



# Computing the Plasma Impedance

## *Introduction*

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Power is supplied to a Capacitively Coupled Plasma (CCP) by a Radio Frequency (RF) power supply connected to the CCP via a match network. The match network enables maximum power transfer to be transferred from the RF supply to the CCP at the operating frequency. The match network is designed using the output impedance of the RF supply, the CCP impedance and the operating frequency. This tutorial shows how the CCP impedance is calculated from the following CCP parameters:

- Gas composition.
- Pressure.
- Power.
- Frequency.
- Secondary emission coefficients.
- Discharge geometry, dimensions, and symmetry.

In practice the relationship between the CCP voltage and current waveform over an RF cycle is nonlinear and the current waveform contain harmonics. The impedance is therefore computed using the amplitudes of the voltage and current waveforms at the fundamental frequency.

## *Model Definition*

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In order to focus on the required modeling steps to compute the plasma impedance, a simple geometry and chemistry is used. The geometry is 1D and consists of two 300 mm diameter parallel plates, separated by 25 mm. A simplified helium chemistry is used consisting of 3 species and 3 reactions. For the ions, the local field approximation is used for the temperature, and a lookup table for the ion mobility. The table describes how the ion mobility changes as a function of the reduced electric field. The electron mobility and other transport properties are automatically computed from the list of electron impact reactions.

Loss of ions to the wall is assumed to be due to migration only. The powered electrode is driven at 13.56 MHz with a fixed value for power. Since the discharge is symmetric (that is, the grounded surface area is the same as the powered electrode surface area), no self DC bias is expected. The symmetry also means two harmonics are expected in the discharge current, one at the fundamental frequency, and one at 3 times the fundamental.

Computing the plasma impedance is done in 3 stages:

- 1 Compute the time periodic solution for the plasma variables.
- 2 Map that solution to the time domain.
- 3 Compute the FFT of the time domain solution.

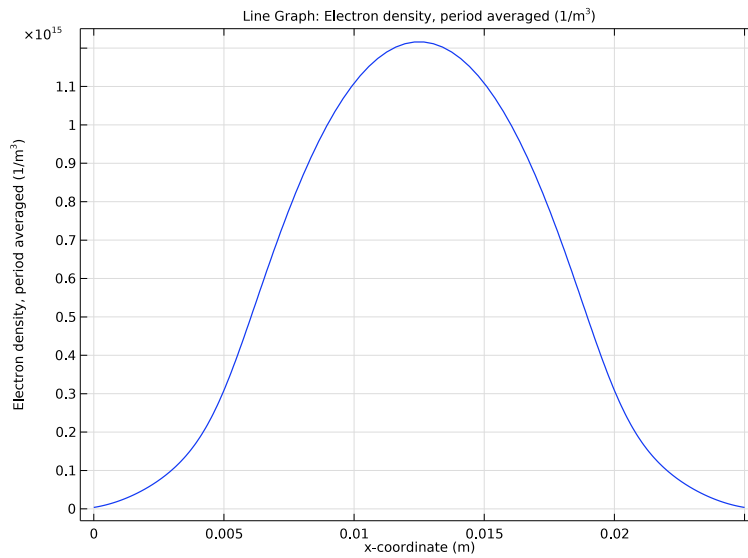
## Results and Discussion

Results of period averaged plasma quantities are shown below in [Figure 1](#) through [Figure 6](#) for the 10 W case. The current and voltage profiles over the period are plotted in [Figure 7](#). The voltage follows a perfect cosine, since this is imposed as a boundary condition in the model. The current looks sinusoidal, and is phase shifted from the voltage. In order to explore the characteristics of the current in more detail, its Fourier transform can be taken. This is plotted in [Figure 8](#), and there is the main harmonic at 13.56 MHz as expected, and a small harmonic at 40.68 MHz. In symmetric discharges, which form no self DC bias, this is expected. If the discharge was non-symmetric, other harmonics would be visible. When the power is increased to 50 W, the magnitude of the third harmonic increases, [Figure 9](#). When more power is deposited into the plasma, the current arriving on the electrode becomes more distorted, and has a greater proportion at the third harmonic. The impedance of the discharge at the fundamental frequency is summarized in the table below:

