explained in the Model Definition section, start with Heat Transfer in Porous Media (ht).

- I In the Model Builder window, under Component I (compl) click Heat Transfer in Porous Media (ht).
- 2 In the Settings window for Heat Transfer in Porous Media, locate the Physical Model section.
- **3** In the T_{ref} text field, type T_initial_w.

Porous Matrix I

- I In the Model Builder window, under Component I (compl)> Heat Transfer in Porous Media (ht)>Porous Medium I click Porous Matrix I.
- 2 In the Settings window for Porous Matrix, locate the Matrix Properties section.
- **3** From the **Define** list, choose **Solid phase properties**, because the properties of a pure solid material are given.

With the following steps you define the residual liquid water volume as product of porosity and predefined residual water saturation.

Porous Medium I

In the Model Builder window, click Porous Medium 1.

Immobile Fluids 1

In the Physics toolbar, click — Attributes and choose Immobile Fluids.

Fluid 1

- I In the Model Builder window, click Fluid I.
- 2 In the Settings window for Fluid, locate the Model Input section.
- **3** From the p_A list, choose **Absolute pressure (dl)**.
- 4 Locate the Heat Convection section. From the u list, choose Darcy's velocity field (dl/ porous I).

Phase Change Material I

In the Physics toolbar, click Attributes and choose Phase Change Material.

Phase Change Material I

- I In the Model Builder window, expand the Component I (compl)> Heat Transfer in Porous Media (ht)>Porous Medium 1>Fluid I node, then click Phase Change Material I.
- 2 In the Settings window for Phase Change Material, click to expand the Sketch section. Inflate the sketch to visualize which material you have to choose for which temperatures. In this case, material 1 should be ice, material 2 water. You will use the function θ_2 later when you define the water saturation as a function of phase change.
- **3** Locate the **Phase Change** section. In the $T_{pc,1\rightarrow 2}$ text field, type T_pc.
- **4** In the $\Delta T_{1\rightarrow 2}$ text field, type 1[K].
- **5** In the $L_{1\rightarrow 2}$ text field, type L.
- 6 Locate the Phase I section. From the Material, phase I list, choose Ice (mat3).

7 Locate the Phase 2 section. From the Material, phase 2 list, choose Water (mat2).

Initial Values 1

- I In the Model Builder window, under Component I (compl)> Heat Transfer in Porous Media (ht) click Initial Values I.
- 2 In the Settings window for Initial Values, locate the Initial Values section.
- **3** In the *T* text field, type T_initial_w.

Initial Values 2

- I In the Physics toolbar, click Domains and choose Initial Values.
- 2 Select Domain 2 only.
- 3 In the Settings window for Initial Values, locate the Initial Values section.
- **4** In the *T* text field, type T_initial_i.

Temberature I

- I In the Physics toolbar, click Boundaries and choose Temperature.
- 2 Select Boundary 1 only.
- 3 In the Settings window for Temperature, locate the Temperature section.
- **4** In the T_0 text field, type T_initial_w.

Outflow I

- I In the Physics toolbar, click Boundaries and choose Outflow.
- 2 Select Boundary 9 only.

Symmetry I

- I In the Physics toolbar, click Boundaries and choose Symmetry.
- 2 Select Boundaries 3, 6, and 8 only.

DEFINITIONS (COMPI)

For Darcy's Law the water saturation and the permeability have to be defined as temperature-dependent variables. More precisely they depend on θ_2 which is the fraction of water due to phase change.

Variables 1

- I In the Model Builder window, expand the Component I (compl)>Definitions node.
- 2 Right-click Component I (compl)>Definitions and choose Variables.
- 3 In the Settings window for Variables, locate the Variables section.

4 In the table, enter the following settings:

Name	Expression	Unit	Description
Sw	Sw_res+(1-Sw_res)* ht.theta2		Water saturation
kr	<pre>max(10^(-Omega* epsilon_p*(1-Sw)),1e- 6)</pre>		Relative permeability
kappa_w	k_int*kr	m²	Permeability

DARCY'S LAW (DL)

- I In the Model Builder window, under Component I (compl) click Darcy's Law (dl).
- 2 In the Settings window for Darcy's Law, click to expand the Discretization section.
- 3 From the Pressure list, choose Linear.

Change the shape function to linear to reduce the computational time and increase numerical stability.

