Total Internal Reflection Thin-Film Achromatic Phase Shifter (TIRTF APS)

Introduction

The capability to alter the polarization of light is crucial to a wide variety of optical devices. For example, the polarization of light has a significant effect on the performance of optical isolators, attenuators, and beam splitters. By assigning a specific polarization to light, most notably linear or circular polarization, it is possible to substantially reduce glare in optical systems. Linearly and circularly polarized light are widely used in metrology applications, such as stress analysis and biological diagnosis, and in imaging applications such as the design of camera and video equipment.

One of the most fundamental methods of manipulating polarization is wave retardation, in which one component of the electric field is subjected to a phase delay, or retarded, relative to the orthogonal electric field component in a propagating beam of light.

Of particular interest is a quarter-wave retarder, which imposes a 90° phase delay between the two orthogonal components of the electric field. Quarter-wave retarders can be used to convert linearly polarized light into circularly or elliptically polarized light. By combining quarter-wave retarders and linear polarizers it is possible, for example, to design an optical isolator that transmits incoming radiation but blocks scattered or reflected radiation from returning to the source.

In this tutorial, the phenomenon of total internal reflection is used to design and model an achromatic phase shifter, or a wave retarder that exhibits nearly uniform phase delay over a wide spectral range. The importance of thin dielectric films in achieving such a uniform wave retardation is discussed.

Model Background

Two types of quarter-wave retarder are discussed in this section: birefringent wave plates and prisms exploiting total internal reflection. The latter is the subject of this tutorial.

BIREFRINGENT WAVE PLATES

A conventional wave plate is a birefringent crystal that causes a phase delay between two orthogonal components of the electric field of a propagating ray by allowing these components to propagate at slightly different speeds. Typically, a wave plate is made of a uniaxial material that introduces a relative phase delay δ (SI unit: rad) between the two orthogonal field components given by

$$\delta = \frac{2\pi \Delta n L}{\lambda_0} \tag{1}$$

where λ_0 (SI unit: m) is the free-space wavelength of the radiation, L is the thickness of the wave plate, and Δn is the difference in refractive index between the ordinary axis and extraordinary axis of the crystal. Because linearly polarized light in the direction of the ordinary or extraordinary axes of a birefringent crystal would propagate at different group velocities, linearly polarized light at a 45° angle to these two axes will show a gradual retardation of one component with respect to the orthogonal component, eventually converting the linearly polarized light to circularly polarized light. This phenomenon is illustrated in Figure 1 below.
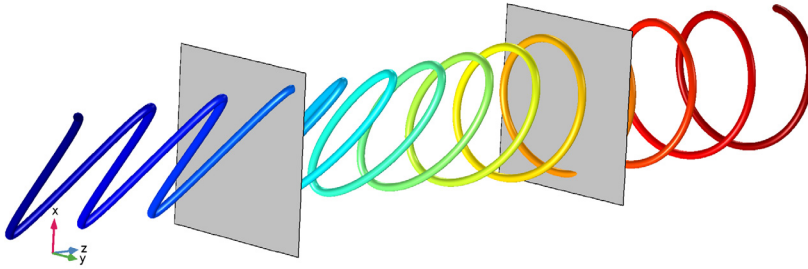

Figure 1: Evolution of the instantaneous electric field amplitude in a wave plate. A birefringent medium exists between the two surfaces, which ordinary and extraordinary axes parallel to the x- and y-axes. As the linearly polarized ray propagates in the +z-direction, it gradually becomes circularly polarized.

A fundamental shortcoming of birefringent wave plates is that they are inherently wavelength-dependent. As shown in Equation 1, the phase shift is inversely proportional to the vacuum wavelength. Typically retarders that introduce phase delays of 90° or 180° are the most useful, and by using a wave plate it would only be possible to get the desired phase delay for specific wavelengths.

An alternative, which is explored in this example, is to use the phenomenon of total internal reflection to design a quarter-wave retarder that is achromatic; that is, a quarter-wave retarder that creates a nearly constant phase delay across a range of vacuum wavelengths.

REFLECTION AND REFRACTION OF LIGHT AT MATERIAL DISCONTINUITIES

To quantify the phase delays introduced by total internal reflections within a glass prism, a preliminary understanding of the fundamentals of reflection and refraction of light is required. The following is explained in more detail in Ref. 1 and in the *Ray Optics Module User's Guide*.

