

Analyzing Porous Structures on the Microscopic Scale

Modeling flow through realistic porous structures is difficult due to the complexity of the structure itself. Resolving the flow field in detail is not feasible in real-life applications. Therefore, macroscopic approaches utilizing averaged quantities of the porous structure, such as porosity and permeability, are used. This example analyzes the flow field at the pore scale in detail. The results are used to validate and adapt the macroscopic description, which in turn are used to model large-scale porous geometries.

Model Definition

The modeled geometry shown in Figure 1 is a representative elementary volume (REV) whose properties are representative for the entire system.

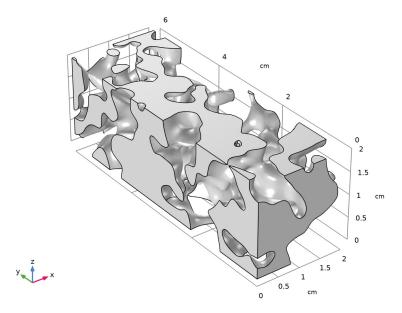


Figure 1: Geometry of the REV's pore volume.

Because only the area of the pore space is required for modeling, the porous matrix is not resolved explicitly. The REV has a quadratic cross section of 2 cm side length and a width of 6 cm. Water flows with a velocity of u = 0.1 mm/s and can flow out freely in the normal direction at the other end. The other boundaries are assumed to be symmetry boundaries. This does not correspond to the actual situation, but for modeling an REV, symmetry boundary conditions are very well suited.

To characterize the flow inside the porous structure one can estimate the Reynolds number according to

$$Re = \frac{\rho u L}{\mu}$$

with the water density $\rho=1000~{\rm kg/m^3}$ and viscosity $\mu=10^{-3}~{\rm Pa\cdot s}$. The cross-section side length serves as the characteristic length scale, L. This results in Re = 2 and the Stokes equation can be used to describe the flow where inertia terms are neglected.

Finally, the goal of the model is to obtain the averaged values for porosity and permeability to describe a macroscopic model with, for example, Darcy's law or the Brinkman equation. The porosity is defined as the fraction of pore space volume V_{fluid} to total volume V_{tot} :

$$\varepsilon = \frac{V_{\text{fluid}}}{V_{\text{tot}}}.$$

To calculate the permeability κ (m²) the following relationship is used:

$$\mathbf{u} = -\frac{\kappa}{\mu} \nabla p$$

Approximating the pressure gradient ∇p by the pressure difference between the inlet and the outlet, Δp , divided by the side length L, and replacing the velocity vector \mathbf{u} by the outlet velocity u_{out} in the flow direction gives the expression

$$\kappa = u_{\text{out}} \mu \frac{L}{\Delta p}$$
.

First of all it is interesting to have a look at the mesh, or rather the mesh quality. Although the mesh quality in itself says nothing about the quality of the results, a good mesh quality is beneficial for convergence. The overall mesh quality as shown in Figure 2 is very good.

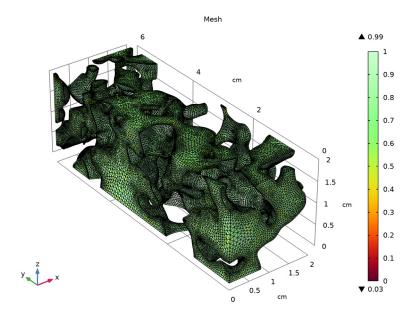


Figure 2: Mesh quality plot.

The velocity field in the REV is shown in Figure 3. Characteristic for the flow in a porous material are areas with high and areas with low velocity. In some areas the flow also stagnates. This behavior is characterized by the permeability.

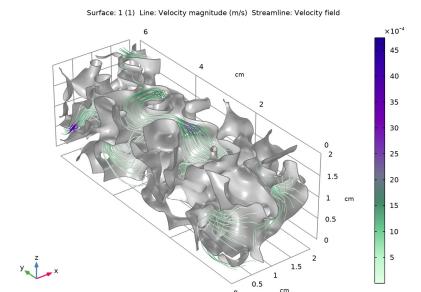


Figure 3: Velocity in the REV.

