

Ion Energy Distribution Function in a Capacitively Coupled Plasma Reactor

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Introduction

Plasma processing techniques are widely used in the industry to modify the chemical and physical properties of surfaces. Some processes require energetic ion bombardment and a high degree of ion velocity anisotropy. Therefore, it is of great value to know the ion energy distribution function (IEDF) and the velocity dispersion at the surface.

In this example, the IEDF at the electrode surface is computed for a commercial capacitively coupled plasma reactor. The computed IEDF is compared with measurements from Ref. 1 and a reasonable agreement is found.

Note: This application requires the Plasma Module and the Particle Tracing Module.

Model Definition

The reactor that was used in Ref. 1 to measure the IEDF is simulated using the COMSOL Multiphysics Plasma Module. The reactor is the Plasmalab System 100 parallel plate, capacitively coupled, RIE plasma tool. It is an asymmetric capacitively coupled reactor with a 200 mm diameter powered electrode, and a gap between electrodes of 4.5 cm. The other dimensions of the reactor were inferred from diagrams and photos. Since the plasma volume and the area ratio between electrodes is not exactly known a direct comparison between simulations and measurements for absorbed power, applied voltage, and DC selfbias is only approximate.

Two physics interfaces are used in two consecutive steps. First, the **Plasma, Time Periodic** (**ptp**) interface is used to solve for the space and time-periodic evolution of the plasma. After, the **Charged Particle Tracing (cpt)** interface is used to obtained the IEDF collected at the powered electrode.

PLASMA SIMULATION

The simulations are for an Argon plasma sustained at a pressure of 20 mTorr with a periodic electric excitation of 13.56 MHz. The model is 2-dimensional and describes the space and time-periodic evolution of several macroscopic properties of the discharge.

The electron mobility and other electron transport properties are automatically computed from the electron impact reactions. For the ions, the mobility is given as a function of the reduced electric field using a lookup table. The ion diffusivity is obtained from the Einstein relation where the ion temperature comes from a local field approximation.

IEDF SIMULATION

To simulate the IEDF at the powered electrode particles representing ions are released from a point in the plasma and the ion trajectories are computed over several RF cycles until all the released particles reach the wall. The ions should be released from a point outside the plasma sheath in order to experience the full range of the sheath electric field. This point can be estimated by looking at the time averaged electric potential and identifying a region where the potential starts to have smaller variations in the axial direction. Also important is to release particles along the excitation period in order to sample the sheath motion.

The ions are released with a random Maxwellian distribution at 300 K. After the release, the ions are accelerated by a time varying electric field toward the surface. Eventually some ions experience collisions with the background gas. In this simulation two different types of collision events are used: elastic and resonant charge exchange collisions. The collisions are specified by energy dependent cross sections. For the charge exchange collision a scattering angle of 5 degrees is arbitrarily defined.

ELECTRIC EXCITATION

The driven electrode has a fixed power and computes the self DC bias. This corresponds to the following expression and set of constraints on the electric potential:

$$V_s = V_a \cos(2\pi f_p t + \alpha) + V_{dc,b}$$
(1)

$$0 = f_p \int_{\partial t \partial \Omega} (\mathbf{n} \cdot \mathbf{J}_i + \mathbf{n} \cdot \mathbf{J}_e + \mathbf{n} \cdot \mathbf{J}_d) dS dt$$
(2)

$$P_{\rm rf} = f_p \int_{\partial t \,\partial\Omega} (V_s - V_{\rm dc,b}) (\mathbf{n} \cdot \mathbf{J}_i + \mathbf{n} \cdot \mathbf{J}_e + \mathbf{n} \cdot \mathbf{J}_{\rm d}) dS dt .$$
(3)

The constraint in Equation 2 is used to compute the self DC bias, $V_{dc,b}$. The constraint in Equation 3 is used to compute the RF potential, V_a such that a fixed amount of power is deposited into the plasma.

PLASMA CHEMISTRY

Argon plasmas have one of the simplest reactions schemes. We use a simplified plasma chemistry that comprises 7 volume reactions involving electrons, atomic ion, and a lumped level representing the argon 4s states (electron impact cross-sections are obtained Ref. 3).

