

# Evaluation of Dynamic Coefficients of a Plain Journal Bearing

# Introduction

When analyzing rotors, it is common that bearings are modeled through their effective dynamic coefficients about a static equilibrium position. This example demonstrates a method to compute such coefficients for a plain journal bearing. Computed coefficients are compared to analytical values obtained from solving Reynolds equation, using a short bearing approximation. To make the comparison meaningful, the length of the bearing is taken to be much smaller than its diameter.

# Model Definition

The plain journal bearing has a radius of 0.1 m, and a length of 0.04 m. The angular velocity of the journal is 1000 rad/s, and the clearance between the journal and the bearing is 0.0001 m. The viscosity and density of the lubricant are taken as 0.02 Pa·s and 866 kg/m<sup>3</sup>, respectively. To find the equilibrium position corresponding to different static loads, the journal weight is varied from 500 N to 50,000 N.

Bearing stiffness and damping coefficients are computed for the equilibrium positions by solving a perturbed form of Reynolds equation.

The dimensionless stiffness and damping coefficients obtained from an analytical solution of Reynolds equation (Ref. 1) are:

$$\begin{split} k_{22} &= \frac{4[16\varepsilon^2 + \pi^2(2 - \varepsilon^2)]}{Q} \\ k_{23} &= \frac{\pi[\pi^2(1 - \varepsilon^2) - 16\varepsilon^4]}{Q\varepsilon\sqrt{1 - \varepsilon^2}} \\ k_{32} &= -\frac{\pi[\pi^2(1 + 2\varepsilon^2)(1 - \varepsilon^2) + 32\varepsilon^2(1 + \varepsilon^2)]}{Q(1 - \varepsilon^2)} \\ k_{33} &= \frac{4[\pi^2(1 + 2\varepsilon^2)(1 - \varepsilon^2) + 32\varepsilon^2(1 + \varepsilon^2)]}{Q(1 - \varepsilon^2)} \end{split}$$

and

$$\begin{split} c_{22} &= \frac{2\pi [\pi^2 (1+2\epsilon^2) - 16\epsilon^2] \sqrt{1-\epsilon^2}}{Q\epsilon} \\ c_{23} &= \frac{8 [16\epsilon^2 - \pi^2 (1+2\epsilon^2)]}{Q} \\ c_{32} &= c_{23} \\ c_{33} &= \frac{2\pi [48\epsilon^2 + \pi^2 (1-\epsilon^2)^2]}{Q\epsilon \sqrt{1-\epsilon^2}} \end{split}$$

The parameter  $\varepsilon$  is the relative eccentricity of the journal. Q is given by

$$Q = [\pi^2 + (16 - \pi^2)\epsilon^2]^{3/2}$$

Scaling factors for the above dimensionless parameters are  $k_0 = W/C$  for stiffness and  $c_0 = W/(C\Omega)$  for damping. The bearing load is W, C is the clearance, and  $\Omega$  is the angular speed of the journal.

# Results and Discussion

Figure 1 shows how the journal eccentricity changes with the static load on the bearing. The figure shows that with increasing load, its effect on eccentricity decreases. This clearly depicts the nonlinear behavior of the bearing.



Figure 1: Eccentricity vs load.

Figure 2 shows the computed attitude angle with respect to loading direction, compared to the analytical curve. For small loads the curves coincide. With increasing loads, the journal becomes increasingly eccentric in the bearing. This produces a difference in shear forces at the minimum and maximum film thickness locations. The difference results in a net force on the journal. In high eccentricity cases, the journal equilibrium location is



determined by the balance of external loads on the bearing, and the pressure and shear forces.

Figure 2: Attitude angle vs load.



The maximum film pressure and minimum film thickness are two important performance parameters for a bearing. These are plotted in Figure 3.

Figure 3: Maximum pressure and minimum film thickness vs load.

Figure 4 and Figure 5 compare the computed values of the dimensionless stiffness and dimensionless damping coefficient with the corresponding analytical values. The computed values match the analytical values.



Figure 4: Dimensionless stiffness.



Figure 5: Dimensionless damping.