When a ray of light reaches the boundary between two dielectric media, a refracted ray is released in a direction determined by Snell's law,

$$n_1 \sin \theta_i = n_2 \sin \theta_t \tag{2}$$

where

- n_1 (dimensionless) is the refractive index of the medium containing the incident ray,
- n_2 (dimensionless) is the refractive index of the medium containing the refracted ray,
- θ_i (SI unit: rad) is the angle of incidence, and
- θ_t (SI unit: rad) is the angle of refraction.

Alternatively, Snell's law may be written as

$$\cos \theta_t = \sqrt{1 - \left(\frac{n_1}{n_2} \sin \theta_i\right)^2} \tag{3}$$

which shows that the direction cosine of the refracted ray is either a pure real number or a pure imaginary number when the refractive indices are real.

In addition to the refracted ray described above, a reflected ray is released with an angle of reflection equal to the angle of incidence. The incident ray, reflected ray, refracted ray, and surface normal all lie within the same plane, called the plane of incidence. These angles are illustrated below.
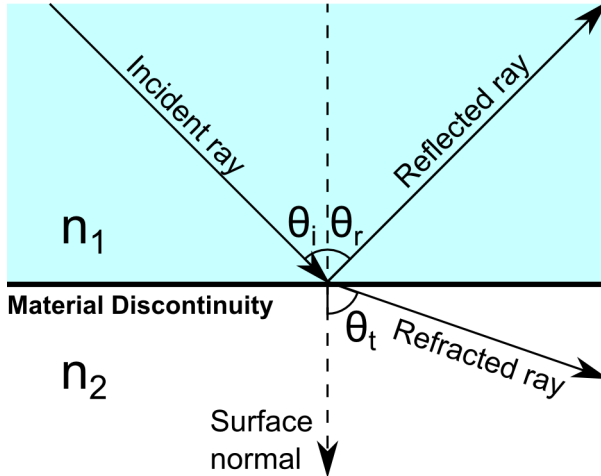

Figure 2: Incident, reflected, and refracted rays at an interface between two materials of different refractive indices.

The electric field amplitudes of the reflected and refracted rays are related to the amplitude of the incident ray by the Fresnel Equations:

$$t_p = \frac{E_p^{(t)}}{E_p^{(i)}} = \frac{2n_1 \cos\theta_i}{n_2 \cos\theta_i + n_1 \cos\theta_t} \qquad t_s = \frac{E_s^{(t)}}{E_s^{(i)}} = \frac{2n_1 \cos\theta_i}{n_1 \cos\theta_i + n_2 \cos\theta_t} \qquad (4)$$

$$r_p = \frac{E_p^{(r)}}{E_p^{(i)}} = \frac{n_2 \cos\theta_i - n_1 \cos\theta_t}{n_2 \cos\theta_i + n_1 \cos\theta_t} \qquad r_s = \frac{E_s^{(r)}}{E_s^{(i)}} = \frac{n_1 \cos\theta_i - n_2 \cos\theta_t}{n_1 \cos\theta_i + n_2 \cos\theta_t} \qquad (5)$$

where

- the subscript p (for "parallel") represents the electric field component parallel to the plane of incidence,
- the subscript s (from the German "senkrecht", or "perpendicular") represents the electric field component perpendicular to the plane of incidence,
- the superscript (i) represents the incident ray,
- the superscript (r) represents the reflected ray, and
- the superscript (t) represents the refracted ray.

The variables r and t are called the reflection coefficient and transmission coefficient, respectively. For example, Equation 4 yields the reflection coefficient of p-polarized radiation, or radiation polarized so that the electric field vector lies in the plane of incidence.

The ratio of the intensities of the transmitted and reflected rays to the intensity of the incident ray are called the transmittance T and reflectance R, respectively, given by

$$R = |r|^2 \qquad (6)$$

$$T = \frac{n_2 \cos\theta_t}{n_1 \cos\theta_i} |t|^2 \qquad (7)$$

By substituting the definitions of the Fresnel coefficients for either s- or p-polarized radiation into Equation 6 and Equation 7, one obtains $R + T = 1$.

