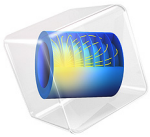

Double-Headed Streamer in Nitrogen

Introduction

Streamers are transient filamentary electric discharges that can develop in a nonconducting background in the presence of an intense electric field. These discharges can attain high electron number density and, consequently, a high concentration of chemical active species that are relevant for numerous applications.

The propagation of streamers is driven by very nonlinear dynamics that involve very steep density gradients and high space-charge density distributed in very thin layers. The charge separation at the front (or head) of the streamer generates intense electric fields that are responsible for sharp ionization fronts propagating into the neutral medium.

In negative (anode directed) streamers, ionizing electrons are accelerated outward by the space-charge (the streamer extends toward the anode). These high energy electrons may have been transported by drift or diffusion, or created by another mechanism that provides pre-ionization ahead of the streamer such as photoionization or ionization from runway electrons. In positive (or cathode directed) streamers the space-charge field in the streamer head accelerates the electrons inward. Consequently, the ionizing electrons must be produced by a pre-ionization mechanism. The pre-ionization of the streamer is a complex subject that is believed to be critical for both negative and positive streamers propagation and is still under investigation. This document is an introduction to streamer modeling with focus on basic concepts of streamer propagation. With that in mind it is followed a simplified approach where all pre-ionization is neglected and only negative streamers are discussed.

This example presents a study of a double-headed streamer. An initial seed of electrons is placed between two electrodes spaced by 1 cm to which a voltage of 52 kV is applied (providing a constant initial electric field of 52 kV/cm). A negative and positive streamers propagate towards the electrodes. The electron density, electric field, and the propagation speed of the streamer agrees well with simulation results from Ref. 1.

Model Definition

The model is two dimensional and describes the transient behavior of an initial electron seed in the presence of a strong electric field using fluid-type equations.

The model solves the electron and ion continuity and momentum equations, in the drift-diffusion approximation, self-consistently coupled with Poisson's equation. The local field approximation is used, which means that transport and source coefficients are assumed to be well parameterized through the reduced electric field ($\boldsymbol{E}/\boldsymbol{N}$). In the local field

approximation the fluid equation for the mean electron energy is not solved, which reduces significantly the complexity of the numerical problem.

The local field approximation is valid in a situation where the rate of electron energy gain from the electric field is locally balanced by the energy loss rate. When this condition is met the electrons are said to be in local equilibrium with the electric field and the electron mean properties can be expressed as a function the reduced electric field.

DOMAIN EQUATIONS

The electron density is computed by solving the drift-diffusion equation for the electron density.

$$\frac{\partial}{\partial t}(n_e) + \nabla \cdot [-n_e(\mu_e \bullet \mathbf{E}) - \mathbf{D}_e \bullet \nabla n_e] = R_e \tag{1}$$

For more detailed information on electron transport, see the section *Theory for the Drift Diffusion Interface* in the *Plasma Module User's Guide*. The source coefficients in the above equation can be determined by the plasma chemistry using rate coefficients

$$R_e = \sum_{j=1}^{M} x_j k_j N_n n_e \tag{2}$$

where x_j is the mole fraction of the target species for reaction j, k_j is the rate coefficient for reaction j (SI unit: m^3/s), and N_n is the total neutral number density (SI unit: $1/\mathrm{m}^3$). For drift-dominated discharges it is better practice to use Townsend coefficients instead of rate coefficients to define reaction rates Ref. 2. When Townsend coefficients are used, the electron source term is given by:

$$R_e = \sum_{j=1}^{M} x_j \alpha_j N_n |\Gamma_e| \tag{3}$$

where α_j is the Townsend coefficient for reaction j (m^2) and Γ_e is the electron flux as defined above ($1/(\mathrm{m}^2 \cdot \mathrm{s})$). Townsend coefficients can increase the stability of the numerical scheme when the electron flux is field driven as is the case with streamers.