# Reference

1. J.S. Rao, *Rotor Dynamics*, section 7.6, pp. 179–191, New Age International (P) Limited, 2014.

**Application Library path:** Rotordynamics\_Module/Verification\_Examples/ journal\_bearing\_dynamic\_coefficients

# Modeling Instructions

From the File menu, choose New.

NEW

In the New window, click 🔗 Model Wizard.

#### MODEL WIZARD

- I In the Model Wizard window, click 间 3D.
- 2 In the Select Physics tree, select Structural Mechanics>Rotordynamics> Hydrodynamic Bearing (hdb).
- 3 Click Add.
- 4 Click  $\bigcirc$  Study.
- 5 In the Select Study tree, select General Studies>Stationary.
- 6 Click **M** Done.

## GLOBAL DEFINITIONS

#### Parameters 1

- I In the Model Builder window, under Global Definitions click Parameters I.
- 2 In the Settings window for Parameters, locate the Parameters section.
- **3** In the table, enter the following settings:

Name	Expression	Value	Description
Rj	0.1[m]	0.1 m	Journal radius
Н	0.04[m]	0.04 m	Journal width
С	1e-4[m]	IE-4 m	Clearance
Omega	1000[rad/s]	1000 rad/s	Angular velocity
muO	0.02[Pa*s]	0.02 Pa·s	Lubricant viscosity
rho0	866[kg/m^3]	866 kg/m³	Lubricant density
W	500[N]	500 N	Static load on bearing

## GEOMETRY I

Cylinder I (cyl1)

- I In the **Geometry** toolbar, click **(D)** Cylinder.
- 2 In the Settings window for Cylinder, locate the Object Type section.
- 3 From the Type list, choose Surface.
- 4 Locate the Size and Shape section. In the Radius text field, type Rj.
- 5 In the Height text field, type H.
- 6 Locate the Axis section. From the Axis type list, choose x-axis.
- 7 Click 🟢 Build All Objects.

Define the variables for the analytical stiffness and damping.

## DEFINITIONS

## Variables I

- I In the Model Builder window, expand the Component I (compl)>Definitions node.
- 2 Right-click **Definitions** and choose **Variables**.
- 3 In the Settings window for Variables, locate the Variables section.
- **4** In the table, enter the following settings:

Name	Expression	Unit	Description
k0	W/C	N/m	Stiffness scaling
c0	W/(C*Omega)	N∙s/m	Damping scaling
е	hdb.hjb1.ec_rel		Eccentricity
phi0	atan2(pi*sqrt(1-e^2), 4*e)	rad	Attitude angle
Q	(pi^2+(16-pi^2)* e^2)^1.5		Auxiliary variable
k22	4*(16*e^2+pi^2*(2- e^2))/Q		Dimensionless stiffness, 22 component
k23	pi*(pi^2*(1-e^2)-16* e^4)/(e*sqrt(1-e^2)*Q)		Dimensionless stiffness, 23 component
k32	-pi*(pi^2*(1+2*e^2)* (1-e^2)+32*e^2*(1+ e^2))/(e*sqrt(1-e^2)* Q)		Dimensionless stiffness, 32 component
k33	4*(pi^2*(1+2*e^2)*(1- e^2)+32*e^2*(1+e^2))/ ((1-e^2)*Q)		Dimensionless stiffness, 33 component
c22	2*pi*sqrt(1-e^2)* (pi^2*(1+2*e^2)-16* e^2)/(e*Q)		Dimensionless damping, 22 component
c23	8*(16*e^2-pi^2*(1+2* e^2))/Q		Dimensionless damping, 23 component
c32	c23		Dimensionless damping, 32 component
c33	2*pi*(48*e^2+pi^2*(1- e^2)^2)/(e*sqrt(1- e^2)*Q)		Dimensionless damping, 33 component

#### HYDRODYNAMIC BEARING (HDB)

- I In the Model Builder window, under Component I (comp1) click Hydrodynamic Bearing (hdb).
- 2 In the Settings window for Hydrodynamic Bearing, locate the Dynamic Coefficients section.
- **3** Select the **Calculate dynamic coefficients** check box.