If the ray is propagating from a medium of higher refractive index to a medium of lower refractive index, that is, $n_2 < n_1$, then there exists a maximum value of the angle of incidence, beyond which no real value of the angle of refraction exists. This is the critical angle θ_c, given by

$$\theta_c = \text{asin}\frac{n_2}{n_1} \tag{8}$$

which is also the angle at which the radicand in Equation 3 equals zero. When the angle of incidence is greater than the critical angle, that is, $\theta_i > \theta_c$, the ray is completely reflected and no refracted ray can propagate in the adjacent domain. This phenomenon is called total internal reflection. Because $\theta_i > \theta_c$, it follows from Equation 3 that $\cos\theta_t$ has a value on the purely imaginary axis in the complex plane. The choice of whether to use the positive or negative imaginary value for $\cos\theta_t$ depends on the sign convention used to write the electromagnetic wave equation; the convention used in the COMSOL optics products would require $\cos\theta_t$ to be a negative imaginary number,

$$\cos\theta_t = -i\sqrt{\left(\frac{n_1}{n_2}\sin\theta_i\right)^2 - 1} \tag{9}$$

Substitution of Equation 9 into Equation 5 yields

$$r_p = \frac{n_2\cos\theta_i + n_1 i\sqrt{\left(\frac{n_1}{n_2}\sin\theta_i\right)^2 - 1}}{n_2\cos\theta_i - n_1 i\sqrt{\left(\frac{n_1}{n_2}\sin\theta_i\right)^2 - 1}} \qquad r_s = \frac{n_1\cos\theta_i + n_2 i\sqrt{\left(\frac{n_1}{n_2}\sin\theta_i\right)^2 - 1}}{n_1\cos\theta_i - n_2 i\sqrt{\left(\frac{n_1}{n_2}\sin\theta_i\right)^2 - 1}}$$

Let $n = n_2/n_1$; then further simplification yields

$$r_p = \frac{n^2\cos\theta_i + i\sqrt{\sin^2\theta_i - n^2}}{n^2\cos\theta_i - i\sqrt{\sin^2\theta_i - n^2}} \qquad r_s = \frac{\cos\theta_i + i\sqrt{\sin^2\theta_i - n^2}}{\cos\theta_i - i\sqrt{\sin^2\theta_i - n^2}} \tag{10}$$

Both r_p and r_s are complex numbers of the form $A(A^*)^{-1}$, where A is the numerator. Each numerator can be expressed in terms of polar coordinates in the complex plane,

$$n^2\cos\theta_i + i\sqrt{\sin^2\theta_i - n^2} = a_p\exp(ib_p) \qquad \cos\theta_i + i\sqrt{\sin^2\theta_i - n^2} = a_s\exp(ib_s)$$

where the phases are

$$\tan b_p = \frac{\sqrt{\sin^2\theta_i - n^2}}{n^2\cos\theta_i} \qquad \tan b_s = \frac{\sqrt{\sin^2\theta_i - n^2}}{\cos\theta_i} \tag{11}$$

so that the reflection coefficients are of the form

$$r_{\mathrm{p}} = \frac{a_{\mathrm{p}}\exp(b_{\mathrm{p}}i)}{a_{\mathrm{p}}\exp(-b_{\mathrm{p}}i)} = \exp(2ib_{\mathrm{p}}) \qquad r_{\mathrm{s}} = \frac{a_{\mathrm{s}}\exp(b_{\mathrm{s}}i)}{a_{\mathrm{s}}\exp(-b_{\mathrm{s}}i)} = \exp(2ib_{\mathrm{s}}) \qquad (12)$$

which confirms that both reflection coefficients have unit magnitude during TIR.

Let δ_{p} and δ_{s} be the arguments of the complex-valued quantities r_{p} and r_{s}, respectively,

$$r_{\mathrm{p}} = \exp(i\delta_{\mathrm{p}}) \qquad r_{\mathrm{s}} = \exp(i\delta_{\mathrm{s}}) \qquad (13)$$

Comparing Equation 12 with Equation 13 and substituting the expressions in Equation 11 yields

$$\tan\frac{\delta_{\mathrm{p}}}{2} = \frac{\sqrt{\sin^2\theta_{\mathrm{i}} - n^2}}{n^2\cos\theta_{\mathrm{i}}} \qquad \tan\frac{\delta_{\mathrm{s}}}{2} = \frac{\sqrt{\sin^2\theta_{\mathrm{i}} - n^2}}{\cos\theta_{\mathrm{i}}} \qquad (14)$$