From the simulation the values for the porosity and permeability are obtained, with $\varepsilon = 0.373$ and $\kappa \approx 3 \cdot 10^{-8} \text{m}^2$.

Notes About the COMSOL Implementation

An STL file of the porous structure is imported and remeshed directly in the meshing sequence. This means that the geometry sequence is kept empty. To resolve the velocity gradients at the boundaries of the porous matrix, add a boundary layer mesh.

Application Library path: Porous_Media_Flow_Module/Fluid_Flow/ pore_scale_flow_3d

From the File menu, choose New.

NEW

In the New window, click Model Wizard.

MODEL WIZARD

- I In the Model Wizard window, click **3D**.
- 2 In the Select Physics tree, select Fluid Flow>Single-Phase Flow>Creeping Flow (spf).
- 3 Click Add.
- 4 Click 🔵 Study.
- 5 In the Select Study tree, select General Studies>Stationary.
- 6 Click **Done**.

GEOMETRY I

- I In the Model Builder window, under Component I (compl) click Geometry I.
- 2 In the Settings window for Geometry, locate the Units section.
- **3** From the **Length unit** list, choose **cm**. Import the STL file and remesh it directly in the meshing sequence. This means that the geometry sequence will be kept empty.

MESH I

The meshing sequence will define the geometric model on which the physics is defined. The mesh is therefore set up completely before assigning material and physics.

Import I

- I In the Mesh toolbar, click Import.
- 2 In the Settings window for Import, locate the Import section.
- 3 Click **Browse**.
- **4** Browse to the model's Application Libraries folder and double-click the file pore_scale_flow_3d.stl.
- 5 From the Boundary partitioning list, choose Detect boundaries.

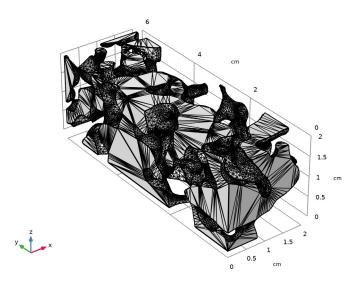
Increase the tolerance to collapse small and sliver mesh elements. This avoids the creation of small boundaries.

6 From the Repair tolerance list, choose Absolute.

7 In the Absolute tolerance text field, type 1e-5.

An increased **Maximum neighbor angle** will aid the algorithm in forming as few boundaries as possible while still recognizing the planar faces.

- 8 Locate the Detect Faces section. In the Maximum neighbor angle text field, type 62.
- **9** Locate the **Import** section. Click **Import**.



Join Entities 1

Join all the boundaries of the free surface to one boundary.

- I In the Mesh toolbar, click Join Entities.

 Instead of clicking on boundaries in the Graphics window, paste the numbers from this document or type them in manually.
- 2 In the Settings window for Join Entities, locate the Geometric Entity Selection section.
- 3 Click Paste Selection.
- 4 In the Paste Selection dialog box, type 14-20 in the Selection text field.
- 5 Click OK.
- 6 In the Settings window for Join Entities, click 📗 Build Selected.

The imported STL file only contains a triangle mesh without domains. Use **Create Domains** to form the fluid domain in the enclosed volume.

Create Domains 1

- I In the Mesh toolbar, click Create Entities and choose Create Domains.
- 2 In the Settings window for Create Domains, click | Build Selected.

Take a look in the Messages log to confirm that the mesh now contains one domain and 37 boundaries.

Next, increase the quality of the triangle mesh by remeshing the faces using a Free **Triangular** operation. This will also result in triangles of more equal size.

Free Triangular I

- I In the Mesh toolbar, click A Boundary and choose Free Triangular.
- 2 In the Settings window for Free Triangular, locate the Boundary Selection section.
- 3 From the Selection list, choose All boundaries.

Size

- I In the Model Builder window, expand the Free Triangular I node, then click Size.
- 2 In the Settings window for Size, locate the Element Size section.
- 3 From the Calibrate for list, choose Fluid dynamics.
- 4 From the Predefined list, choose Fine.