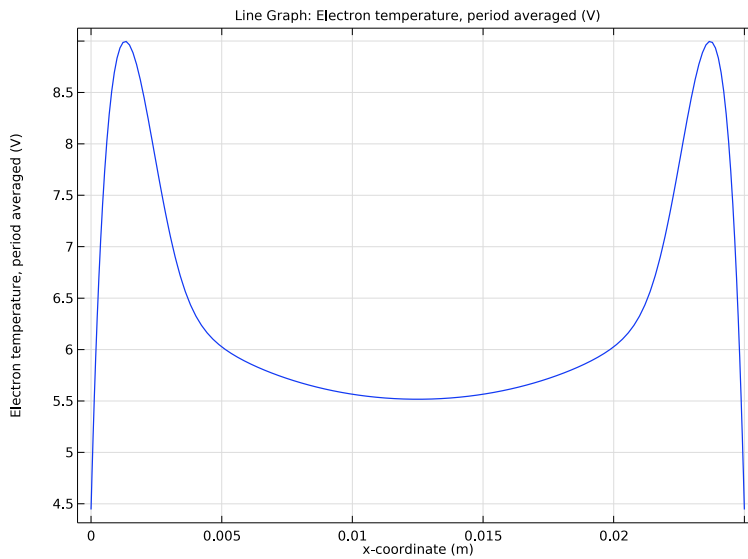
TABLE 1: COMPUTED PLASMA IMPEDANCE AND HARMONIC CONTENT.

POWER (W)	IMPEDANCE (OHM)	HARMONIC FRACTION (%)
10	42.696 - 156.62j	1.9
50	29.175 - 126.47j	4.5

As the power is increased, the discharge becomes less resistive, so tuning a matching network for the 10 W case will result in a mismatch when run at 50 W.



*Figure 1: Plot of the period averaged electron density.*



*Figure 2: Plot of the period averaged electron temperature.*

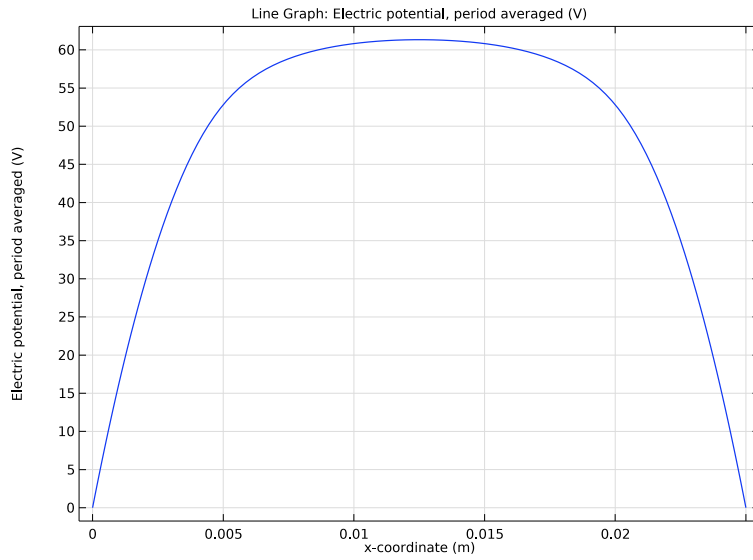


Figure 3: Plot of the period averaged electric potential.