The phase delay introduced by the total internal reflection is the difference between the two arguments,

$$\delta = \delta_{\mathrm{p}} - \delta_{\mathrm{s}}$$

Using the difference identity

$$\tan\frac{\delta}{2} = \frac{\tan\dfrac{\delta_{\mathrm{p}}}{2} - \tan\dfrac{\delta_{\mathrm{s}}}{2}}{1 - \tan\dfrac{\delta_{\mathrm{p}}}{2}\tan\dfrac{\delta_{\mathrm{s}}}{2}}$$

and substituting Equation 14 and yields an expression for the phase delay,

$$\delta = 2\operatorname{atan}\left(\frac{\cos\theta_i\sqrt{\sin^2\theta_{\mathrm{i}} - n^2}}{\sin^2\theta_{\mathrm{i}}}\right) \qquad (15)$$

Equation 15 is an expression for the phase delay that doesn't depend on the vacuum wavelength, and therefore can be considered an achromatic phase shifter. However, several technical complications arise when using total internal reflection to induce a 90° phase shift:

- Most common optical materials don't have sufficiently large refractive indices to induce a 90° phase shift with a single reflection. For example, if the region surrounding the prism is an air or vacuum domain ($n_2 = 1$), then the smallest real value of the refractive index is approximately $n_1 = 2.414$ for an angle of incidence of about $\theta_{\mathrm{i}} = 32.9°$. If the

angle of incidence is 45° or greater, then no real finite value of n_2 will yields a 90° phase delay. Therefore, most practical designs would require light to be reflected at least twice, with a smaller phase delay (say, 45°) during each reflection. The Fresnel rhomb, for example, subjects light to two consecutive total internal reflections to cause a total phase delay of 90°.

• Equation 15 can still depend on the vacuum wavelength indirectly because of its dependence on the refractive index ratio n. Most real media are dispersive to some extent; that is, the refractive index is a function of the vacuum wavelength.

• Equation 15 is heavily dependent on the angle of incidence θ_i, and will therefore yield dramatically different phase delays if the incident beam is not collimated; that is, if not all of the incoming rays reach the surface at exactly the specified angle of incidence.

In this tutorial, the dispersive medium and the dependence on the angle of incidence are not considered. However, it will be shown that a greater phase delay can be achieved during a single total internal reflection by coating the surface of the prism with thin dielectric layers.

THIN DIELECTRIC FILMS

The interface between two media can be coated with one or more thin dielectric films to change the coefficients of reflection and transmission. By "thin" it is usually meant that the film is comparable in thickness to the wavelength of the radiation, but for the following analysis the only requirement is that the thickness of the film is much smaller than the coherence length of the radiation. When the incident ray enters a thin dielectric film, it is reflected back and forth at the two adjacent surfaces, producing many refracted rays in each region.
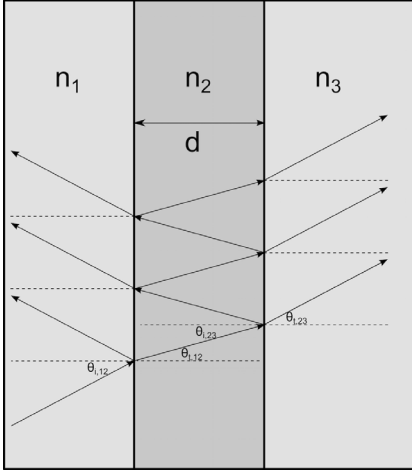
Figure 3: Visualization of the reflected and refracted rays that are created as an incident ray crosses a thin dielectric film.

All of the rays that propagate into either region can interfere constructively or destructively with each other due to the coherent nature of the radiation. The net effect of superposing all of these escaping rays can be considered by replacing the thin film with a fictitious surface with complex Fresnel coefficients:

$$r_{eq} = \frac{r_{12} + r_{23}\exp(-2i\beta)}{1 + r_{12}r_{23}\exp(-2i\beta)} \tag{16}$$

$$t_{eq} = \frac{t_{12}t_{23}\exp(-i\beta)}{1 + r_{12}r_{23}\exp(-2i\beta)} \tag{17}$$

$$\beta = \frac{2\pi n_2 d\cos\theta_2}{\lambda_0} \tag{18}$$

Because the Fresnel coefficients are complex, they induce phase delays on the reflected and refracted rays. Furthermore, simply changing the thickness of the film affects the arguments of the reflection and transmission coefficients. Therefore, thin films offer much more precise control over the phase delays at air-dielectric interfaces compared to uncoated prisms. However, as shown in Equation 18, the thin films introduce a degree of wavelength-dependence that is not seen during total internal reflection at uncoated boundaries. This can be compensated for by using multilayer films, which offer much more precise control of the phase delay over a range of wavelengths.