Free Triangular I

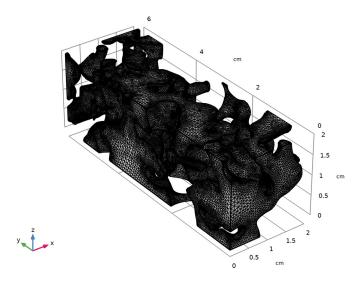
Set an even finer mesh size on the free surface.

Size 1

- I In the Model Builder window, right-click Free Triangular I and choose Size.
- 2 In the Settings window for Size, locate the Geometric Entity Selection section.
- 3 Click Clear Selection.
- 4 Click Paste Selection.
- 5 In the Paste Selection dialog box, type 14 36 37 in the Selection text field.
- 6 Click OK.
- 7 In the Settings window for Size, locate the Element Size section.
- 8 Click the **Custom** button.
- 9 Locate the Element Size Parameters section. Select the Maximum element size check box.
- 10 In the associated text field, type 0.04[cm].

II Click **Build Selected**.

The surface mesh is now of good quality.



Next, generate a tetrahedral mesh in the domain of the porous structure.

Free Tetrahedral I

In the Mesh toolbar, click \bigwedge Free Tetrahedral.



Size

- I In the Model Builder window, expand the Free Tetrahedral I node, then click Size.
- 2 In the Settings window for Size, locate the Element Size section.
- 3 From the Calibrate for list, choose Fluid dynamics.
- 4 From the Predefined list, choose Fine.
- 5 Click **Build Selected**.

Boundary Layers 1

In order to resolve the large velocity gradients at the boundaries to the porous matrix properly, add a boundary layer mesh.

In the Mesh toolbar, click Boundary Layers.

Boundary Layer Properties

- I In the Model Builder window, click Boundary Layer Properties.
- 2 In the Settings window for Boundary Layer Properties, locate the Geometric Entity Selection section.
- 3 Click Paste Selection.
- 4 In the Paste Selection dialog box, type 14 36 37 in the Selection text field.
- 5 Click OK.
- 6 In the Settings window for Boundary Layer Properties, locate the Layers section.
- 7 In the Number of layers text field, type 1.
- 8 In the Thickness adjustment factor text field, type 10.
- 9 Click 🖷 Build Selected.
- 10 In the Mesh toolbar, click A Plot.

RESULTS

Mesh I

Compare with Figure 2. The overall mesh quality is very good. The amount of mesh elements with low quality (skewness) is low. Mesh quality is not an indication for the accuracy of the solution, but it affects the convergence behavior.

GLOBAL DEFINITIONS

Add some parameters that are used to set up the model.

Parameters 1

- I In the Model Builder window, under Global Definitions click Parameters I.
- 2 In the Settings window for Parameters, locate the Parameters section.
- **3** In the table, enter the following settings:

Name	Expression	Value	Description
rho_f	1000[kg/m^3]	1000 kg/m³	Fluid density
mu_f	1e-3[Pa*s]	0.001 Pa·s	Fluid viscosity
u_in	1e-4[m/s]	IE-4 m/s	Inlet velocity
width	2[cm]	0.02 m	REV width
length	6[cm]	0.06 m	REV length
V_tot	width^2*length	2.4E-5 m ³	Total REV volume

MATERIALS

Water

- I In the Model Builder window, under Component I (compl) right-click Materials and choose Blank Material.
- 2 In the Settings window for Material, type Water in the Label text field.
- **3** Locate the **Material Contents** section. In the table, enter the following settings:

Property	Variable	Value	Unit	Property group
Density	rho	rho_f	kg/m³	Basic
Dynamic viscosity	mu	mu_f	Pa·s	Basic

CREEPING FLOW (SPF)

- I In the Model Builder window, under Component I (compl) click Creeping Flow (spf).
- 2 In the Settings window for Creeping Flow, click to expand the Discretization section.
- 3 From the Discretization of fluids list, choose PI+PI.