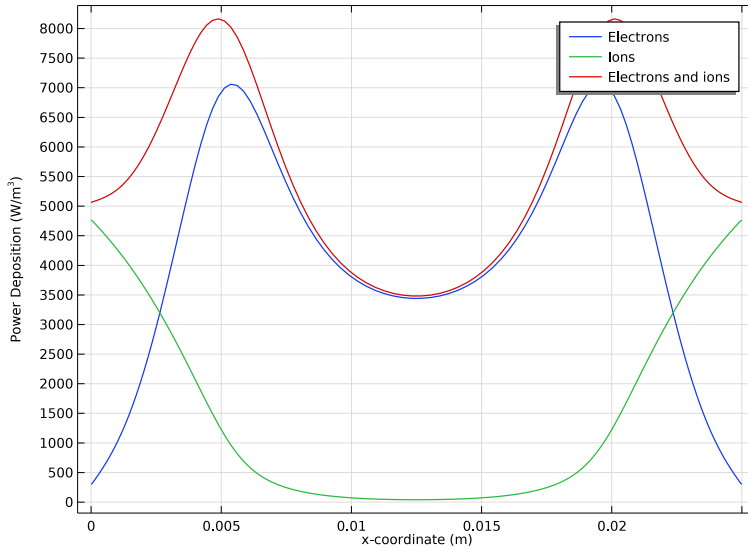
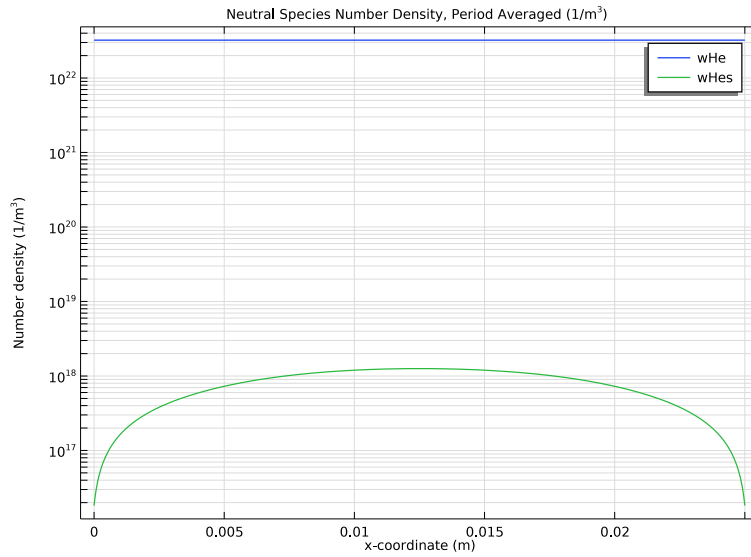
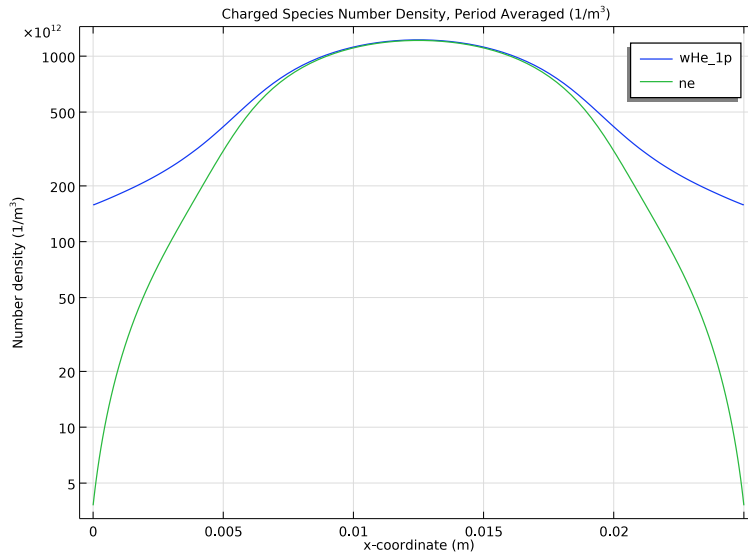


Figure 4: Plot of the period averaged power deposition.