Porous Medium I

- I In the Model Builder window, under Component I (compl)>Darcy's Law (dl) click Porous Medium 1.
- 2 In the Settings window for Porous Medium, locate the Porous Medium section.
- 3 From the Storage model list, choose User defined. In the $S_{
 m p}$ text field, type Sw* epsilon p*beta.

Porous Matrix I

- I In the Model Builder window, click Porous Matrix I.
- 2 In the Settings window for Porous Matrix, locate the Matrix Properties section.
- **3** From the ε_p list, choose **User defined**. In the associated text field, type epsilon_p*Sw.

Mass Source I

I In the Physics toolbar, click **Domains** and choose Mass Source.

The mass source accounts for the addition of water during melting as described in the Model Definition section.

- 2 In the Settings window for Mass Source, locate the Domain Selection section.
- 3 From the Selection list, choose All domains.
- **4** Locate the Mass Source section. In the $Q_{
 m m}$ text field, type dl.epsilon*(rho_irho_w)*d(Sw,t). Remember that dl.epsilon was defined as epsilon_p*Sw to represent only the pore space that is filled with water.

The flow is driven by an imposed gradient of the hydraulic head. This is defined as a parameter and could be varied in a parametric sweep in future simulations (as it was done in the INTERFROST project, see References).

Hydraulic Head I

- I In the Physics toolbar, click Boundaries and choose Hydraulic Head.
- 2 Select Boundary 1 only.
- 3 In the Settings window for Hydraulic Head, locate the Hydraulic Head section.
- **4** In the H_0 text field, type delta H dLx*3[m].

Hydraulic Head 2

- I In the Physics toolbar, click Boundaries and choose Hydraulic Head.
- 2 Select Boundary 9 only.

Symmetry I

- I In the Physics toolbar, click Boundaries and choose Symmetry.
- 2 Select Boundaries 3, 6, and 8 only.

MATERIALS

Having defined the physics, now fill in the empty expressions in the Materials node.

Porous Material I (pmat I)

- I In the Model Builder window, under Component I (compl)>Materials click Porous Material I (pmat I).
- 2 In the Settings window for Porous Material, locate the Homogenized Properties section.
- **3** In the table, enter the following settings:

Variable	Value	Unit	Property group
kappa_iso ; kappaii = kappa_iso,	kappa_w	m²	Basic
	kappa_iso ; kappaii = kappa_iso,	kappa_iso; kappa_w kappaii =	kappa_iso; kappa_w m² kappaii = kappa_iso,

Solid Matrix (mat I)

- I In the Model Builder window, click Solid Matrix (matl).
- 2 In the Settings window for Material, locate the Material Contents section.

3 In the table, enter the following settings:

Property	Variable	Value	Unit	Property group
Density	rho	rho_s	kg/m³	Basic
Thermal conductivity	k_iso ; kii = k_iso, kij = 0	lambda_s	W/(m·K)	Basic
Heat capacity at constant pressure	Ср	Cs	J/(kg·K)	Basic

Water (mat2)

- I In the Model Builder window, click Water (mat2).
- 2 In the Settings window for Material, locate the Material Contents section.
- **3** In the table, enter the following settings:

Property	Variable	Value	Unit	Property group
Density	rho	rho_w	kg/m³	Basic
Dynamic viscosity	mu	mu_w	Pa·s	Basic
Thermal conductivity	k_iso ; kii = k_iso, kij = 0	lambda_w	W/(m·K)	Basic
Heat capacity at constant pressure	Ср	Cw	J/(kg·K)	Basic

Ice (mat3)

- I In the Model Builder window, click Ice (mat3).
- 2 In the Settings window for Material, locate the Material Contents section.
- **3** In the table, enter the following settings:

Property	Variable	Value	Unit	Property group
Thermal conductivity	k_iso ; kii = k_iso, kij = 0	lambda_i	W/(m·K)	Basic
Density	rho	rho_i	kg/m³	Basic
Heat capacity at constant pressure	Ср	Ci	J/(kg·K)	Basic

MESH I

When generating the mesh, note that it should be fine enough to capture strong gradients within the phase change region. In this case, a maximum element size of 5 mm is enforced in the domain of the frozen inclusion.

Free Triangular I

- I In the Mesh toolbar, click Free Triangular.
- 2 In the Settings window for Free Triangular, locate the Domain Selection section.
- 3 From the Geometric entity level list, choose Domain.
- **4** Select Domain 2 only.