Linear elements reduce the number of degrees of freedom to be solved. Because the geometry already requires a fine mesh, using linear elements decreases computational time and memory requirements while maintaining sufficient accuracy.

Inlet I

- I In the Physics toolbar, click **Boundaries** and choose Inlet.
- **2** Select Boundary 2 only.
- 3 In the Settings window for Inlet, locate the Velocity section.
- **4** In the U_0 text field, type u_in.
- 5 Locate the Boundary Selection section. Click 堶 Create Selection.
- 6 In the Create Selection dialog box, type Inlet in the Selection name text field.
- 7 Click OK.

Outlet I

- I In the Physics toolbar, click **Boundaries** and choose **Outlet**.
- **2** Select Boundaries 16 and 21 only.
- 3 In the Settings window for Outlet, locate the Pressure Conditions section.
- 4 Select the Normal flow check box.
- 5 Locate the Boundary Selection section. Click \(\bigsigma\) Create Selection.
- 6 In the Create Selection dialog box, type Outlet in the Selection name text field.

7 Click OK.

Create a few more selections to use them throughout the model set up.

DEFINITIONS

Wall

- I In the Model Builder window, expand the Component I (compl)>Definitions node.
- 2 Right-click **Definitions** and choose **Selections>Explicit**.
- 3 In the Settings window for Explicit, type Wall in the Label text field.
- 4 Locate the Input Entities section. From the Geometric entity level list, choose Boundary.
- **5** Select Boundaries 4, 10, and 30 only.

It might be easier to select the correct boundary by using the Selection List window. To open this window, in the Home toolbar click Windows and choose Selection List. (If you are running the cross-platform desktop, you find **Windows** in the main menu.)

Note that the boundary numbering differs compared to the numbering in the meshing sequence. This is due to the associativity update that runs when the Finalize node is built.

All boundaries

- I In the **Definitions** toolbar, click **\(\bigcap_{\text{a}} \) Explicit**.
- 2 In the Settings window for Explicit, locate the Input Entities section.
- 3 From the Geometric entity level list, choose Boundary.
- 4 Select the All boundaries check box.
- 5 In the Label text field, type All boundaries.

Symmetry

- I In the **Definitions** toolbar, click Difference.
- 2 In the Settings window for Difference, type Symmetry in the Label text field.
- 3 Locate the Geometric Entity Level section. From the Level list, choose Boundary.
- 4 Locate the Input Entities section. Under Selections to add, click + Add.
- 5 In the Add dialog box, select All boundaries in the Selections to add list.
- 6 Click OK.
- 7 In the Settings window for Difference, locate the Input Entities section.
- 8 Under Selections to subtract, click + Add.
- 9 In the Add dialog box, in the Selections to subtract list, choose Inlet, Outlet, and Wall.

IO Click OK.

CREEPING FLOW (SPF)

Symmetry I

- I In the Physics toolbar, click **Boundaries** and choose Symmetry.
- 2 In the Settings window for Symmetry, locate the Boundary Selection section.
- 3 From the Selection list, choose Symmetry.

STUDY I

- I In the Model Builder window, click Study I.
- 2 In the Settings window for Study, locate the Study Settings section.
- 3 Clear the Generate default plots check box.
- 4 In the Home toolbar, click **Compute**.

Create the plot as shown in Figure 3.

5 In the Results toolbar, click More Datasets and choose Surface.

RESULTS

Surface I

- I In the Settings window for Surface, locate the Selection section.
- 2 From the Selection list, choose Wall.

Velocity

- I In the Results toolbar, click **and 3D Plot Group**.
- 2 In the Settings window for 3D Plot Group, type Velocity in the Label text field.
- 3 Locate the Plot Settings section. Clear the Plot dataset edges check box.

Surface I

- I Right-click Velocity and choose Surface.
- 2 In the Settings window for Surface, locate the Data section.
- 3 From the Dataset list, choose Surface 1.
- **4** Locate the **Expression** section. In the **Expression** text field, type 1.
- 5 Locate the Coloring and Style section. From the Coloring list, choose Uniform.
- 6 From the Color list, choose Gray.