*Figure 5: Plot of the period averaged number density of the neutral species.*



*Figure 6: Plot of the period averaged charged species number densities. The y-axis is on a log scale.*

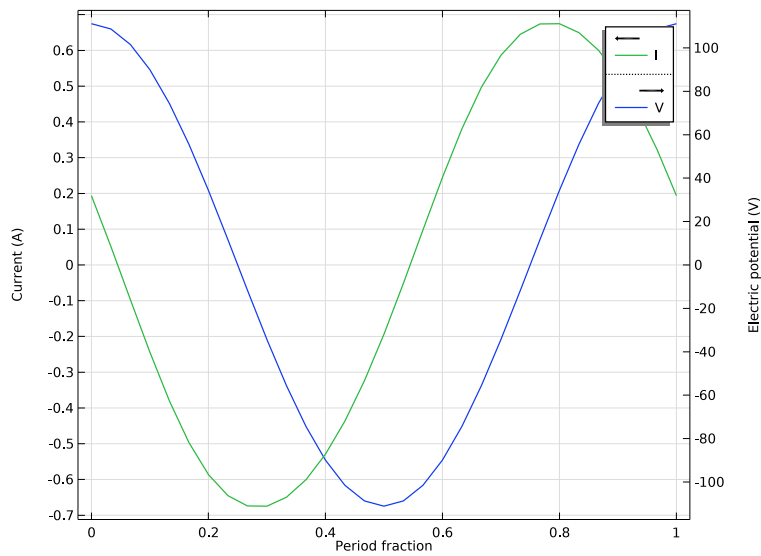


Figure 7: Plot of current and voltage over one period.

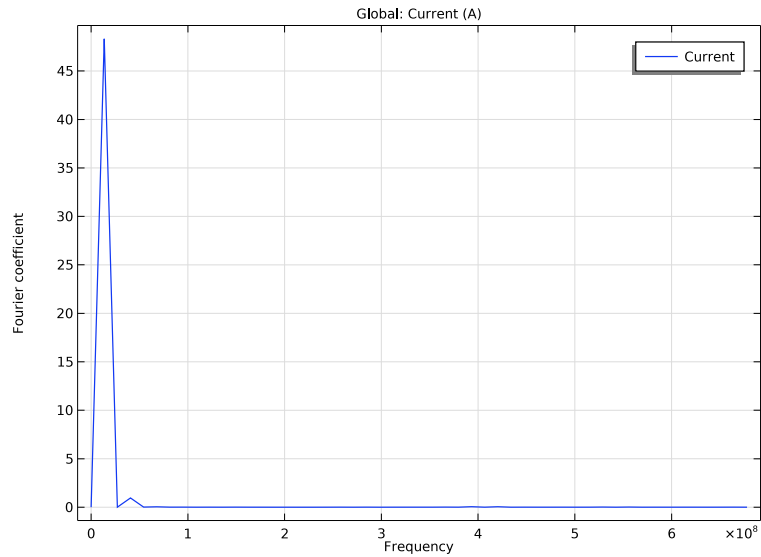


Figure 8: Fourier transform of the discharge current for a power of 10 W. The contribution at the third harmonic is around 1.9 % of the fundamental.

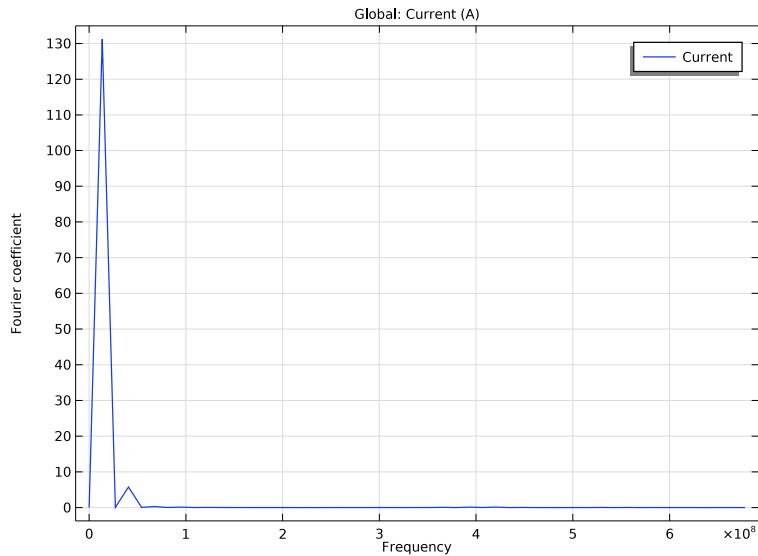


Figure 9: Fourier transform of the discharge current for a power of 50 W. The contribution at the third harmonic is around 4.5 % of the fundamental.