Size 1

- I Right-click Free Triangular I and choose Size.
- 2 In the Settings window for Size, locate the Element Size section.
- **3** Click the **Custom** button.
- 4 Locate the Element Size Parameters section. Select the Maximum element size check box.
- **5** In the associated text field, type 0.005.
- 6 Click Build Selected.

Free Triangular 2

In the Mesh toolbar, click Free Triangular.

Size

- I In the Model Builder window, click Size.
- 2 In the Settings window for Size, locate the Element Size section.
- 3 From the Predefined list, choose Extra fine.

Free Triangular 2

In the Model Builder window, right-click Free Triangular 2 and choose Build All.

STUDY I

Step 1: Stationary

- I In the Model Builder window, under Study I click Step I: Stationary.
 - The initial pressure and velocity field results from a steady state flow simulation which is performed in this first step.
- 2 In the Settings window for Stationary, locate the Physics and Variables Selection section.
- 3 In the table, clear the Solve for check box for Heat Transfer in Porous Media (ht).

4 In the Home toolbar, click **Compute**.

Time Dependent

Now add the time dependent study step. To ensure that the phase change is represented properly, the time steps have to be chosen small enough to catch the increase of temperature as soon as the ice has totally melted. This is done by defining the output times accordingly and by forcing the time step to be smaller than the output intervals.

- I In the Study toolbar, click Study Steps and choose Time Dependent> Time Dependent.
- 2 In the Settings window for Time Dependent, locate the Study Settings section.
- **3** From the **Time unit** list, choose **h**.
- 4 In the Output times text field, type range (0,5,60) [min] range (1.5,0.5,18) range(18.1,0.05,22) range(22.5,0.5,28) range(29,1,56).
- **5** From the **Tolerance** list, choose **User controlled**.
- 6 In the Relative tolerance text field, type 0.005.

Solution I (soll)

- I In the Study toolbar, click Show Default Solver.
- 2 In the Model Builder window, expand the Solution I (soll) node, then click Time-Dependent Solver I.
- 3 In the Settings window for Time-Dependent Solver, click to expand the Time Stepping section.
- 4 From the Steps taken by solver list, choose Strict This ensures that the time step cannot be bigger than the output intervals. However, it can still be smaller if necessary.
- **5** Select the **Initial step** check box.
- 6 In the associated text field, type 1[s].

Step 2: Time Dependent

In the Model Builder window, under Study I right-click Step 2: Time Dependent and choose **Compute Selected Step.**

RESULTS

The default plot shows the pressure distribution and velocity field at the final time. To see the pressure and velocity field for the full model domain at an earlier time, when there is still some ice present (see Figure 2), follow the steps below. First, create the full dataset from the simplified symmetric model domain.

Mirror 2D I

- I In the Model Builder window, expand the Results>Datasets node.
- 2 Right-click Results>Datasets and choose More 2D Datasets>Mirror 2D.
- 3 In the Settings window for Mirror 2D, locate the Axis Data section.
- 4 In row Point I, set Y to 0.5.
- 5 In row Point 2, set X to 3.
- 6 In row Point 2, set Y to 0.5.
- 7 Click to expand the Advanced section. Select the Remove elements on the symmetry axis check box.

Pressure (dl)

- I In the Model Builder window, under Results click Pressure (dl).
- 2 In the Settings window for 2D Plot Group, locate the Data section.
- 3 From the Dataset list, choose Mirror 2D 1.
- 4 From the Time (h) list, choose 9.

Streamline 1

- I In the Model Builder window, expand the Pressure (dl) node, then click Streamline I.
- 2 In the Settings window for Streamline, locate the Streamline Positioning section.
- 3 From the Positioning list, choose Magnitude controlled.
- **4** In the **Density** text field, type 10.
- 5 In the Pressure (dl) toolbar, click Plot.

Temberature

Add the surface plot of the temperature field as shown in Figure 3.

- I In the Home toolbar, click **Add Plot Group** and choose **2D Plot Group**.
- 2 In the Settings window for 2D Plot Group, type Temperature in the Label text field.
- 3 Locate the Data section. From the Dataset list, choose Mirror 2D 1.
- 4 From the Time (h) list, choose 9.

Surface I

- I Right-click Temperature and choose Surface.
- 2 In the Settings window for Surface, locate the Expression section.
- **3** In the **Expression** text field, type T.
- 4 From the Unit list, choose degC.