For heavy species, the following equation is solved for the mass fraction of each species:

$$\rho \frac{\partial}{\partial t}(w_k) + \rho(\mathbf{u} \cdot \nabla) w_k = \nabla \cdot \mathbf{j}_k + R_k \tag{4}$$

For detailed information on the transport of the heavy species, see the section *Theory for the Heavy Species Transport Interface* in the *Plasma Module User's Guide*.

The electrostatic field is computed using Poisson's equation

$$-\nabla \cdot \varepsilon_0 \varepsilon_r \nabla V = \rho \tag{5}$$

The space charge density ρ is automatically computed based on the plasma chemistry specified in the model using the formula

$$\rho = q\left(\sum_{k=1}^{N} Z_k n_k - n_e\right) \tag{6}$$

For detailed information about electrostatics, see *Theory for the Electrostatics Interface* in the *Plasma Module User's Guide*.

Boundary Conditions

The present simulation is arranged in a way that the charged particle interaction with the surfaces does not influence much the streamer propagation. Nevertheless boundary conditions must be given. For electrons a fixed electron density of 10^{14} cm^{-3} is set at the electrodes and the default zero flux boundary condition is set at the right boundary. For ions the default zero flux is applied to all boundaries.

For Poisson's equation, an electric potential of 0 V and 52 kV is set at the bottom and top electrodes, respectively, and the default zero charge boundary condition is applied at the right boundary.

PLASMA CHEMISTRY

The chemistry of a plasma sustained in nitrogen can be very complex and a detailed study of the main excited states could easily have hundreds of reactions. The main goal of this model is to study the charge particle density profiles in the presence of strong electric fields. With this in mind, the model includes a single ionization reaction, as presented in Table 1, that correctly describes the creation of charged species in a background of nitrogen. This work uses the Townsend coefficient as a function of reduced electric field provided in Ref. 1.

TABLE 1: IONIZATION REACTION.

Reaction	Formula	Type	Δε (eV)
1	e+M=>2e+M+	Ionization	10

Results and Discussion

The results in this section are for a double-headed streamer propagating in a background gas kept at a constant density as obtained by the ideal gas law at atmospheric pressure and at a temperature of 293.15 K. There is a general good agreement with the results from the model here presented and the ones from Ref. 1.

Figure 1, Figure 2, and Figure 3 show 2d plots of the electron density, space charge density, and electric potential at 2.5 ns. Figure 4 and Figure 5 present the spatial distribution of the electron and ion densities, and the z component of the electric field for several instants during the streamer simulation.

Starting from the center, two streamers develop toward the electrodes. These streamers have different propagation mechanisms that result in different morphology and propagation speeds. The top streamer is anode directed and develops a negative space-charge density since the electric field pulls electrons ahead of the streamer. The bottom streamer is cathode directed and the electrons are drifting in the opposite direction of the streamer propagation. The propagation of the cathode-directed streamer is only possible because it is given a high enough background electron density. Note how the quasineutral streamer body shields the electric field to very small values causing the electrons to cool down.

Figure 3 and Figure 5 can be compared with figures 6 and 7 of Ref. 1. In general, the agreement is very good with the velocity of propagation and the spread being captured accurately.

