

Mixed-Mode Debonding of a Laminated Composite

Interfacial failure by delamination or debonding is one of the main failure modes of laminate structures. Such failures can be simulated with a cohesive zone model (CZM). A key ingredient of a cohesive zone model is a traction-separation law that describes the softening in the cohesive zone near the delamination tip. This example shows how to model debonding using the decohesion model in COMSOL Multiphysics. The capabilities of the CZM to predict mixed-mode softening and delamination propagation are demonstrated in a model of the mixed-mode bending of a composite beam.

Model Definition

COHESIVE ZONE MODEL (CZM)

The CZM is in this example defined using the displacement-based damage model available in the **Decohesion** node under **Contact**. The model is used to predict crack propagation at the interface of a laminated composite beam under mixed-mode loading. The material properties needed for this constitutive model are summarized in Table 1.

| PROPERTY | SYMBOL | VALUE |
|---|-----------------------|-----------------------------------|
| Normal tensile strength | $\sigma_{ m t}$ | 80 MPa |
| Shear strength | σ_{s} | 100 MPa |
| Penalty stiffness | $p_{ m n}$ | 10 ⁶ N/mm ³ |
| Critical energy release rate, tension | $G_{ m ct}$ | 969 J/m ² |
| Critical energy release rate, shear | $G_{ m cs}$ | 1719 J/m ² |
| Exponent of the Benzeggagh and Kenane (B-K) criterion | α | 2.284 |

The CZM is defined using a bilinear traction-separation law. Traction increases linearly with a stiffness p_n until the opening crack reaches a damage initiation displacement u_0 . When the crack opens beyond u_0 , the material softens irreversibly and the stiffness decreases as a function of increasing damage d. The material fails once the stiffness has decreased to zero, that is, when d = 1. This happens at the ultimate displacement u_f .

The values of u_0 and u_f depend on whether the separation displacement is normal (mode I) or tangential (mode II and III) to an interface. For the mixed mode, a combination is used. For the displacement based damage model, two different criteria are available to define this combination. Here the model by Benzeggagh and Kenane is used.

MIXED-MODE BENDING OF A LAMINATED COMPOSITE BEAM

A commonly used method to measure the delamination resistance of composite materials is the mixed-mode bending (MMB) test, see Ref. 1 and Ref. 2. This experimental procedure is here modeled to demonstrate the capabilities of the CZM.

The geometry of the test specimen is illustrated in Figure 1. It consists of a beam cracked along a ply interface halfway through its thickness. The initial crack length is c_1 . The beam is supported at the outermost bottom edges. A mixed-mode bending load is produced as the result of forces applied to the top edges at the cracked end and at the center of the beam.

Because of the symmetry, only half of the beam is modeled and a Symmetry boundary condition is applied.

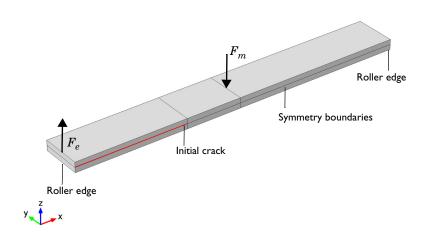


Figure 1: The geometry of the test specimen.

The material properties are those of AS4/PEEK unidirectional laminates. The orthotropic linear elastic properties assume that the longitudinal direction is aligned with the global X direction. The material properties of the laminate composite are listed in Table 2.

TABLE 2: LAMINATED COMPOSITE MATERIAL PROPERTIES.

| PROPERTY | SYMBOL | VALUE |
|--------------------------------|-------------------------------------|-----------|
| Young's modulus, along fibers | $E_{ m X}$ | 122.7 GPa |
| Young's modulus, across fibers | $E_{ m Y}\!\!=\!\!E_{ m Z}$ | 10.1 GPa |
| Poisson's ratio | $v_{ m YZ}$ | 0.45 |
| Poisson's ratio | $v_{\mathrm{XY}} = v_{\mathrm{XZ}}$ | 0.25 |
| Shear modulus | $G_{ m YZ}$ | 3.7 GPa |
| Shear modulus | $G_{\mathrm{XY}} = G_{\mathrm{XZ}}$ | 5.5 GPa |

The beam is supported on the bottom at its outer edges. A lever that sits on top of the beam applies a load. The lever is also attached to the cracked end and swivels around a contact area at the center of the beam. The lever is pushed down at the opposite free end, thereby simultaneously applying mode I and mode II loads on the test specimen. Arbitrary ratios of mixed-mode loading can be adjusted by varying the length of the lever l_1 .