5 In the Temperature toolbar, click Plot.

Liquid Water Saturation

To reproduce the plot of the liquid water saturation (Figure 4), follow the instructions below.

- I In the Home toolbar, click Add Plot Group and choose 2D Plot Group.
- 2 In the Settings window for 2D Plot Group, type Liquid Water Saturation in the Label text field.
- 3 Locate the Data section. From the Dataset list, choose Mirror 2D 1.
- 4 From the Time (h) list, choose 9.

Contour I

- I Right-click Liquid Water Saturation and choose Contour.
- 2 In the Settings window for Contour, locate the Expression section.
- 3 In the Expression text field, type Sw.
- 4 Locate the Levels section. From the Entry method list, choose Levels.
- 5 In the Levels text field, type 0.05 0.5 1.
- 6 Locate the Coloring and Style section. From the Contour type list, choose Filled.
- 7 From the Color table list, choose JupiterAuroraBorealis.
- 8 From the Color table transformation list, choose Reverse.
- **9** In the Liquid Water Saturation toolbar, click **1** Plot.

Surface Minimum 1

To reproduce the next plots the resulting data has to be further evaluated. Follow the instructions below to derive the minimum temperature as a function of time.

- I In the Results toolbar, click 8.85 More Derived Values and choose Minimum> Surface Minimum.
- 2 In the Settings window for Surface Minimum, locate the Selection section.
- 3 From the Selection list, choose All domains.
- 4 Click Replace Expression in the upper-right corner of the Expressions section. From the menu, choose Component I (compl)>Heat Transfer in Porous Media>Temperature>T -Temperature - K.

5 Locate the **Expressions** section. In the table, enter the following settings:

Expression	Unit	Description
T	degC	Temperature

6 Click **= Evaluate**.

TABLE

- I Go to the **Table** window.
- 2 Click Table Graph in the window toolbar.

RESULTS

Minimum Temperature

- I In the Model Builder window, under Results click ID Plot Group 4.
- 2 In the Settings window for ID Plot Group, type Minimum Temperature in the Label text field.

Line Integration I

Following the next steps you will derive and plot the total heat flux exiting the system via the downstream boundary as a function of time.

- I In the Results toolbar, click 8.85 More Derived Values and choose Integration> Line Integration.
- 2 Select Boundary 9 only.
- 3 In the Settings window for Line Integration, click Replace Expression in the upper-right corner of the Expressions section. From the menu, choose Component I (compl)> Heat Transfer in Porous Media>Boundary fluxes>ht.ntflux - Normal total heat flux - W/m2.
- **4** Locate the **Expressions** section. In the table, enter the following settings:

Expression	Unit	Description
ht.ntflux*dl.d*2	W	Total Heat Flux

As the model is a 2D approximation of the thawing process and due to symmetry only half the domain was modeled, the heat flux value has to be multiplied by the thickness of the model, dl.d, and by a factor of 2 to get the correct value.

5 Click ▼ next to **= Evaluate**, then choose **New Table**.

TABLE

I Go to the Table window.

2 Click Table Graph in the window toolbar.

RESULTS

Total Heat Flux

- I In the Model Builder window, under Results click ID Plot Group 5.
- 2 In the Settings window for ID Plot Group, type Total Heat Flux in the Label text field.

Surface Integration 2

The final plot, the evolution of total liquid water volume in the domain as shown in Figure 7 can be produced as follows.

- I In the Results toolbar, click 8.85 More Derived Values and choose Integration> Surface Integration.
- 2 In the Settings window for Surface Integration, locate the Selection section.
- 3 From the Selection list, choose All domains.
- 4 Locate the Expressions section. Click \ Clear Table.
- **5** In the table, enter the following settings:

Expression	Unit	Description
Sw*epsilon_p*dl.d*2	m^3	Total liquid water volume

Sw*epsilon_p gives the pore space filled with water, dl.d is the thickness of the model and the factor of 2 is due to symmetry.

6 Click ▼ next to **= Evaluate**, then choose **New Table**.

TABLE

- I Go to the **Table** window.
- 2 Click Table Graph in the window toolbar.

RESULTS

Total Liquid Water Volume

- I In the Model Builder window, under Results click ID Plot Group 6.
- 2 In the Settings window for ID Plot Group, type Total Liquid Water Volume in the **Label** text field.

Compare your results with those presented in Figure 7.