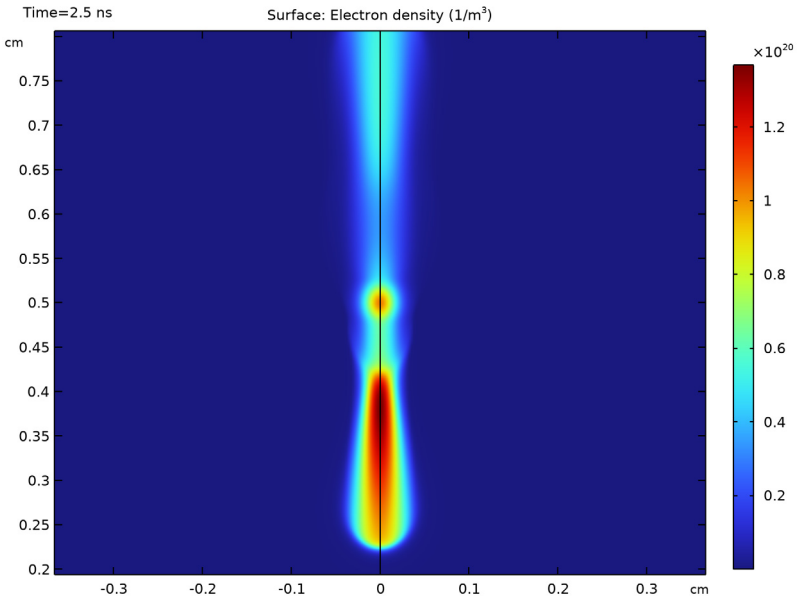

Figure 1: Electron number density at 2.5 ns.

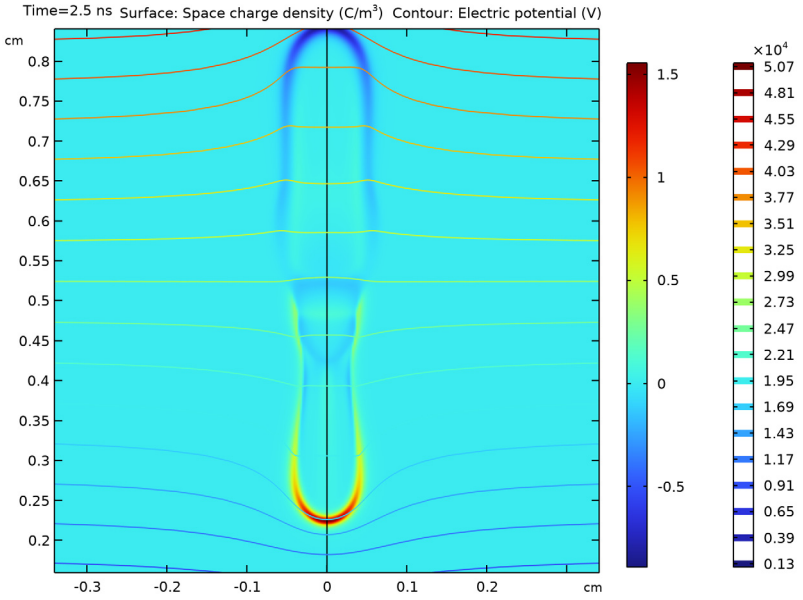

Figure 2: Space charge density and electric potential at 2.5 ns.

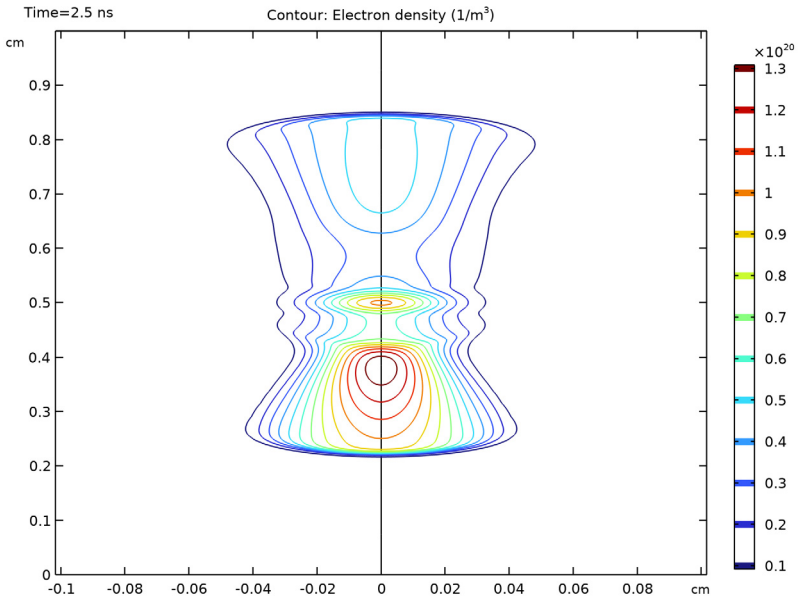

Figure 3: Contours of the electron number density at 2.5 ns. Compare with figure 6 of Ref. 1.