In this example, the lever is omitted. Instead, the forces that the lever transmits to the beam are applied directly. A pulling force F_e is acting on the cracked side of the beam. At the center, a force $F_{\rm m}$ pushes down. The desired mixed-mode ratio $m_{\rm m}$ regulates the ratio of their magnitudes l_r via

$$l_{\rm r} = 8 \Biggl(\frac{6 m_{\rm m} + \sqrt{3 m_{\rm m} (1 - m_{\rm m})}}{3 + 9 m_{\rm m} + 8 \sqrt{3 m_{\rm m} (1 - m_{\rm m})}} \Biggr) \,. \label{eq:lr}$$

Further details on the background of the equation above can be found in Ref. 1 and Ref. 2.

Results and Discussion

The model is analyzed for a mixed-mode ratio of 50%. The von Mises stress distribution of the last computed parameter step is shown in Figure 2. At this step the initial crack has propagated along the interface as shown in Figure 3. The crack now extends beyond the center of the beam.

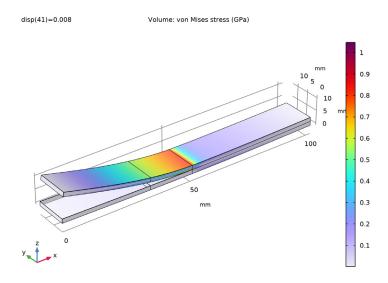


Figure 2: The Von Mises stress distribution at the last computation step.

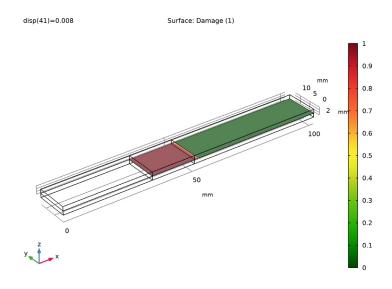


Figure 3: Plot showing the health of the laminate interface. The debonded part is shown in red, the intact part in green.

One of the outputs of the MMB test is a load-displacement curve. Both the load and displacement are measured at the endpoint of the lever that is used to apply the load to the test specimen. Since the lever is not explicitly modeled, the load-displacement data has to be deduced from the simulation results. Details of the analysis are contained in Ref. 1 and Ref. 2, with the following result.

The force F_{lp} at the load point of the lever can be determined from the load applied to the cracked edge in the model $F_{\rm e}$ and the lengths of the test beam $l_{\rm b}$ and load lever $l_{\rm l}$:

$$F_{\rm lp} = F_{\rm e} \frac{l_{\rm b}/2}{l_{\rm l}}.$$

The length of the load lever above depends on the desired mode mixture m_m :

$$l_1 = \frac{(l_{\rm b}/2)\!\!\left(\!\frac{1}{2}\sqrt{\!3\frac{1-m_{\rm m}}{m_{\rm m}}}+1\!\right)}{3\!-\!\frac{1}{2}\sqrt{\!3\frac{1-m_{\rm m}}{m_{\rm m}}}}\,.$$

 l_1 measures the length from the center of the test specimen to the free end of the load lever.

The displacement at the load point u_{lp} is computed from the mode I opening at the cracked edge $u_{\rm Ie}$ and the z-displacement at the center of the beam $w_{\rm c}$ according to

$$u_{\rm lp} = \left(\frac{3l_{\rm l} - l_{\rm b}/2}{4l_{\rm b}/2}\right) u_{\rm Ie} + \left(\frac{l_{\rm l} + l_{\rm b}/2}{l_{\rm b}/2}\right) (-w_{\rm c} + u_{\rm Ie}/4).$$

The resulting load-displacement curve is shown in Figure 4. The curve confirms what Figure 3 displayed. The maximum load that the beam with the initial crack can carry is exceeded and delamination occurs. After exceeding a peak load, the load decreases until the displacement reaches around 7 mm. This point approximately corresponds to when

the crack reaches the center of the specimen. Thereafter, the load starts to increase again, but with a much lower stiffness than before delamination.

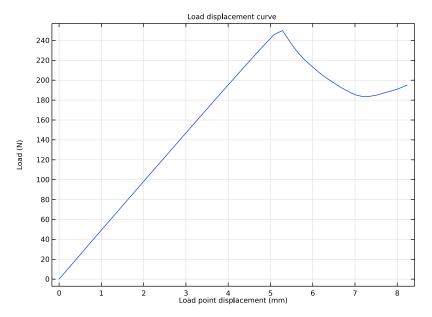


Figure 4: Load-displacement curve of the MMB test at 50% mixed-mode loading.