## Reference


1. M.A. Lieberman and A.J. Lichtenberg, *Principles of Plasma Discharges and Materials Processing*, John Wiley & Sons, 2005.

**Application Library path:** Plasma\_Module/Capacitively\_Coupled\_Plasmas/  
computing\_plasma\_impedance

## Modeling Instructions




From the **File** menu, choose **New**.

### NEW

In the **New** window, click  **Model Wizard**.



## MODEL WIZARD

- 1 In the **Model Wizard** window, In order to illustrate the modeling steps required to model a simple capacitive discharge and compute its impedance at the fundamental frequency, this example will be 1D with a simple plasma chemistry. We first illustrate this procedure at low power, then again at high power.
- 2 click  **ID**.
- 3 In the **Select Physics** tree, select **Plasma>Plasma, Time Periodic (ptp)**.
- 4 Click **Add**.
- 5 Click  **Study**.
- 6 In the **Select Study** tree, select **Preset Studies for Selected Physics Interfaces>Time Periodic**.
- 7 Click  **Done**.

The **Time Periodic** study computes the periodic steady state solution of the plasma. Add some parameters for the geometric size, power, frequency and pressure.

## GLOBAL DEFINITIONS

*Parameters I*

- 1 In the **Model Builder** window, under **Global Definitions** click **Parameters I**.
- 2 In the **Settings** window for **Parameters**, locate the **Parameters** section.
- 3 In the table, enter the following settings:

Name	Expression	Value	Description
L	0.025[m]	0.025 m	Discharge gap
de	0.3[m]	0.3 m	Electrode diameter
As	$0.25 \cdot \pi \cdot de^2$	0.070686 m <sup>2</sup>	Electrode area
P0	10[W]	10 W	Input power
f0	13.56E6[Hz]	1.356E7 Hz	Frequency
p0	1[torr]	133.32 Pa	Pressure
T0	300[K]	300 K	Temperature



## GEOMETRY I

*Interval I (il)*

- 1 In the **Model Builder** window, under **Component I (comp1)** right-click **Geometry I** and choose **Interval**.

- 2 In the **Settings** window for **Interval**, locate the **Interval** section.
- 3 In the table, enter the following settings:

Coordinates (m)
0
L



- 4 Click  **Build All Objects**.
- 5 Click the  **Zoom Extents** button in the **Graphics** toolbar.

#### PLASMA, TIME PERIODIC (PTP)

- 1 In the **Model Builder** window, under **Component 1 (comp1)** click **Plasma, Time Periodic (ptp)**.
- 2 In the **Settings** window for **Plasma, Time Periodic**, locate the **Out-of-Plane Thickness** section.
- 3 In the  $A$  text field, type  $A_s$ .
- 4 Locate the **Plasma Properties** section. Select the **Use reduced electron transport properties** check box.
- 5 Locate the **Extra Dimension Settings** section. In the  $P_{xd}$  text field, type  $1/f0$ .
- 6 In the  $N$  text field, type 30.

Next, import cross section data for helium.

#### Cross Section Import 1

- 1 In the **Physics** toolbar, click  **Global** and choose **Cross Section Import**.
- 2 In the **Settings** window for **Cross Section Import**, locate the **Cross Section Import** section.
- 3 Click  **Browse**.
- 4 Browse to the model's Application Libraries folder and double-click the file `He_xsecs.txt`.