If multiple dielectric layers are adjacent to each other, the equivalent Fresnel coefficients for the multilayer film can be computed recursively, repeatedly using Equation 16 and Equation 17 while replacing r_{23} with the value of r_{eq} from the previous iteration.

Model Definition

In this tutorial, a linearly polarized ray reaches the surface of a prism at an angle of incidence of 45°. The plane of polarization is at a 45° angle to the plane of incidence, so that the incoming ray is equal parts s- and p-polarized. Therefore, a 90° phase shift should produce circularly polarized light.

As explained in the previous section, no transparent dielectric material can induce a 90° phase delay for light at a 45° angle of incidence, so thin dielectric films must be used to achieve the desired phase delay with only one reflection. Following Ref. 2, the refractive index of the glass prism is n_p = 1.509 and the following combinations of thin films are used:

1 Single layer with n_1 = 2.0535 and t_1 = 0.089λ_0.
2 Triple layer with n_1 = 2.346, n_2 = 1.664, n_3 = 1.496, t_1 = 0.0447λ_0, t_2 = 0.111λ_0, and t_3 = 0.3856λ_0.

Results and Discussion

For the two combinations of thin dielectric films described in the previous section, the phase retardation as a function of free space wavelength is plotted in Figure 4. As is often the case with phase shifters using total internal reflection, a multilayer film offers more precise control over the retardance and reduces its wavelength dependence more than a single-layer film.

The phase retardation in Figure 4 may be compared to Fig. 7 in Ref. 2. However, note that Ref. 2 uses the opposite sign convention for the propagation of a plane electromagnetic wave, compared to the sign convention used in the Ray Optics Module. Hence Figure 4 in this document and Fig. 7 in Ref. 2 will show phase delays of the same magnitude but opposite in sign.
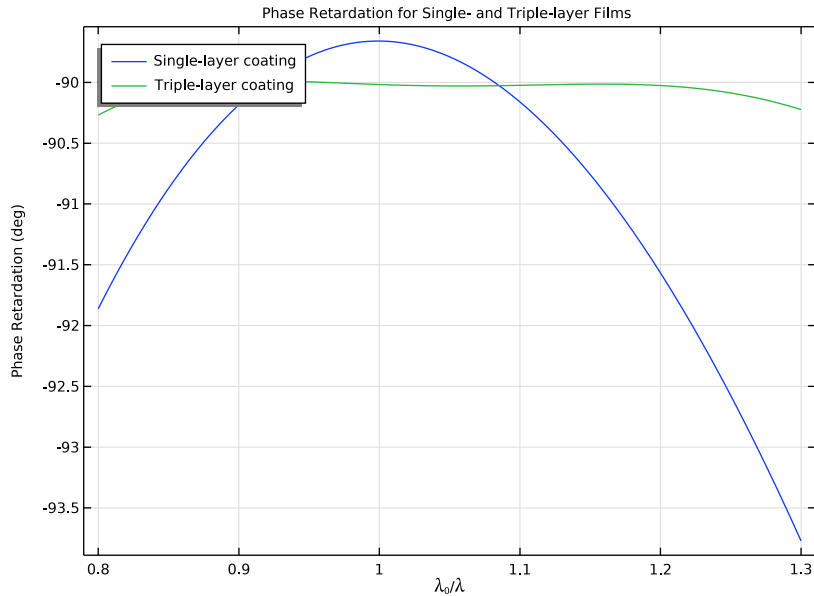
Phase Retardation for Single- and Triple-layer Films

Figure 4: Phase retardation between the s- and p-polarized components of the electric field amplitude are plotted as a function of free space wavelength.

References

1. M. Born and E. Wolf, *Principles of Optics*, 7th ed., Cambridge, 1999.

2. Spiller, Eberhard. "Totally reflecting thin-film phase retarders." Applied Optics 23, no. 20 (1984): 3544-3549.

3. King, R. J. "Quarter-wave retardation systems based on the Fresnel rhomb principle." Journal of Scientific Instruments 43, no. 9 (1966): 617.

Application Library path: `Ray_Optics_Module/Prisms_and_Coatings/`
`achromatic_phase_shifter`

Modeling Instructions

From the **File** menu, choose **New**.

In the **New** window, click **Model Wizard**.

MODEL WIZARD

1 In the **Model Wizard** window, click **3D**.

2 In the **Select Physics** tree, select **Optics>Ray Optics>Geometrical Optics (gop)**.

3 Click **Add**.

4 Click **Study**.

5 In the **Select Study** tree, select **Preset Studies for Selected Physics Interfaces>Ray Tracing**.

6 Click **Done**.

GLOBAL DEFINITIONS

Parameters 1

Load a file containing parameters for the refractive index and thickness of each thin film.