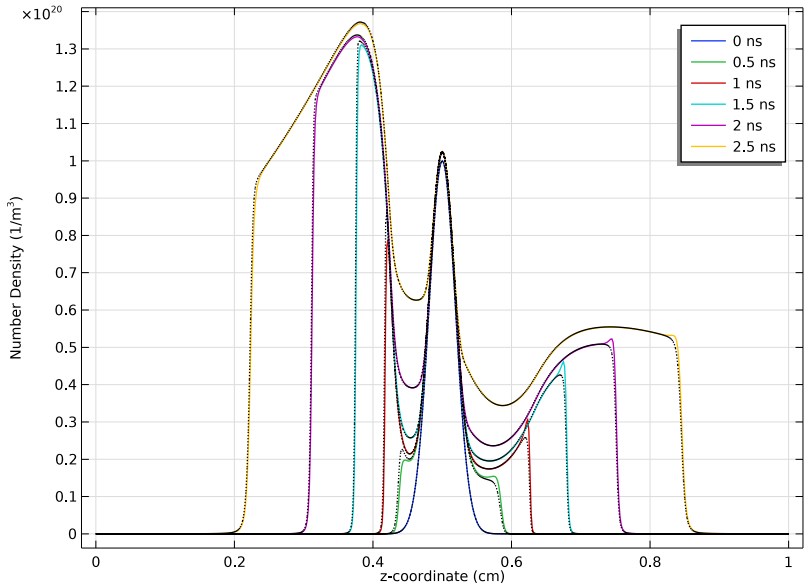

Figure 4: Spatial distribution along the axis of symmetry of the electron (colored solid lines) and ion number density (black dashed lines) for several time instants during the streamer propagation.

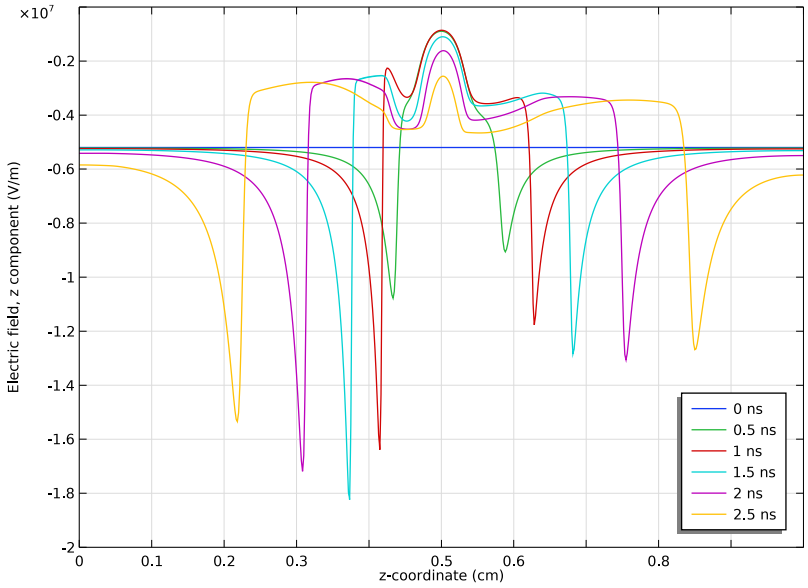

Figure 5: Spatial distribution along the axis of symmetry of the z component of the electric field for several time instants during the streamer propagation. Compare with figure 7 of Ref. 1

References

1. D. Bessieres, J. Paillol, A. Bourdon, P. Segur, and E. Marode, "A new one-dimensional moving mesh method applied to the simulation of streamer discharges," *J. Phys. D: Appl. Phys.*, vol. 40, pp. 6559–6570, 2007.

2. M.A. Lieberman and A.J. Lichtenberg, *Principles of Plasma Discharges and Materials Processing*, John Wiley & Sons, 2005.

Application Library path: `Plasma_Module/Direct_Current_Discharges/streamer_2d`

Modeling Instructions

From the **File** menu, choose **New**.

NEW

In the **New** window, click **Model Wizard**.