#### Species: He

- 1 In the **Model Builder** window, click **Species: He**.
- 2 In the **Settings** window for **Species**, locate the **Species Formula** section.
- 3 Select the **From mass constraint** check box.
- 4 Locate the **General Parameters** section. From the **Preset species data** list, choose **He**.

#### Species: Hes

- 1 In the **Model Builder** window, click **Species: Hes**.

- 2 In the **Settings** window for **Species**, locate the **General Parameters** section.
- 3 From the **Preset species data** list, choose **He**.

The electric fields generated will be rather high in the sheath, so use the local field approximation for the ion temperature, and a lookup table for the ion mobility.

*Species: He+*

- 1 In the **Model Builder** window, click **Species: He+**.
- 2 In the **Settings** window for **Species**, locate the **General Parameters** section.
- 3 From the **Preset species data** list, choose **He**.
- 4 Locate the **Species Formula** section. Select the **Initial value from electroneutrality constraint** check box.
- 5 Click to expand the **Mobility and Diffusivity Expressions** section. From the **Specification** list, choose **Specify mobility, compute diffusivity**.
- 6 From the **Ion temperature** list, choose **Use local field approximation**.
- 7 Click to expand the **Mobility Specification** section. From the **Specify using** list, choose **Helium ion in helium**.


*Plasma Model I*

- 1 In the **Model Builder** window, click **Plasma Model I**.
- 2 In the **Settings** window for **Plasma Model**, locate the **Model Inputs** section.
- 3 In the  $T$  text field, type  $T_0$ .
- 4 In the  $p_A$  text field, type  $p_0$ .

The electron mobility and other transport parameters can be automatically computed from the set of electron impact reactions.

Add surface reactions for the loss of ions and electronically excited helium atoms at the walls.


*Surface Reaction I*

- 1 In the **Physics** toolbar, click  **Boundaries** and choose **Surface Reaction**.
- 2 In the **Settings** window for **Surface Reaction**, locate the **Boundary Selection** section.
- 3 From the **Selection** list, choose **All boundaries**.
- 4 Locate the **Reaction Formula** section. In the **Formula** text field, type  $\text{He}^+ \Rightarrow \text{He}$ .
- 5 Locate the **Reaction Parameters** section. In the  $\gamma_f$  text field, type 0.
- 6 Locate the **Secondary Emission Parameters** section. In the  $\gamma_i$  text field, type 0.1.
- 7 In the  $\epsilon_i$  text field, type 5.8.

2:  $\text{He} + \text{e} \Rightarrow \text{He}$

- 1 Right-click **Surface Reaction 1** and choose **Duplicate**.
- 2 In the **Settings** window for **Surface Reaction**, locate the **Reaction Formula** section.
- 3 In the **Formula** text field, type  $\text{He} + \text{e} \Rightarrow \text{He}$ .
- 4 Locate the **Reaction Parameters** section. In the  $\gamma_f$  text field, type 1.

*Wall 1*


- 1 In the **Physics** toolbar, click  **Boundaries** and choose **Wall**.
- 2 In the **Settings** window for **Wall**, locate the **Boundary Selection** section.
- 3 From the **Selection** list, choose **All boundaries**.

*Ground 1*

- 1 In the **Physics** toolbar, click  **Boundaries** and choose **Ground**.
- 2 Select Boundary 2 only.


The most stable way of driving the electrode is to use a fixed power.

*Metal Contact 1*

- 1 In the **Physics** toolbar, click  **Boundaries** and choose **Metal Contact**.
- 2 Select Boundary 1 only.
- 3 In the **Settings** window for **Metal Contact**, locate the **Terminal** section.
- 4 From the **Source** list, choose **RF**.
- 5 Locate the **RF Source** section. In the  $P_{\text{rf}}$  text field, type P0.
- 6 In the  $f_p$  text field, type f0.

**MESH 1**

*Edge 1*


In the **Mesh** toolbar, click  **Edge**.