1 In the **Model Builder** window, under **Global Definitions** click **Parameters 1**.

2 In the **Settings** window for **Parameters**, locate the **Parameters** section.

3 Click **Load from File**.

4 Browse to the model's Application Libraries folder and double-click the file
   achromatic_phase_shifter_parameters.txt.

GEOMETRY 1

The geometry is a simple glass prism in which light is incident on the surface at a 45-degree angle.

Work Plane 1 (wp1)
In the **Geometry** toolbar, click **Work Plane**.

Work Plane 1 (wp1)>Plane Geometry
In the **Model Builder** window, click **Plane Geometry**.

Work Plane 1 (wp1)>Polygon 1 (pol1)

1 In the **Work Plane** toolbar, click **Polygon**.

2 In the **Settings** window for **Polygon**, locate the **Coordinates** section.

3 In the table, enter the following settings:

xw (m)	yw (m)
0	0
1	1
0	1

4 Click 🔳 **Build Selected**.

Extrude 1 (ext1)

1 In the **Model Builder** window, under **Component 1 (comp1)>Geometry 1** right-click **Work Plane 1 (wp1)** and choose **Extrude**.

2 In the **Settings** window for **Extrude**, click 🔳 **Build Selected**.

3 Click the ⊕ **Zoom Extents** button in the **Graphics** toolbar.

MATERIALS

Material 1 (mat1)

1 In the **Materials** toolbar, click ▦ **Blank Material**.

2 In the **Settings** window for **Material**, locate the **Material Contents** section.

3 In the table, enter the following settings:

Property	Variable	Value	Unit	Property group
Refractive index, real part	n_iso ; nii = n_iso, nij = 0	np	1	Refractive index

GEOMETRICAL OPTICS (GOP)

Adjust the physics interface settings to allow different wavelengths to be released. This provides a more rigorous test of the achromaticity of the polarizer.

1 In the **Model Builder** window, under **Component 1 (comp1)** click **Geometrical Optics (gop)**.

2 In the **Settings** window for **Geometrical Optics**, locate the **Ray Release and Propagation** section.

3 From the **Wavelength distribution of released rays** list, choose **Polychromatic, specify vacuum wavelength**.

4 In the **Maximum number of secondary rays** text field, type 0. Secondary rays are not needed because all of the primary rays undergo total internal reflection at the material discontinuity.

5 Locate the **Intensity Computation** section. From the **Intensity computation** list, choose **Compute intensity**. When the ray intensity is computed, information about the ray polarization is also available because the intensity calculation method allocates degrees of freedom for all four Stokes parameters.

Release from Grid 1

1 In the **Physics** toolbar, click **Global** and choose **Release from Grid**.

2 In the **Settings** window for **Release from Grid**, locate the **Initial Coordinates** section.

3 In the $q_{x,0}$ text field, type 0.2.

4 In the $q_{y,0}$ text field, type 0.5.

5 In the $q_{z,0}$ text field, type 0.5.

6 Locate the **Ray Direction Vector** section. Specify the $\mathbf{L}_0$ vector as

1	x
0	y
0	z

Specify the wavelength range of the released rays. One ray per wavelength value will be released at the specified point.

7 Locate the **Vacuum Wavelength** section. From the **Distribution function** list, choose **List of values**.

8 Click **Range**.

9 In the **Range** dialog box, choose **Number of values** from the **Entry method** list.

10 In the **Start** text field, type lam0/0.8.

11 In the **Stop** text field, type lam0/1.3.

12 In the **Number of values** text field, type 100.

13 Click **Replace**.

Change the polarization of the incident ray so that it is linearly polarized at a 45-degree angle to the plane of incidence.

14 In the **Settings** window for **Release from Grid**, locate the **Initial Polarization** section.

15 From the **Initial polarization type** list, choose **Fully polarized**.

16 From the **Initial polarization** list, choose **User defined**.

17 In the $a_{2,0}$ text field, type 1.

Because the default value of the **Initial phase difference** is zero, the release feature produces rays that are linearly polarized at a 45-degree angle to the direction specified in the **Initial polarization reference direction** field.

Create two instances of the **Material Discontinuity** boundary condition on the internally reflecting surface. These two features will be given different combinations of thin dielectric films and used in two separate studies.