MODEL WIZARD

1 In the **Model Wizard** window, click **2D Axisymmetric**.

2 In the **Select Physics** tree, select **Plasma>Plasma (plas)**.

3 Click **Add**.

4 Click **Study**.

5 In the **Select Study** tree, select **General Studies>Time Dependent**.

6 Click **Done**.

GEOMETRY 1

Create the simulation geometry. Add a layer to define a region where the mesh is going to be refined.

1 In the **Model Builder** window, under **Component 1 (comp1)** click **Geometry 1**.

2 In the **Settings** window for **Geometry**, locate the **Units** section.

3 From the **Length unit** list, choose **cm**.

Rectangle 1 (r1)

1 In the **Geometry** toolbar, click **Rectangle**.

2 In the **Settings** window for **Rectangle**, click to expand the **Layers** section.

3 In the table, enter the following settings:

Layer name	Thickness (cm)
Layer 1	0.05

4 Select the **Layers to the left** check box.

5 Clear the **Layers on bottom** check box.

6 Click **Build All Objects**.

Mesh Control Edges 1 (mce1)

1 In the **Geometry** toolbar, click **Virtual Operations** and choose **Mesh Control Edges**.

2 On the object **fin**, select Boundary 4 only.

3 In the **Geometry** toolbar, click **Build All**.

Import a file that contains variables to be used in the model.

DEFINITIONS

Variables 1

1 In the **Model Builder** window, under **Component 1 (comp1)** right-click **Definitions** and choose **Variables**.

2 In the **Settings** window for **Variables**, locate the **Variables** section.

3 Click **Load from File**.

4 Browse to the model's Application Libraries folder and double-click the file `streamer_2d_variables.txt`.

Define the ionization Townsend coefficient using an analytic function.

Townsend coefficient

1 In the **Home** toolbar, click **Functions** and choose **Local>Analytic**.

2 In the **Settings** window for **Analytic**, type `Townsend coefficient` in the **Label** text field.

3 In the **Function name** text field, type `alpha`.

4 Locate the **Definition** section. In the **Expression** text field, type `5.7*760*exp(-260*760/x)`.

5 Locate the **Units** section. In the table, enter the following settings:

Argument	Unit
x	V/cm

6 In the **Function** text field, type `cm^-1`.

7 Locate the **Plot Parameters** section. In the table, enter the following settings:

Argument	Lower limit	Upper limit	Unit
x	1[kV/cm]	150e3[kV/cm]	V/m

Select to use the Local Field Approximation. This means that the electron mean energy equation is not solved.

Select to use Finite element (quadratic shape function). With this option streamline stabilization is automatically added.

PLASMA (PLAS)

1 In the **Model Builder** window, under **Component 1 (comp1)** click **Plasma (plas)**.

2 In the **Settings** window for **Plasma**, locate the **Plasma Properties** section.

3 From the **Mean electron energy** list, choose **Local field approximation**.

4 Click to expand the **Discretization** section. From the **Formulation** list, choose **Finite element (quadratic shape function)**.

Create an electron impact ionization reaction that uses the Townsend coefficient previously defined.

Set the species M to be computed from mass constraint.

Set the species M+ to use initial condition from electronneutrality and set the ion mobility.

Electron Impact Reaction 1

1 In the **Physics** toolbar, click **Domains** and choose **Electron Impact Reaction**.

2 In the **Settings** window for **Electron Impact Reaction**, locate the **Reaction Formula** section.

3 In the **Formula** text field, type `e+M=>e+e+M+`.

4 Locate the **Collision Type** section. From the **Collision type** list, choose **Ionization**.

5 In the Δε text field, type `10`.

6 Locate the **Reaction Parameters** section. In the k^{f} text field, type `Ri`.

Species: M

1 In the **Model Builder** window, click **Species: M**.

2 In the **Settings** window for **Species**, locate the **Species Formula** section.

3 Select the **From mass constraint** check box.

4 Locate the **General Parameters** section. From the **Preset species data** list, choose **N2**.

Species: M+

1 In the **Model Builder** window, click **Species: M+**.

2 In the **Settings** window for **Species**, locate the **Species Formula** section.

3 Select the **Initial value from electroneutrality constraint** check box.

4 Locate the **General Parameters** section. From the **Preset species data** list, choose **N2**.

5 Locate the **Mobility and Diffusivity Expressions** section. From the **Specification** list, choose **Specify mobility, compute diffusivity**.