Reaction	Formula	Туре	$\Delta\epsilon(eV)$
I	e+Ar=>e+Ar	Elastic	0
2	e+Ar=>e+Ars	Excitation	11.5
3	e+Ars=>e+Ar	Superelastic	-11.5
4	e+Ar=>2e+Ar+	Ionization	15.8
5	e+Ars=>2e+Ar+	Ionization	4.24
6	Ars+Ars =>e+Ar+Ar+	Penning ionization	-
7	Ars+Ar => Ar+Ar	Metastable quenching	-

TABLE I: TABLE OF COLLISIONS AND REACTIONS MODELED.

In addition to volumetric reactions, the following surface reactions are implemented:

TABLE 2: TABLE OF SURFACE REACTIONS.

Reaction	Formula	Sticking coefficient	Secondary emission coefficient	Mean energy of secondary electrons (V)
1	Ars=>Ar	I	0.07	5.8
2	Ar+=>Ar	0	0.07	5.8

When an excited states make contact with the wall, they revert to the ground state argon with some probability. At the metal electrodes, the ions and the excited state use their internal energy to extract one electron from the wall with a probability of 0.07 and a mean energy of 5.8 eV. For the ions, the sticking coefficient is zero meaning that losses to the wall are assumed to be due to migration only.

Results and Discussion

Figure 1, Figure 2, and Figure 3 show the time averaged electron density, electron temperature and electric potential for a plasma sustained at 20 mTorr and 30 W. The period averaged electron density is largest in magnitude in the reactor center, decreasing considerable along the radial and axial directions as expected. The period averaged electron temperature is highest near the electrodes where intense electric fields exist. Since it is an asymmetric reactor the smaller electrode (the powered electrode in this model) has a sheath with more intense electric fields. This is reflected in higher electron temperature

at the power electrode. In Figure 3 it is possible to observe the DC self–bias potential developed in order to ensure that there is no period average conduction current through the electrode.

Figure 4 and Figure 5 presents the IEDF and the angular dispersion of the ion velocity at the power electrode. A double peak structure is observed at high energies meaning that the most energetic ions cross the sheath in a time comparable with the RF cycle. The midpoint energy between the two energetic peaks corresponds to the energy that the ions would gain while accelerating through the time-averaged electric potential. From Figure 3 it is possible to estimate that the midpoint energy should be around 350 V (potential difference from -200 V at the wall to 150 V in the discharge bulk), which is in agreement with the results of Figure 4.

The width between the two high energy peaks ΔE can be estimated from analytical models such as the model from Charles and others (from Ref. 1)

$$\Delta E = V_{\rm pp} \left(1 + \left(\frac{2\pi\tau_{\rm i}}{3\tau_{\rm rf}} \right)^2 \right)^{-1/2} \tag{4}$$

and the model from Sobolewski and others (from Ref. 1)

$$\Delta E = V_{\rm pp} \left(1 + \pi^2 \left(\frac{2T_e}{V_{\rm s}} \right)^{1/2} \left(\frac{\tau_{\rm i}}{\tau_{\rm rf}} \right)^2 \right)^{-1/2}$$
(5)

where $V_{\rm pp}$ is the peak-to-peak sheath voltage, $V_{\rm s}$ is the time-averaged sheath potential, $\tau_{\rm i}$ is the ion transit time through the sheath, $\tau_{\rm rf}$ is the RF period, and T_e is the electron temperature. The ΔE computed form Equation 4 and Equation 5 gives 42 V and 71 V, respectively, which is in good agreement with the 70 V from the simulated IEDF. The values used to compute ΔE are obtain from the simulations and are presented in Table 3.

TABLE 3: VALUES OBTAINED FROM SIMULATIONS AND USED TO COMPUTE $\Delta {\rm E}.$

T _e	4 V
V _s	340 V
V _{pp}	1070
τ_{i}	900 ns

The ions that cross the sheath without colliding (or with a collision in the beginning of the trajectory) contribute to the high-energy part of the IEDF. The low-energy part of the IEDF also has several well-defined peaks. These structures are created by ions that undergo resonant charge exchange collisions along their motion in direction to the wall. After such

collision events, a slow ion is created within the sheath, and consequently it experiences only part of the electric field range.

In Figure 5 it is possible to observe that the high energy ions have a -5 degree tilt relative to their axial velocity. This is a consequence of the existence of important radial electric fields in this region. Also from Figure 5 you can see that there is a dispersion angle at the low-energy range. This is because it is used an arbitrary 5 degree scattering angle for the resonant charge exchange collision. To obtain realistic results for the ion dispersion velocity, a physical model for the scattering angle is needed.



Figure 1: Plot of the period averaged electron density.