Notes About the COMSOL Implementation

To implement a cohesive zone model in COMSOL Multiphysics, use the Contact node with an Adhesion subnode and a Decohesion subnode. Notice that Decohesion requires an active Adhesion subnode.

Since no large relative deformations of the laminates are expected, the contact search can be made in the initial configuration. Setting the Mapping method to Initial configuration in the **Contact Pair** node significantly improves the performance of the model.

To trace the solution after the peak load, the simulation must be displacement controlled. This is obtained by using a global equation, in which the distributed load is controlled by the sweep parameter, which is the displacement at the free edge.

The resolve the decohesion at the interface, a dense mesh is used in the zone where delamination is expected. If the mesh elements were too large, the crack could jump several elements in one parameter step, resulting in an unstable solution.

- 1. P.P. Camanho, C.G. Davila, and M.F. De Moura, "Numerical Simulation of Mixedmode Progressive Delamination in Composite Materials," Journal of composite materials 37.16 (2003): 1415-1438.
- 2. J.R. Reeder, and J.R. Crews Jr., "Mixed-mode bending method for delamination testing," AiAA Journal 28.7 (1990): 1270-1276.

Application Library path: Structural Mechanics Module/ Contact and Friction/cohesive zone debonding

Modeling Instructions

From the File menu, choose New.

NEW

In the New window, click Model Wizard.

MODEL WIZARD

- I In the Model Wizard window, click **3D**.
- 2 In the Select Physics tree, select Structural Mechanics>Solid Mechanics (solid).
- 3 Click Add.
- 4 Click Study.
- 5 In the Select Study tree, select General Studies>Stationary.
- 6 Click M Done.

GLOBAL DEFINITIONS

Load all model parameters from a file containing parameters for the geometry, material properties and boundary conditions.

Parameters 1

- I In the Model Builder window, under Global Definitions click Parameters I.
- 2 In the Settings window for Parameters, locate the Parameters section.
- 3 Click Load from File.

4 Browse to the model's Application Libraries folder and double-click the file cohesive_zone_debonding_parameters.txt.

GEOMETRY I

- I In the Model Builder window, under Component I (compl) click Geometry I.
- 2 In the Settings window for Geometry, locate the Units section.
- 3 From the Length unit list, choose mm.

Now draw the model geometry with three layered blocks.

Block I (blk I)

- I In the Geometry toolbar, click Block.
- 2 In the Settings window for Block, locate the Size and Shape section.
- **3** In the **Width** text field, type 1b.
- 4 In the Depth text field, type wb/2.
- 5 In the Height text field, type hb/2.

Block 2 (blk2)

- I In the Geometry toolbar, click Block.
- 2 In the Settings window for Block, locate the Size and Shape section.
- 3 In the Width text field, type 1b/2-cl.
- 4 In the **Depth** text field, type wb/2.
- 5 In the Height text field, type hb/2.
- 6 Locate the Position section. In the x text field, type c1.

Union I (uni I)

- I In the Geometry toolbar, click Booleans and Partitions and choose Union.
- 2 Click in the **Graphics** window and then press Ctrl+A to select both objects.

Copy I (copy I)

- I In the Geometry toolbar, click \(\sum_{\text{in}} \) Transforms and choose Copy.
- **2** Select the object **unil** only.
- 3 In the Settings window for Copy, locate the Displacement section.
- 4 In the z text field, type hb/2.

Form Union (fin)

I In the Model Builder window, under Component I (compl)>Geometry I click Form Union (fin).

- 2 In the Settings window for Form Union/Assembly, locate the Form Union/Assembly section.
- 3 From the Action list, choose Form an assembly.
- 4 From the Pair type list, choose Contact pair.
- 5 In the Geometry toolbar, click **Build All**.

DEFINITIONS

Load variables for the load point from files.

Load Point Variables

- I In the Home toolbar, click ∂ = Variables and choose Local Variables.
- 2 In the Settings window for Variables, type Load Point Variables in the Label text field.
- 3 Locate the Variables section. Click **Load from File.**
- **4** Browse to the model's Application Libraries folder and double-click the file cohesive zone debonding load point variables.txt.