6 Locate the **Mobility Specification** section. In the u_{m} text field, type `mui`.

Set electron mobility and diffusivity.

Plasma Model 1

1 In the **Model Builder** window, click **Plasma Model 1**.

2 In the **Settings** window for **Plasma Model**, locate the **Electron Density and Energy** section.

3 In the μ_{e} text field, type `mue`.

4 From the **Electron transport properties** list, choose **Specify all**.

5 From the list, choose **Diagonal**.

6 In the D_e table, enter the following settings:

DeL	0
0	DeT

Set the initial conditions to use a profile defined in the variables.

Initial Values 1

1 In the **Model Builder** window, click **Initial Values 1**.

2 In the **Settings** window for **Initial Values**, locate the **Initial Values** section.

3 In the $n_{e,0}$ text field, type ne0.

Apply a ground and a voltage to the bottom and top electrodes

The interaction of the streamer with the surface is not going to be modeled here. That is why Wall and Surface Reaction features, that define flux boundary conditions for electrons and ions, are not applied at the electrodes.

A fixed electron density is applied at the electrodes for stability reasons.

Ground 1

1 In the **Physics** toolbar, click **Boundaries** and choose **Ground**.

2 Select Boundary 2 only.

Metal Contact 1

1 In the **Physics** toolbar, click **Boundaries** and choose **Metal Contact**.

2 In the **Settings** window for **Metal Contact**, locate the **Terminal** section.

3 In the V_0 text field, type V0.

4 Select Boundary 3 only.

Electron Density and Energy 1

1 In the **Physics** toolbar, click **Boundaries** and choose **Electron Density and Energy**.

2 Select Boundaries 2 and 3 only.

3 In the **Settings** window for **Electron Density and Energy**, locate the **Electron Density and Energy** section.

4 Select the **Fix electron density** check box.

5 In the n_{ew} text field, type `ne0_min`.

Define a mesh that is very refined in the axis of symmetry where the streamer is going to propagate, and is coarse everywhere else.

MESH 1

1 In the **Model Builder** window, under **Component 1 (comp1)** click **Mesh 1**.

2 In the **Settings** window for **Mesh**, locate the **Sequence Type** section.

3 From the list, choose **User-controlled mesh**.

Size

1 In the **Model Builder** window, under **Component 1 (comp1)>Mesh 1** click **Size**.

2 In the **Settings** window for **Size**, locate the **Element Size** section.

3 From the **Calibrate for** list, choose **General physics**.

4 From the **Predefined** list, choose **Normal**.

Mapped 1

1 In the **Mesh** toolbar, click  **Mapped**.

2 Right-click **Mapped 1** and choose **Move Up**.

3 In the **Settings** window for **Mapped**, locate the **Domain Selection** section.

4 From the **Geometric entity level** list, choose **Domain**.

5 Select Domain 1 only.

6 Click to expand the **Control Entities** section. Clear the **Smooth across removed control entities** check box.

7 Click to expand the **Reduce Element Skewness** section.

Distribution 1

1 Right-click **Mapped 1** and choose **Distribution**.

2 Select Boundaries 3 and 5 only.

3 In the **Settings** window for **Distribution**, locate the **Distribution** section.

4 From the **Distribution type** list, choose **Predefined**.

5 In the **Number of elements** text field, type `30`.

6 In the **Element ratio** text field, type `5`.

7 From the **Growth rate** list, choose **Exponential**.

Distribution 2

1 In the **Model Builder** window, right-click **Mapped 1** and choose **Distribution**.

2 Select Boundaries 1 and 7 only.

3 In the **Settings** window for **Distribution**, locate the **Distribution** section.

4 In the **Number of elements** text field, type 600.

Free Triangular 1

1 In the **Model Builder** window, under **Component 1 (comp1)>Mesh 1** click **Free Triangular 1**.

2 In the **Settings** window for **Free Triangular**, click to expand the **Control Entities** section.

3 Clear the **Smooth across removed control entities** check box.

4 Click **Build All**.

Create a View that defines the aspect ratio used in the paper from where this example is based.