Figure 2: Plot of the period averaged electron temperature.



Figure 3: Plot of the period averaged electric potential.



Figure 4: IEDF at the powered electrode.



Figure 5: IEDF with velocity angular dispersion at the powered electrode.

References

1. D. Gahan, S. Daniels, C. Hayden, D. O'Sullivan, and M.B. Hopkins, "Characterization of an asymmetric parallel plate radio-frequency discharge using a retarding field energy analyzer," *Plasma Sources Sci. Technol.*, vol. 21, p. 015002 (12pp), 2012.

2. M.A. Lieberman and A.J. Lichtenberg, *Principles of Plasma Discharges and Materials Processing*, John Wiley & Sons, 2005.

3. Phelps database, www.lxcat.net, retrieved 2017.

Application Library path: Plasma_Module/Capacitively_Coupled_Plasmas/ ccp_ion_energy_distribution_function

Modeling Instructions

The following instructions show how to create a 2D model of a RF capacitively coupled plasma and how to obtain the IEDF at the electrode surface using the **Charge Particle Tracing (cpt)** interface. Three studies are needed:

- A Time Periodic study that computes the periodic steady-state solution of the plasma.
- A **Time Periodic to Time Dependent** study that converts the periodic steady-state solution to the time domain.
- A **Time Dependent** study that is used to compute the ion trajectories through the sheath.

From the File menu, choose New.

NEW

In the New window, click 🙆 Model Wizard.

MODEL WIZARD

- I In the Model Wizard window, Select the Plasma, Time Periodic (ptp) interface and the Time Periodic study to compute the periodic steady state solution of the plasma.
- 2 click 🚈 2D Axisymmetric.
- 3 In the Select Physics tree, select Plasma>Plasma, Time Periodic (ptp).
- 4 Click Add.
- 5 Click \bigcirc Study.

- 6 In the Select Study tree, select Preset Studies for Selected Physics Interfaces> Time Periodic.
- 7 Click 🗹 Done.

GEOMETRY I

- I In the Model Builder window, under Component I (compl) click Geometry I.
- 2 In the Settings window for Geometry, locate the Units section.
- 3 From the Length unit list, choose mm.

Add some parameters for the reactor dimensions, power, pressure, and excitation frequency.

GLOBAL DEFINITIONS

Parameters 1

I In the Model Builder window, under Global Definitions click Parameters I.

2 In the Settings window for Parameters, locate the Parameters section.

3 In the table, enter the following settings:

Name	Expression	Value	Description
r0	160[mm]	0.16 m	
h0	4.5[cm]	0.045 m	
r1	110[mm]	0.11 m	
h1	40[mm]	0.04 m	
Prf	1[W]	I W	
fO	13.56[MHz]	1.356E7 Hz	
p0	0.02[torr]	2.6664 Pa	

GEOMETRY I

Rectangle 1 (r1)

- I In the Geometry toolbar, click Rectangle.
- 2 In the Settings window for Rectangle, locate the Size and Shape section.
- 3 In the Width text field, type r0.
- **4** In the **Height** text field, type h0.

Rectangle 2 (r2)

I In the Geometry toolbar, click Rectangle.

- 2 In the Settings window for Rectangle, locate the Size and Shape section.
- **3** In the **Width** text field, type r0-r1.
- 4 In the **Height** text field, type h1.
- **5** Locate the **Position** section. In the **r** text field, type **r1**.
- 6 In the z text field, type -h1.

Union I (unil)

- I In the Geometry toolbar, click 📕 Booleans and Partitions and choose Union.
- 2 Click in the Graphics window and then press Ctrl+A to select both objects.
- 3 In the Settings window for Union, locate the Union section.
- 4 Clear the Keep interior boundaries check box.
- 5 Click 🟢 Build All Objects.

Chamfer I (chaI)

- I In the Geometry toolbar, click 🧹 Chamfer.
- 2 On the object unil, select Point 4 only.
- 3 In the Settings window for Chamfer, locate the Distance section.
- 4 In the **Distance from vertex** text field, type 10.

Fillet I (fill)

- I In the **Geometry** toolbar, click *Fillet*.
- 2 On the object chal, select Points 3 and 5 only.
- 3 In the Settings window for Fillet, locate the Radius section.
- 4 In the Radius text field, type 6.