## Hydrodynamic Journal Bearing 1

- I In the Model Builder window, under Component I (compl)>Hydrodynamic Bearing (hdb) click Hydrodynamic Journal Bearing I.
- **2** In the **Settings** window for **Hydrodynamic Journal Bearing**, locate the **Bearing Properties** section.
- **3** In the *C* text field, type **C**.
- 4 Locate the Journal Properties section. From the Specify list, choose Load.
- **5** Specify the **W**<sub>i</sub> vector as

0	x
0	у
- W	z

**6** Specify the **u**<sub>J0</sub> vector as

0	x
0	у
0	z

**7** In the  $\Omega$  text field, type Omega.

Choose the **Gümbel** boundary condition to consider the film with positive pressure only to participate in the load equilibrium.

- 8 Locate the Film Boundary Condition section. From the Film type list, choose Gümbel.
- **9** Locate the Fluid Properties section. From the  $\mu$  list, choose User defined. In the associated text field, type mu0.
- **IO** From the  $\rho$  list, choose **User defined**. In the associated text field, type rho0.

Use a mapped mesh to resolve the pressure.

#### MESH I

#### Mapped I

- I In the Mesh toolbar, click  $\bigwedge$  Boundary and choose Mapped.
- 2 In the Settings window for Mapped, locate the Boundary Selection section.
- 3 From the Selection list, choose All boundaries.

## Distribution I

- I Right-click Mapped I and choose Distribution.
- 2 Select Edges 1, 2, 4, and 6 only.
- 3 In the Settings window for Distribution, locate the Distribution section.
- 4 In the Number of elements text field, type 15.

#### Distribution 2

- I In the Model Builder window, right-click Mapped I and choose Distribution.
- 2 Select Edge 7 only.
- 3 In the Settings window for Distribution, locate the Distribution section.
- 4 In the Number of elements text field, type 4.
- 5 Click 📗 Build All.

## STUDY I

Step 1: Stationary

- I In the Model Builder window, under Study I click Step I: Stationary.
- 2 In the Settings window for Stationary, click to expand the Study Extensions section.
- **3** Select the **Auxiliary sweep** check box.
- 4 Click + Add.
- **5** In the table, enter the following settings:

Parameter name	Parameter value list	Parameter unit
W (Static load on bearing)	range(500,500,50000)	Ν

6 In the **Home** toolbar, click **= Compute**.

## RESULTS

## Fluid Pressure (hdb)

In the Fluid Pressure (hdb) toolbar, click 💿 Plot.

Use the following instructions to plot the eccentricity versus load curve shown in Figure 1.

## Eccentricity

- I In the Home toolbar, click 🚛 Add Plot Group and choose ID Plot Group.
- 2 In the Settings window for ID Plot Group, type Eccentricity in the Label text field.

## Global I

- I Right-click Eccentricity and choose Global.
- In the Settings window for Global, click Replace Expression in the upper-right corner of the y-Axis Data section. From the menu, choose Component I (compl)>
  Hydrodynamic Bearing>Hydrodynamic Journal Bearing I>Eccentricity and attitude angle>
  hdb.hjbl.ec\_rel Relative eccentricity.
- 3 Click to expand the Coloring and Style section. In the Width text field, type 2.
- **4** In the **Eccentricity** toolbar, click **I** Plot.
- **5** Click the **Show Legends** button in the **Graphics** toolbar.

#### Eccentricity

- I In the Model Builder window, click Eccentricity.
- 2 In the Settings window for ID Plot Group, click to expand the Title section.
- 3 From the Title type list, choose None.
- **4** In the **Eccentricity** toolbar, click **I** Plot.

To compare the computed and analytical attitude angles shown in Figure 2, follow the below instructions.

## Attitude Angle

- I In the Home toolbar, click 🚛 Add Plot Group and choose ID Plot Group.
- 2 In the Settings window for ID Plot Group, type Attitude Angle in the Label text field.