DEFINITIONS

View 2

1 In the **Model Builder** window, under **Component 1 (comp1)** right-click **Definitions** and choose **View**.

2 In the **Settings** window for **View**, locate the **View** section.

3 Select the **Lock axis** check box.

Axis

1 In the **Model Builder** window, expand the **View 2** node, then click **Axis**.

2 In the **Settings** window for **Axis**, locate the **Axis** section.

3 In the **r minimum** text field, type -0.06.

4 In the **r maximum** text field, type 0.06.

5 In the **z minimum** text field, type 0.

6 In the **z maximum** text field, type 1.

7 From the **View scale** list, choose **Manual**.

8 In the **y scale** text field, type 0.17.

9 Click **Update**.

All plots are going to be created manually. Also, create a mirror data set to have a better view of the streamer.

STUDY 1

1 In the **Model Builder** window, click **Study 1**.

2 In the **Settings** window for **Study**, locate the **Study Settings** section.

3 Clear the **Generate default plots** check box.

4 In the **Study** toolbar, click **Get Initial Value**.

RESULTS

In the **Model Builder** window, expand the **Results** node.

Mirror 2D 1

1 In the **Model Builder** window, expand the **Results>Datasets** node.

2 Right-click **Results>Datasets** and choose **More 2D Datasets>Mirror 2D**.

Electron Number Density

1 In the **Results** toolbar, click **2D Plot Group**.

2 In the **Settings** window for **2D Plot Group**, type `Electron Number Density` in the **Label** text field.

3 Locate the **Data** section. From the **Dataset** list, choose **Mirror 2D 1**.

4 Locate the **Plot Settings** section. From the **View** list, choose **View 1**.

Surface 1

1 Right-click **Electron Number Density** and choose **Surface**.

2 In the **Settings** window for **Surface**, locate the **Expression** section.

3 In the **Expression** text field, type `plas.ne`.

4 Click the **Zoom In** button in the **Graphics** toolbar.

5 In the **Electron Number Density** toolbar, click **Plot**.

Set the output times and choose to view the results while solving.

STUDY 1

Step 1: Time Dependent

1 In the **Model Builder** window, under **Study 1** click **Step 1: Time Dependent**.

2 In the **Settings** window for **Time Dependent**, locate the **Study Settings** section.

3 From the **Time unit** list, choose **ns**.

4 In the **Output times** text field, type `range(0,0.5,2.5)`.

5 Click to expand the **Results While Solving** section. Select the **Plot** check box.

6 From the **Update at** list, choose **Time steps taken by solver**.

7 In the **Home** toolbar, click **Compute**.

Create more plots to show the electric potential, space charge density, and electric fields.

Start by creating a plot of the space charge density with superimposed electric potential contour lines.

RESULTS

Electric Potential and Space Charge Density

1 In the **Home** toolbar, click **Add Plot Group** and choose **2D Plot Group**.

2 In the **Settings** window for **2D Plot Group**, type `Electric Potential and Space Charge Density` in the **Label** text field.

3 Locate the **Data** section. From the **Dataset** list, choose **Mirror 2D 1**.

4 Locate the **Plot Settings** section. From the **View** list, choose **View 1**.

Surface 1

1 Right-click **Electric Potential and Space Charge Density** and choose **Surface**.

2 In the **Settings** window for **Surface**, locate the **Expression** section.

3 In the **Expression** text field, type `plas.scharge`.

4 In the **Electric Potential and Space Charge Density** toolbar, click **Plot**.

Contour 1

1 In the **Model Builder** window, right-click **Electric Potential and Space Charge Density** and choose **Contour**.

2 In the **Settings** window for **Contour**, locate the **Expression** section.

3 In the **Expression** text field, type `V`.

4 Click the **Zoom In** button in the **Graphics** toolbar.

5 In the **Electric Potential and Space Charge Density** toolbar, click **Plot**.

Create an electron density contour plot to be compared with the figure from the reference paper.