*Distribution 1*

- 1 Right-click **Edge 1** and choose **Distribution**.
- 2 In the **Settings** window for **Distribution**, locate the **Distribution** section.
- 3 From the **Distribution type** list, choose **Predefined**.
- 4 In the **Number of elements** text field, type 125.
- 5 In the **Element ratio** text field, type 10.
- 6 Select the **Symmetric distribution** check box.

7 Click  **Build All**.

## TIME PERIODIC



- 1 In the **Model Builder** window, click **Study 1**.
- 2 In the **Settings** window for **Study**, type Time Periodic in the **Label** text field.
- 3 In the **Home** toolbar, click  **Compute**.

## RESULTS

### *Electron Density, Period Averaged (ptp)*

So far, we have computed the periodic steady state solution only. In order to see the time dependent behavior of the plasma, we need to convert the solution to the time domain. To do this, use the **Time Periodic to Time Dependent** study.


## ADD STUDY


- 1 In the **Home** toolbar, click  **Add Study** to open the **Add Study** window.
- 2 Go to the **Add Study** window.
- 3 Find the **Studies** subsection. In the **Select Study** tree, select **Preset Studies for Selected Physics Interfaces>Time Periodic to Time Dependent**.
- 4 Click **Add Study** in the window toolbar.
- 5 In the **Home** toolbar, click  **Add Study** to close the **Add Study** window.

## STUDY 2

### *Step 1: Time Periodic to Time Dependent*

The final output time should correspond to 1 RF cycle. The number of output times should typically be around 100. When computing the time periodic solution, only 30 points were used in the (hidden) time axis. When converting to the time domain, COMSOL will use linear interpolation of the solution between these points.


- 1 In the **Settings** window for **Time Periodic to Time Dependent**, locate the **Study Settings** section.
- 2 Click  **Range**.
- 3 In the **Range** dialog box, choose **Number of values** from the **Entry method** list.
- 4 In the **Stop** text field, type  $1/f0$ .
- 5 In the **Number of values** text field, type 101.
- 6 Click **Replace**.

- 7 In the **Settings** window for **Time Periodic to Time Dependent**, click to expand the **Values of Dependent Variables** section.
- 8 Find the **Values of variables not solved for** subsection. From the **Settings** list, choose **User controlled**.
- 9 From the **Method** list, choose **Solution**.
- 10 From the **Study** list, choose **Time Periodic, Time Periodic**.
- 11 In the **Model Builder** window, click **Study 2**.
- 12 In the **Settings** window for **Study**, type Time Periodic to Time Dependent in the **Label** text field.
- 13 Locate the **Study Settings** section. Clear the **Generate convergence plots** check box.
- 14 Clear the **Generate default plots** check box.
- 15 In the **Home** toolbar, click  **Compute**.




Next, create a FFT plot of the electrode current, so the relative magnitude of the harmonic content can be visualized.

## RESULTS

### *Harmonic Content of the Current*

- 1 In the **Home** toolbar, click  **Add Plot Group** and choose **ID Plot Group**.
- 2 In the **Settings** window for **ID Plot Group**, type Harmonic Content of the Current in the **Label** text field.
- 3 Locate the **Data** section. From the **Dataset** list, choose **Time Periodic to Time Dependent/ Solution 2 (sol2)**.



### *Global I*

- 1 Right-click **Harmonic Content of the Current** and choose **Global**.
- 2 In the **Settings** window for **Global**, click **Replace Expression** in the upper-right corner of the **y-Axis Data** section. From the menu, choose **Component 1 (comp1)>Plasma, Time Periodic>Metal Contact 1>ptp.mct1.I - Current - A**.
- 3 In the **Harmonic Content of the Current** toolbar, click  **Plot**.
- 4 Locate the **x-Axis Data** section. From the **Parameter** list, choose **Discrete Fourier transform**.
- 5 From the **Show** list, choose **Frequency spectrum**.
- 6 In the **Harmonic Content of the Current** toolbar, click  **Plot**.
- 7 Click the  **Zoom Extents** button in the **Graphics** toolbar.