Global I

- I Right-click Attitude Angle and choose Global.
- In the Settings window for Global, click Replace Expression in the upper-right corner of the y-Axis Data section. From the menu, choose Component I (compl)>
  Hydrodynamic Bearing>Hydrodynamic Journal Bearing I>Eccentricity and attitude angle>
  hdb.hjbl.phia Attitude angle rad.
- 3 Locate the y-Axis Data section. In the table, enter the following settings:

Expression	Unit	Description
hdb.hjb1.phia	deg	Attitude angle, COMSOL

4 Locate the Coloring and Style section. In the Width text field, type 3.

#### Global 2

- I In the Model Builder window, right-click Attitude Angle and choose Global.
- 2 In the Settings window for Global, locate the y-Axis Data section.
- **3** In the table, enter the following settings:

Expression	Unit	Description
phi0	deg	Attitude angle, Analytical

- **4** Locate the **Coloring and Style** section. Find the **Line style** subsection. From the **Line** list, choose **None**.
- 5 Find the Line markers subsection. From the Marker list, choose Cycle.
- 6 In the Number text field, type 50.
- 7 In the Attitude Angle toolbar, click **I** Plot.

#### Attitude Angle

- I In the Model Builder window, click Attitude Angle.
- 2 In the Settings window for ID Plot Group, locate the Plot Settings section.
- **3** Select the **y-axis label** check box.
- 4 In the associated text field, type Attitude Angle (deg).
- 5 Locate the Title section. From the Title type list, choose None.

Duplicate the eccentricity plot and follow the instructions below to plot the maximum pressure, and minimum film thickness curves, as shown in Figure 3.

## Pressure and Film Thickness

- I In the Model Builder window, right-click Eccentricity and choose Duplicate.
- 2 In the Settings window for ID Plot Group, type Pressure and Film Thickness in the Label text field.

#### Global I

- I In the Model Builder window, expand the Pressure and Film Thickness node, then click Global I.
- In the Settings window for Global, click Replace Expression in the upper-right corner of the y-Axis Data section. From the menu, choose Component I (compl)>
  Hydrodynamic Bearing>Pressure>hdb.hjbl.p\_max Maximum bearing pressure Pa.

#### Global 2

- I Right-click Results>Pressure and Film Thickness>Global I and choose Duplicate.
- In the Settings window for Global, click Replace Expression in the upper-right corner of the y-Axis Data section. From the menu, choose Component I (compl)>
  Hydrodynamic Bearing>Journal and bearing properties>Film thickness and clearance>
  hdb.hjbl.h\_min Minimum film thickness m.

#### Pressure and Film Thickness

- I In the Model Builder window, click Pressure and Film Thickness.
- 2 In the Settings window for ID Plot Group, locate the Plot Settings section.
- **3** Select the **Two y-axes** check box.
- 4 In the table, select the Plot on secondary y-axis check box for Global 2.
- **5** Click the **Show Legends** button in the **Graphics** toolbar.
- 6 In the Pressure and Film Thickness toolbar, click 💽 Plot.

Figure 4 compares the computed dimensionless stiffness to its analytical counterpart. Follow the instructions below to generate this plot.

## Bearing Stiffness

- I In the Home toolbar, click 🚛 Add Plot Group and choose ID Plot Group.
- 2 In the Settings window for ID Plot Group, type Bearing Stiffness in the Label text field.

## Global I

- I Right-click Bearing Stiffness and choose Global.
- In the Settings window for Global, click Replace Expression in the upper-right corner of the y-Axis Data section. From the menu, choose Component I (compl)>
  Hydrodynamic Bearing>Dynamic coefficients>hdb.hjbl.k22 Bearing stiffness, local yy component N/m.
- 3 Click Add Expression in the upper-right corner of the y-Axis Data section. From the menu, choose Component I (comp1)>Hydrodynamic Bearing>Dynamic coefficients> hdb.hjb1.k23 Bearing stiffness, local yz component N/m.
- 4 Click Add Expression in the upper-right corner of the y-Axis Data section. From the menu, choose Component I (comp1)>Hydrodynamic Bearing>Dynamic coefficients> hdb.hjb1.k32 Bearing stiffness, local zy component N/m.
- 5 Click Add Expression in the upper-right corner of the y-Axis Data section. From the menu, choose Component I (comp1)>Hydrodynamic Bearing>Dynamic coefficients> hdb.hjb1.k33 Bearing stiffness, local zz component N/m.