Electron Density Contour

1 In the **Home** toolbar, click **Add Plot Group** and choose **2D Plot Group**.

2 In the **Settings** window for **2D Plot Group**, type `Electron Density Contour` in the **Label** text field.

3 Locate the **Data** section. From the **Dataset** list, choose **Mirror 2D 1**.

Contour 1

1 Right-click **Electron Density Contour** and choose **Contour**.

2 In the **Settings** window for **Contour**, locate the **Expression** section.

3 In the **Expression** text field, type `plas.ne`.

4 Locate the **Levels** section. From the **Entry method** list, choose **Levels**.

5 In the **Levels** text field, type `range(1.0e19,1.0e19,1.3e20)`.

6 In the **Electron Density Contour** toolbar, click **Plot**.

Electron Density Contour

1 In the **Model Builder** window, click **Electron Density Contour**.

2 In the **Settings** window for **2D Plot Group**, locate the **Plot Settings** section.

3 From the **View** list, choose **View 2**.

4 In the **Electron Density Contour** toolbar, click **Plot**.

Create a 1D plot of the charged species number densities along the axis of symmetry.

Charged species number density

1 In the **Home** toolbar, click **Add Plot Group** and choose **1D Plot Group**.

2 In the **Settings** window for **1D Plot Group**, type `Charged species number density` in the **Label** text field.

3 Click to expand the **Title** section. From the **Title type** list, choose **None**.

4 Locate the **Plot Settings** section. Select the **y-axis label** check box.

5 In the associated text field, type `Number Density (1/m<sup>3</sup>)`.

Line Graph 1

1 Right-click **Charged species number density** and choose **Line Graph**.

2 In the **Settings** window for **Line Graph**, locate the **y-Axis Data** section.

3 In the **Expression** text field, type `plas.ne`.

4 Select Boundary 1 only.

5 Locate the **x-Axis Data** section. From the **Parameter** list, choose **Expression**.

6 In the **Expression** text field, type `z`.

7 Click to expand the **Legends** section. Select the **Show legends** check box.

8 Click to expand the **Quality** section. From the **Resolution** list, choose **No refinement**.

9 In the **Charged species number density** toolbar, click **Plot**.

Line Graph 2

1 Right-click **Line Graph 1** and choose **Duplicate**.

2 In the **Settings** window for **Line Graph**, locate the **Legends** section.

3 Clear the **Show legends** check box.

4 Locate the **y-Axis Data** section. In the **Expression** text field, type `plas.n_wM_1p`.

5 Click to expand the **Coloring and Style** section. Find the **Line style** subsection. From the **Line** list, choose **Dotted**.

6 From the **Color** list, choose **Black**.

Create a 1D plot of the z component of the electric field along the axis of symmetry.

Electric field

1 In the **Home** toolbar, click  **Add Plot Group** and choose **1D Plot Group**.

2 In the **Settings** window for **1D Plot Group**, type `Electric field` in the **Label** text field.

3 Locate the **Title** section. From the **Title type** list, choose **None**.

4 Locate the **Axis** section. Select the **Manual axis limits** check box.

5 In the **x minimum** text field, type `0`.

6 In the **y minimum** text field, type `-2e7`.

7 In the **y maximum** text field, type `0`.

8 Locate the **Legend** section. From the **Position** list, choose **Lower right**.

Line Graph 1

1 Right-click **Electric field** and choose **Line Graph**.

2 Select Boundary 1 only.

3 In the **Settings** window for **Line Graph**, locate the **y-Axis Data** section.

4 In the **Expression** text field, type `plas.Ez`.

5 Locate the **x-Axis Data** section. From the **Parameter** list, choose **Expression**.

6 In the **Expression** text field, type `z`.

7 Locate the **Legends** section. Select the **Show legends** check box.

8 In the **Electric field** toolbar, click  **Plot**.