Single-layer Coating

1 In the **Physics** toolbar, click 🔲 **Boundaries** and choose **Material Discontinuity**.

2 Select Boundary 2 only.

3 In the **Settings** window for **Material Discontinuity**, type Single-layer Coating in the **Label** text field.

4 Locate the **Coatings** section. From the **Thin dielectric films on boundary** list, choose **Add layers to surface**.

Thin Dielectric Film 1

1 In the **Physics** toolbar, click 🔲 **Attributes** and choose **Thin Dielectric Film**.

2 In the **Settings** window for **Thin Dielectric Film**, locate the **Film Properties** section.

3 In the n text field, type nf1.

4 In the t text field, type tf1.

Triple-layer Coating

1 In the **Physics** toolbar, click 🔲 **Boundaries** and choose **Material Discontinuity**.

2 Select Boundary 2 only.

3 In the **Settings** window for **Material Discontinuity**, type Triple-layer Coating in the **Label** text field.

4 Locate the **Coatings** section. From the **Thin dielectric films on boundary** list, choose **Add layers to surface**.

Thin Dielectric Film 1

1 In the **Physics** toolbar, click 🔲 **Attributes** and choose **Thin Dielectric Film**.

2 In the **Settings** window for **Thin Dielectric Film**, locate the **Film Properties** section.

3 In the n text field, type nf2.

4 In the t text field, type tf2.

Triple-layer Coating
In the **Model Builder** window, click **Triple-layer Coating**.

Thin Dielectric Film 2
1 In the **Physics** toolbar, click ![icon] **Attributes** and choose **Thin Dielectric Film**.

2 In the **Settings** window for **Thin Dielectric Film**, locate the **Film Properties** section.

3 In the *n* text field, type nf3.

4 In the *t* text field, type tf3.

Triple-layer Coating
In the **Model Builder** window, click **Triple-layer Coating**.

Thin Dielectric Film 3
1 In the **Physics** toolbar, click ![icon] **Attributes** and choose **Thin Dielectric Film**.

2 In the **Settings** window for **Thin Dielectric Film**, locate the **Film Properties** section.

3 In the *n* text field, type nf4.

4 In the *t* text field, type tf4.

STUDY 1

Step 1: Ray Tracing
1 In the **Model Builder** window, under **Study 1** click **Step 1: Ray Tracing**.

2 In the **Settings** window for **Ray Tracing**, locate the **Study Settings** section.

3 From the **Time-step specification** list, choose **Specify maximum path length**.

4 In the **Lengths** text field, type 0 0.6.

   In the first study, the single-layer coating is used, so the triple-layer coating must be disabled.

5 Locate the **Physics and Variables Selection** section. Select the **Modify model configuration for study step** check box.

6 In the tree, select **Component 1 (Comp1)>Geometrical Optics (Gop)>Triple-Layer Coating**.

7 Click ![icon] **Disable**.

8 In the **Home** toolbar, click ![icon] **Compute**.

RESULTS

Polarization Ellipses, Single-layer Coating
In the **Settings** window for **3D Plot Group**, type Polarization Ellipses, Single-layer Coating in the **Label** text field.

Ray Trajectories 1

1 In the **Model Builder** window, expand the **Polarization Ellipses, Single-layer Coating** node, then click **Ray Trajectories 1**.

2 In the **Settings** window for **Ray Trajectories**, locate the **Coloring and Style** section.

3 Find the **Point style** subsection. From the **Type** list, choose **Ellipse**.

Color Expression 1

1 In the **Model Builder** window, expand the **Ray Trajectories 1** node, then click **Color Expression 1**.

2 In the **Settings** window for **Color Expression**, locate the **Expression** section.

3 In the **Expression** text field, type `gop.s3/gop.s0`. This expression gives the degree of circular polarization, which has a magnitude of zero for linearly polarized or unpolarized radiation and unit magnitude for circularly polarized radiation.

4 In the **Polarization Ellipses, Single-layer Coating** toolbar, click ▣ **Plot**.

5 In the **Graphics** window toolbar, click ▼ next to ↳ **Go to Default View**, then choose **Go to ZX View**. The polarization ellipses appear as line segments for the incident linearly polarized rays and circles for the reflected rays. The polarization ellipses and color expression confirm that the reflected rays are all nearly circularly polarized.