Integration Edge

- I In the Definitions toolbar, click / Nonlocal Couplings and choose Integration. The following nonlocal integration couplings make values of the selected points globally available.
- 2 In the Settings window for Integration, type Integration Edge in the Label text field.
- 3 Locate the Source Selection section. From the Geometric entity level list, choose Point.
- **4** Select Point 17 only.
- 5 Locate the Advanced section. From the Frame list, choose Material (X, Y, Z).

Integration Center

- I In the Definitions toolbar, click M Nonlocal Couplings and choose Integration.
- 2 In the Settings window for Integration, type Integration Center in the Label text field.
- 3 Locate the Source Selection section. From the Geometric entity level list, choose Point.
- **4** Select Point 26 only.
- 5 Locate the Advanced section. From the Frame list, choose Material (X, Y, Z).

Contact Pair I (ab I)

Remove the boundaries that correspond to the initial crack from the contact pair.

- I In the Model Builder window, click Contact Pair I (ap I).
- 2 In the Settings window for Pair, locate the Pair Type section.

- 3 Select the Manual control of selections and pair type check box.
- **4** Locate the **Source Boundaries** section. Click to select the **Activate Selection** toggle button.
- **5** In the list, select **4**.
- 6 Click Remove from Selection.
- **7** Select Boundaries 9 and 14 only.
- 8 Locate the **Destination Boundaries** section. Click to select the **Destination Boundaries** section toggle button.
- **9** In the list, select **19**.
- 10 Click Remove from Selection.
- II Select Boundaries 24 and 29 only.

Since no large relative deformations of the laminates are expected, the contact search can be made in the initial configuration. This setting results in a significant speed up of the problem.

- 12 Locate the Advanced section. From the Mapping method list, choose Initial configuration.
- 13 In the Extrapolation tolerance text field, type 1e-2.

MATERIALS

Create a new material containing the orthotropic linear elastic properties of AS4/PEEK.

AS4/PEEK

- I In the Materials toolbar, click **Blank Material**.
- 2 In the Settings window for Material, type AS4/PEEK in the Label text field.
- 3 Click to expand the Material Properties section. In the Material properties tree, select Solid Mechanics>Linear Elastic Material>Orthotropic>Young's modulus (Evector).
- 4 Click + Add to Material.

5 Locate the **Material Contents** section. In the table, enter the following settings:

| Property | Variable | Value | Unit | Property group |
|-----------------|--------------------------------------|------------------------------|-------|----------------|
| Young's modulus | {Evector1, Evector2, Evector3} | {122.7e9, 10.1e9, 10.1e9} | Pa | Orthotropic |
| Poisson's ratio | {nuvector I, nuvector2, nuvector3} | {0.25, 0.45, 0.25} | I | Orthotropic |
| Shear modulus | {Gvector I, Gvector2, Gvector3 } | {5.5e9, 3.7e9, 5.5e9} | N/m² | Orthotropic |
| Density | rho | 1570 | kg/m³ | Basic |

SOLID MECHANICS (SOLID)

Linear Elastic Material I

- I In the Model Builder window, under Component I (compl)>Solid Mechanics (solid) click Linear Elastic Material I.
- 2 In the Settings window for Linear Elastic Material, locate the Linear Elastic Material section.
- 3 From the Solid model list, choose Orthotropic.

Add the CZM using a **Contact** node.

Contact I

- I In the Model Builder window, click Contact I.
- 2 In the Settings window for Contact, locate the Contact Pressure Penalty Factor section.
- 3 From the Penalty factor control list, choose User defined.
- 4 In the p_n text field, type pn.

Adhesion I

- I In the Physics toolbar, click 🕞 Attributes and choose Adhesion.
- 2 In the Settings window for Adhesion, locate the Adhesive Activation section.
- 3 From the Activation criterion list, choose Always active.

4 Locate the **Adhesive Stiffness** section. In the n_{τ} text field, type 1.

Contact I

In the Model Builder window, click Contact 1.

Decohesion I

- I In the Physics toolbar, click 🕞 Attributes and choose Decohesion.
- 2 In the Settings window for Decohesion, locate the Decohesion section.
- **3** In the σ_t text field, type sigmat.
- 4 In the σ_s text field, type sigmas.
- **5** In the $G_{\rm ct}$ text field, type Gct.
- **6** In the $G_{\rm cs}$ text field, type Gcs.
- 7 From the Mixed mode criterion list, choose Benzeggagh-Kenane.
- **8** In the α text field, type alpha.

Symmetry I

- I In the Physics toolbar, click **Boundaries** and choose Symmetry.
- **2** Select Boundaries 2, 7, 12, 18, 23, and 28 only.