Line 1

I In the Model Builder window, right-click Velocity and choose Line.

- 2 In the Settings window for Line, locate the Data section.
- 3 From the Dataset list, choose Surface 1.
- 4 Locate the Coloring and Style section. From the Coloring list, choose Uniform.
- 5 From the Color list, choose Black.
- 6 In the **Velocity** toolbar, click **Plot**.
- **7** From the **Color** list, choose **Custom**.
- **8** On Windows, click the colored bar underneath, or if you are running the crossplatform desktop — the Color button. Choose a darker shade of gray.

Streamline 1

- I Right-click Velocity and choose Streamline.
- 2 In the Settings window for Streamline, locate the Selection section.
- **3** From the **Selection** list, choose **Inlet**.
- 4 Locate the Streamline Positioning section. In the Number text field, type 40.
- 5 Locate the Coloring and Style section. Find the Line style subsection. From the Type list, choose Tube.
- 6 Select the Radius scale factor check box.
- 7 In the associated text field, type 0.0075.

Color Expression 1

- I Right-click Streamline I and choose Color Expression.
- 2 In the Settings window for Color Expression, locate the Coloring and Style section.
- 3 From the Color table list, choose AuroraBorealis.

Next, determine the porosity and permeability for our REV in order to perform simulations at the macroscopic level.

DEFINITIONS

Integration I (intob1)

- I In the Definitions toolbar, click // Nonlocal Couplings and choose Integration.
- 2 In the Settings window for Integration, locate the Source Selection section.
- 3 From the Selection list, choose All domains.

Average Inlet

- I In the **Definitions** toolbar, click **M Nonlocal Couplings** and choose **Average**.
- 2 In the Settings window for Average, type Average Inlet in the Label text field.

- 3 Locate the Source Selection section. From the Geometric entity level list, choose Boundary.
- 4 From the Selection list, choose Inlet.

Average Outlet

- I In the Definitions toolbar, click A Nonlocal Couplings and choose Average.
- 2 In the Settings window for Average, type Average Outlet in the Label text field.
- 3 Locate the Source Selection section. From the Geometric entity level list, choose Boundary.
- 4 From the Selection list, choose Outlet.

Variables 1

- I In the Model Builder window, right-click Definitions and choose Variables.
- 2 In the Settings window for Variables, locate the Variables section.
- **3** In the table, enter the following settings:

Name	Expression	Unit	Description
por	intop1(1)/V_tot		Porosity
dPdL	-(aveop2(p)-aveop1(p))/ length	N/m³	Pressure drop
u_out	<pre>spf.out1.massFlowRate/ rho_f/width^2</pre>	m/s	Superficial outlet velocity
kappa	u_out*mu_f/dPdL	m²	Permeability

Since you have introduced new variables, the solution needs to be updated. It is not necessary to compute the study again.

STUDY I

In the Study toolbar, click C Update Solution.

RESULTS

Global Evaluation 1

- I In the Results toolbar, click (8.5) Global Evaluation.
- 2 In the Settings window for Global Evaluation, click Add Expression in the upper-right corner of the Expressions section. From the menu, choose Component I (compl)> Definitions>Variables>por Porosity.
- 3 Click Add Expression in the upper-right corner of the Expressions section. From the menu, choose Component I (compl)>Definitions>Variables>kappa Permeability m².

4 Click **= Evaluate**.

The results are shown in the **Table** window. The porosity is 0.371 and the permeability is about 3e-8m². These results can now be used for calculations of large scale models.

Streamline 1

To make the results even more descriptive, create an animation.

- I In the Model Builder window, under Results>Velocity click Streamline I.
- 2 In the Settings window for Streamline, locate the Coloring and Style section.
- 3 Find the Point style subsection. From the Type list, choose Interactive arrow.
- 4 In the Extra release times text field, type 2000.

Animation I

I In the Velocity toolbar, click Animation and choose Player.

You can adjust the number of frames to obtain a smooth animation of the interactive arrows. This animation visualizes very nicely that there are regions in the porous medium with both very slow and very high velocity.