Phase Retardation vs. Wavelength

1 In the **Home** toolbar, click ▦ **Add Plot Group** and choose **1D Plot Group**.

2 In the **Settings** window for **1D Plot Group**, type `Phase Retardation vs. Wavelength` in the **Label** text field.

3 Locate the **Data** section. From the **Dataset** list, choose **Ray 1**.

4 From the **Time selection** list, choose **Last**.

5 Click to expand the **Title** section. From the **Title type** list, choose **Manual**.

6 In the **Title** text area, type `Phase Retardation for Single- and Triple-layer Films`.

7 Locate the **Plot Settings** section. Select the **x-axis label** check box.

8 In the associated text field, type `\lambda<sub>0</sub>/\lambda`.

9 Select the **y-axis label** check box.

10 In the associated text field, type `Phase Retardation (deg)`.

11 Locate the **Legend** section. From the **Position** list, choose **Upper left**.

Ray 1

1 In the **Phase Retardation vs. Wavelength** toolbar, click 〰 **More Plots** and choose **Ray**.

It is convenient to use the **Ray** plot because it can be used to compare any two variables that are defined for all rays.

2 In the **Settings** window for **Ray**, locate the **y-Axis Data** section.

3 In the **Expression** text field, type gop.delta.

4 From the **Unit** list, choose **°**.

5 Locate the **x-Axis Data** section. From the **Parameter** list, choose **Expression**.

6 In the **Expression** text field, type lam0/gop.lambda0.

7 Click to expand the **Legends** section. Select the **Show legends** check box.

8 From the **Legends** list, choose **Manual**.

9 In the table, enter the following settings:

Legends
Single-layer coating

10 In the **Phase Retardation vs. Wavelength** toolbar, click ▣ **Plot**.

The plot shows the phase shift as a function of the free-space wavelength of the radiation. When a single-layer film is used, the phase shift varies by about 4 degrees over the simulated frequency range. Now repeat the analysis using a multilayer film.

ADD STUDY

1 In the **Home** toolbar, click ⚲ **Add Study** to open the **Add Study** window.

2 Go to the **Add Study** window.

3 Find the **Studies** subsection. In the **Select Study** tree, select
Preset Studies for Selected Physics Interfaces>Ray Tracing.

4 Click **Add Study** in the window toolbar.

5 In the **Home** toolbar, click ⚲ **Add Study** to close the **Add Study** window.

STUDY 2

Step 1: Ray Tracing

1 In the **Settings** window for **Ray Tracing**, locate the **Study Settings** section.

2 From the **Time-step specification** list, choose **Specify maximum path length**.

3 In the **Lengths** text field, type 0 0.6.

In the second study, the triple-layer coating is used, so the single-layer coating must be disabled.

4 Locate the **Physics and Variables Selection** section. Select the **Modify model configuration for study step** check box.

5 In the tree, select **Component 1 (Comp1)>Geometrical Optics (Gop)>Single-Layer Coating**.

6 Click ⊘ **Disable**.

7 In the **Home** toolbar, click ▬ **Compute**.

RESULTS

Polarization Ellipses, Triple-layer Coating
In the **Settings** window for **3D Plot Group**, type Polarization Ellipses, Triple-layer Coating in the **Label** text field.

Ray Trajectories 1
1 In the **Model Builder** window, expand the **Polarization Ellipses, Triple-layer Coating** node, then click **Ray Trajectories 1**.

2 In the **Settings** window for **Ray Trajectories**, locate the **Coloring and Style** section.

3 Find the **Point style** subsection. From the **Type** list, choose **Ellipse**.

Color Expression 1
1 In the **Model Builder** window, expand the **Ray Trajectories 1** node, then click **Color Expression 1**.

2 In the **Settings** window for **Color Expression**, locate the **Expression** section.

3 In the **Expression** text field, type gop.s3/gop.s0.

4 In the **Polarization Ellipses, Triple-layer Coating** toolbar, click ▣ **Plot**. The polarization ellipse and color expression confirm that the reflected rays are all nearly circularly polarized.

Ray 2
1 In the **Model Builder** window, under **Results>Phase Retardation vs. Wavelength** right-click **Ray 1** and choose **Duplicate**.

2 In the **Settings** window for **Ray**, locate the **Data** section.

3 From the **Dataset** list, choose **Ray 2**.

4 From the **Time selection** list, choose **Last**.

5 Locate the **Legends** section. In the table, enter the following settings:

Legends
Triple-layer coating

6 In the **Phase Retardation vs. Wavelength** toolbar, click � **Plot**. Compare the resulting plot to Figure 4 in the model documentation and to Fig.7 in Ref. 2.