Load on Cracked Edge (Fe)

- I In the Physics toolbar, click Edges and choose Edge Load.
- 2 In the Settings window for Edge Load, type Load on Cracked Edge (Fe) in the Label text field.
- **3** Select Edge 32 only.
- 4 Locate the Force section. From the Load type list, choose Total force.
- **5** Specify the \mathbf{F}_{tot} vector as

| 0 | x |
|-------|---|
| 0 | у |
| force | z |

Load on Middle Edge (Fm)

- I In the Physics toolbar, click Edges and choose Edge Load.
- 2 In the Settings window for Edge Load, type Load on Middle Edge (Fm) in the Label text field.
- **3** Select Edge 48 only.
- 4 Locate the Force section. From the Load type list, choose Total force.

5 Specify the \mathbf{F}_{tot} vector as

| 0 | x |
|-----------|---|
| 0 | у |
| -lr*force | z |

Prescribed Displacement I

- I In the Physics toolbar, click **Edges** and choose **Prescribed Displacement**.
- 2 In the Settings window for Prescribed Displacement, locate the Edge Selection section.
- 3 Click Paste Selection.
- 4 In the Paste Selection dialog box, type 2, 26 in the Selection text field.
- 5 Click OK.
- 6 In the Settings window for Prescribed Displacement, locate the Prescribed Displacement
- 7 Select the Prescribed in z direction check box.

Prescribed Displacement 2

- I In the Physics toolbar, click Points and choose Prescribed Displacement.
- **2** Select Point 1 only.
- 3 In the Settings window for Prescribed Displacement, locate the Prescribed Displacement section.
- 4 Select the Prescribed in x direction check box.
- 5 Click the Show More Options button in the Model Builder toolbar.
- 6 In the Show More Options dialog box, in the tree, select the check box for the node Physics>Equation-Based Contributions.
- 7 Click OK.

This is to make **Global Equations** accessible. Add a global equation to control the applied load with a monotonically increasing parameter.

Global Equations 1

- I In the Physics toolbar, click A Global and choose Global Equations.
- 2 In the Settings window for Global Equations, locate the Global Equations section.
- **3** In the table, enter the following settings:

| Name | f(u,ut,utt,t) (1) | Initial value (u_0) (1) | Initial value (u_t0) (1/s) |
|-------|-------------------|-------------------------|----------------------------|
| force | disp-u_Ie | 0 | 0 |

- 4 Locate the Units section. Click Select Dependent Variable Quantity.
- 5 In the Physical Quantity dialog box, type force in the text field.
- 6 Click **Filter**.
- 7 In the tree, select General>Force (N).
- 8 Click OK.
- 9 In the Settings window for Global Equations, locate the Units section.
- 10 Click Select Source Term Quantity.
- II In the Physical Quantity dialog box, type length in the text field.
- 12 Click **Filter**.
- 13 In the tree, select General>Length (m).
- I4 Click OK.

MESH I

Mapped I

- I In the Mesh toolbar, click A Boundary and choose Mapped.
- **2** Select Boundaries 4, 9, 14, 19, 24, and 29 only.

Distribution I

- I Right-click Mapped I and choose Distribution.
- **2** Select Edges 4 and 30 only.
- 3 In the Settings window for Distribution, locate the Distribution section.
- 4 In the Number of elements text field, type 3.

Distribution 2

- I In the Model Builder window, right-click Mapped I and choose Distribution.
- 2 Select Edges 5 and 31 only.
- 3 In the Settings window for Distribution, locate the Distribution section.
- 4 In the Number of elements text field, type 10.

Distribution 3

- I Right-click Mapped I and choose Distribution.
- 2 Select Edges 13 and 39 only.
- 3 In the Settings window for Distribution, locate the Distribution section.
- 4 In the Number of elements text field, type 50.

Distribution 4

- I Right-click Mapped I and choose Distribution.
- 2 In the Settings window for Distribution, locate the Distribution section.
- 3 From the Distribution type list, choose Predefined.
- **4** Select Edges 21, 24, 47, and 51 only.
- 5 In the Number of elements text field, type 20.
- 6 In the Element ratio text field, type 5.

Swept I

In the Mesh toolbar, click A Swept.

Distribution I

- I Right-click Swept I and choose Distribution.
- 2 In the Settings window for Distribution, locate the Distribution section.
- **3** In the **Number of elements** text field, type 2.
- 4 Click Build All.

STUDY I

Configure the solver to enable tracking of the post peak behavior of the beam.