For this low power value, the harmonic at 3 times the fundamental is only around 1.9% of the fundamental.

Next, use the FFT study so that the impedance of the discharge can be computed at the fundamental.



#### ADD STUDY

- 1 In the **Home** toolbar, click  **Add Study** to open the **Add Study** window.
- 2 Go to the **Add Study** window.
- 3 Find the **Studies** subsection. In the **Select Study** tree, select **Empty Study**.
- 4 Click **Add Study** in the window toolbar.
- 5 In the **Home** toolbar, click  **Add Study** to close the **Add Study** window.

#### FFT STUDY TO COMPUTE PLASMA IMPEDANCE

In the **Settings** window for **Study**, type FFT Study to Compute Plasma Impedance in the **Label** text field.

*Time to Frequency FFT*

- 1 In the **Study** toolbar, click  **Study Steps** and choose **Frequency Domain>Time to Frequency FFT**.
- 2 In the **Settings** window for **Time to Frequency FFT**, locate the **Study Settings** section.
- 3 In the **End time** text field, type  $1/f_0$ .
- 4 In the **Maximum output frequency** text field, type  $f_0$ .
- 5 From the **Input study** list, choose **Time Periodic to Time Dependent, Time Periodic to Time Dependent**.
- 6 In the **Study** toolbar, click  **Compute**.

#### RESULTS



*Impedance (ptp, dset4)*

After computing, a **Global Evaluation** feature is automatically generated, with the default expression corresponding to the plasma impedance. Evaluate this to the results table.

- 1 In the **Settings** window for **Global Evaluation**, click  **Evaluate**.

In order to re-compute the plasma impedance for different operating conditions, it is necessary to run all three studies. To make this easier, a fourth study can be added, which uses the **Study reference** feature.

## ADD STUDY



- 1 In the **Study** toolbar, click  **Add Study** to open the **Add Study** window.
- 2 Go to the **Add Study** window.
- 3 Find the **Studies** subsection. In the **Select Study** tree, select **Empty Study**.
- 4 Click **Add Study** in the window toolbar.
- 5 In the **Study** toolbar, click  **Add Study** to close the **Add Study** window.

## RUN ALL STUDIES


In the **Settings** window for **Study**, type Run all Studies in the **Label** text field.

Now reference the first three studies. When computing this study, it will run the other three studies in order.

### *No Study*

- 1 In the **Study** toolbar, click  **Study Reference**.
- 2 In the **Settings** window for **Study Reference**, locate the **Study Reference** section.
- 3 From the **Study reference** list, choose **Time Periodic**.
- 4 In the **Study** toolbar, click  **Study Reference**.

### *No Study*

- 1 In the **Settings** window for **Study Reference**, locate the **Study Reference** section.
- 2 From the **Study reference** list, choose **Time Periodic to Time Dependent**.
- 3 In the **Study** toolbar, click  **Study Reference**.

### *No Study*

- 1 In the **Settings** window for **Study Reference**, locate the **Study Reference** section.
- 2 From the **Study reference** list, choose **FFT Study to Compute Plasma Impedance**.

Now change the input power and re-compute the studies.

## GLOBAL DEFINITIONS


### *Parameters I*

- 1 In the **Model Builder** window, under **Global Definitions** click **Parameters I**.
- 2 In the **Settings** window for **Parameters**, locate the **Parameters** section.
- 3 In the table, enter the following settings:

Name	Expression	Value	Description
P0	50[W]	50 W	Input power



## RUN ALL STUDIES

In the **Study** toolbar, click  **Compute**.

Observe how the harmonic at 3 times the fundamental frequency has now increased to around 4.5% of the fundamental. This will cause significant problems with any potential matching network.

## RESULTS

*Harmonic Content of the Current*

Finally, re-evaluate the plasma impedance at this new power value.

*Impedance (ptp, dset4)*

In the **Model Builder** window, under **Results>Derived Values** right-click **Impedance (ptp, dset4)** and choose **Evaluate>Impedance -** .