6 Locate the y-Axis Data section. In the table, enter the following settings:

Expression	Unit	Description
hdb.hjb1.k22/k0	1	k <sub>22</sub> , COMSOL
hdb.hjb1.k23/k0	1	k <sub>23</sub> , COMSOL
hdb.hjb1.k32/k0	1	k <sub>32</sub> , COMSOL
hdb.hjb1.k33/k0	1	k <sub>33</sub> , COMSOL

7 Locate the Coloring and Style section. In the Width text field, type 3.

Global 2

I In the Model Builder window, right-click Bearing Stiffness and choose Global.

2 In the Settings window for Global, locate the y-Axis Data section.

**3** In the table, enter the following settings:

Expression	Unit	Description
k22	1	k <sub>22</sub> , Analytical
k23	1	k <sub>23</sub> , Analytical
k32	1	k <sub>32</sub> , Analytical
k33	1	k <sub>33</sub> , Analytical

- **4** Locate the **Coloring and Style** section. Find the **Line style** subsection. From the **Line** list, choose **None**.
- 5 Find the Line markers subsection. From the Marker list, choose Cycle.
- 6 In the Number text field, type 50.

## **Bearing Stiffness**

- I In the Model Builder window, click Bearing Stiffness.
- 2 In the Settings window for ID Plot Group, locate the Plot Settings section.
- 3 Select the y-axis label check box.
- 4 In the associated text field, type Dimensionless Stiffness.
- 5 Locate the Title section. From the Title type list, choose None.
- 6 Locate the Legend section. From the Position list, choose Lower right.

Figure 5 compares the computed dimensionless damping to its analytical counterpart. Follow the instructions below to generate this plot.

## Bearing Damping Coefficient

I In the Home toolbar, click 🚛 Add Plot Group and choose ID Plot Group.

2 In the Settings window for ID Plot Group, type Bearing Damping Coefficient in the Label text field.

Global I

- I Right-click Bearing Damping Coefficient and choose Global.
- 2 In the Settings window for Global, click Replace Expression in the upper-right corner of the y-Axis Data section. From the menu, choose Component I (compl)>
  Hydrodynamic Bearing>Dynamic coefficients>hdb.hjbl.c22 Bearing damping coefficient, local yy component N·s/m.
- 3 Locate the y-Axis Data section. In the table, enter the following settings:

Expression	Unit	Description
hdb.hjb1.c22/c0	1	c <sub>22</sub> , COMSOL
hdb.hjb1.c23/c0	1	c <sub>23</sub> , COMSOL
hdb.hjb1.c32/c0	1	c <sub>32</sub> , COMSOL
hdb.hjb1.c33/c0	1	c <sub>33</sub> , COMSOL

4 Locate the Coloring and Style section. In the Width text field, type 3.

#### Global 2

- I In the Model Builder window, right-click Bearing Damping Coefficient and choose Global.
- 2 In the Settings window for Global, locate the y-Axis Data section.
- **3** In the table, enter the following settings:

Expression	Unit	Description
c22	1	c <sub>22</sub> , Analytical
c23	1	c <sub>23</sub> , Analytical
c32	1	c <sub>32</sub> , Analytical
c33	1	c <sub>33</sub> , Analytical

- **4** Locate the **Coloring and Style** section. Find the **Line style** subsection. From the **Line** list, choose **None**.
- 5 Find the Line markers subsection. From the Marker list, choose Cycle.
- 6 In the Number text field, type 50.
- 7 In the Bearing Damping Coefficient toolbar, click 💿 Plot.

#### Bearing Damping Coefficient

- I In the Model Builder window, click Bearing Damping Coefficient.
- 2 In the Settings window for ID Plot Group, locate the Title section.

- **3** From the **Title type** list, choose **None**.
- 4 Locate the **Plot Settings** section. Select the **y-axis label** check box.
- **5** In the associated text field, type Dimensionless Damping.