Step 1: Stationary

- I In the Model Builder window, under Study I click Step I: Stationary.
- 2 In the Settings window for Stationary, click to expand the Study Extensions section.
- 3 Select the Auxiliary sweep check box.
- 4 Click + Add.
- **5** In the table, enter the following settings:

| Parameter name | Parameter value list |
|-------------------------------|----------------------|
| disp (Displacement parameter) | range(0,2e-4,8e-3) |

6 Click to expand the Results While Solving section. Select the Plot check box.

Solution I (soll)

- I In the Study toolbar, click Show Default Solver.
- 2 In the Model Builder window, expand the Solution I (soll) node, then click Stationary Solver 1.
- 3 In the Settings window for Stationary Solver, locate the General section.

- 4 In the Relative tolerance text field, type 1e-4.
- 5 In the Model Builder window, expand the Study I>Solver Configurations> Solution I (sol1)>Dependent Variables I node, then click State variable force (comp1.0DEI).
- 6 In the Settings window for State, locate the Scaling section.
- 7 From the Method list, choose Manual.
- 8 In the Scale text field, type 200.

Use a linear predictor.

- 9 In the Model Builder window, expand the Study I>Solver Configurations> Solution I (soll)>Stationary Solver I node, then click Parametric I.
- 10 In the Settings window for Parametric, click to expand the Continuation section.
- II Select the Tuning of step size check box.
- 12 In the Minimum step size text field, type 1e-6.
- 13 From the Predictor list, choose Linear.
 Switch to an undamped Newton method.
- 14 In the Model Builder window, under Study I>Solver Configurations>Solution I (soll)> Stationary Solver I click Fully Coupled 1.
- **I5** In the **Settings** window for **Fully Coupled**, click to expand the **Method and Termination** section.
- 16 From the Nonlinear method list, choose Constant (Newton).
- 17 In the Study toolbar, click **Compute**.

RESULTS

Volume 1

- I In the Model Builder window, expand the Stress (solid) node, then click Volume I.
- 2 In the Settings window for Volume, locate the Expression section.
- 3 From the Unit list, choose GPa.
- 4 In the Stress (solid) toolbar, click Plot.

Now, add a plot showing the state of debonding at the laminate interface.

Interface Health

I In the Home toolbar, click Add Plot Group and choose 3D Plot Group.

2 In the Settings window for 3D Plot Group, type Interface Health in the Label text field.

Surface I

- I Right-click Interface Health and choose Surface.
- 2 In the Settings window for Surface, click Replace Expression in the upper-right corner of the Expression section. From the menu, choose Component I (compl)>Solid Mechanics> Contact>Adhesion>solid.bdmg - Damage.
- 3 Locate the Coloring and Style section. From the Color table list, choose Traffic. This choice plots the debonded part in red while the healthy part remains green.
- 4 In the Interface Health toolbar, click **Plot**.

The following plot corresponds to the load-displacement output of the MMB test.

Load Displacement Curve

- I In the Home toolbar, click 📭 Add Plot Group and choose ID Plot Group.
- 2 In the Settings window for ID Plot Group, type Load Displacement Curve in the Label text field.
- 3 Click to expand the Title section. From the Title type list, choose Manual.
- 4 In the **Title** text area, type Load displacement curve.

Global I

- I Right-click Load Displacement Curve and choose Global.
- 2 In the Settings window for Global, locate the y-Axis Data section.
- **3** In the table, enter the following settings:

| Expression | Unit | Description |
|------------|------|-------------|
| 2*F_1p | N | Load |

The factor 2 is needed to compensate for the model symmetry.

- 4 Locate the x-Axis Data section. From the Parameter list, choose Expression.
- 5 In the Expression text field, type u 1p.
- 6 From the Unit list, choose mm.
- 7 Click to expand the **Legends** section. Clear the **Show legends** check box.

Load Displacement Curve

I In the Model Builder window, click Load Displacement Curve.

Finally, evaluate the maximal load that this beam can carry under this loading condition.

Global Evaluation 1

- I In the Results toolbar, click (8.5) Global Evaluation.
- 2 In the Settings window for Global Evaluation, locate the Expressions section.
- **3** In the table, enter the following settings:

| Expression | Unit | Description |
|------------|------|-------------|
| 2*F_1p | N | Load |

Again, the factor 2 is due to model symmetry.

- 4 Locate the Data Series Operation section. From the Transformation list, choose Maximum.
- 5 Click **= Evaluate**.