Point I (ptl)

- I In the **Geometry** toolbar, click **Point**.
- 2 In the Settings window for Point, locate the Point section.
- **3** In the **r** text field, type **97.5**.
- 4 In the z text field, type h0.

Line Segment 1 (Is1)

- I In the Geometry toolbar, click 🚧 More Primitives and choose Line Segment.
- 2 On the object fill, select Point 3 only.
- 3 In the Settings window for Line Segment, locate the Endpoint section.
- 4 Find the End vertex subsection. Click to select the 🔲 Activate Selection toggle button.

- 5 On the object **pt1**, select Point 1 only.
- 6 Click 틤 Build Selected.

Mesh Control Edges 1 (mcel)

- I In the Geometry toolbar, click 🏷 Virtual Operations and choose Mesh Control Edges.
- 2 On the object fin, select Boundary 4 only.
- 3 In the Geometry toolbar, click 🟢 Build All.

Set the period and the number of elements in the extra dimension.

PLASMA, TIME PERIODIC (PTP)

- I In the Model Builder window, under Component I (compl) click Plasma, Time Periodic (ptp).
- **2** In the **Settings** window for **Plasma**, **Time Periodic**, locate the **Extra Dimension Settings** section.
- **3** In the $P_{\rm xd}$ text field, type 1/f0.
- 4 In the *N* text field, type 30.
- 5 From the Heavy species selection list, choose Base geometry.

Import cross section data for Argon.

Cross Section Import 1

- I In the Physics toolbar, click 🖗 Global and choose Cross Section Import.
- 2 In the Settings window for Cross Section Import, locate the Cross Section Import section.
- 3 Click **Browse**.
- **4** Browse to the model's Application Libraries folder and double-click the file Ar_xsecs.txt.

Add other reactions to complete the plasma chemistry.

Reaction I

- I In the Physics toolbar, click **Domains** and choose **Reaction**.
- 2 In the Settings window for Reaction, locate the Reaction Formula section.
- 3 In the Formula text field, type Ars+Ars=>e+Ar+Ar+.
- 4 Locate the **Reaction Parameters** section. In the k^{f} text field, type 2.3E7.

Reaction 2

- I In the Physics toolbar, click **Domains** and choose **Reaction**.
- 2 In the Settings window for Reaction, locate the Reaction Formula section.

- **3** In the **Formula** text field, type Ars+Ar=>Ar+Ar.
- **4** Locate the **Reaction Parameters** section. In the k^{f} text field, type 1807.

Species: Ar

- I In the Model Builder window, click Species: Ar.
- 2 In the Settings window for Species, locate the Species Formula section.
- **3** Select the **From mass constraint** check box.
- 4 Locate the General Parameters section. From the Preset species data list, choose Ar.

Species: Ars

- I In the Model Builder window, click Species: Ars.
- 2 In the Settings window for Species, locate the General Parameters section.
- 3 From the Preset species data list, choose Ar.

The electric fields generated will be rather high in the sheath, so use the local field approximation for the ion temperature, and a lookup table for the ion mobility.

Species: Ar+

- I In the Model Builder window, click Species: Ar+.
- 2 In the Settings window for Species, locate the Species Formula section.
- **3** Select the **Initial value from electroneutrality constraint** check box.
- 4 Locate the General Parameters section. From the Preset species data list, choose Ar.
- **5** Click to expand the **Mobility and Diffusivity Expressions** section. From the **Specification** list, choose **Specify mobility, compute diffusivity**.
- 6 From the lon temperature list, choose Use local field approximation.
- 7 Click to expand the Mobility Specification section. From the Specify using list, choose Argon ion in argon.

Define surface losses for ions and argon excited state. For the ions, set the **Forward sticking coefficient** to zero to assume that ion losses are due to migration only, and use the default **Secondary emission coefficient** of 0.07.

The argon excited state also uses the default Secondary emission coefficient of 0.07.

Electron secondary emission is made only possible on the electrodes.

Surface Reaction 1

- I In the Physics toolbar, click Boundaries and choose Surface Reaction.
- 2 In the Settings window for Surface Reaction, locate the Reaction Formula section.

- **3** In the **Formula** text field, type Ar+=>Ar.
- **4** Locate the **Reaction Parameters** section. In the γ_f text field, type **0**.
- **5** Select Boundaries 2 and 3 only.

2: Ar+=>Ar

- I Right-click Surface Reaction I and choose Duplicate.
- 2 In the Settings window for Surface Reaction, locate the Boundary Selection section.
- 3 Click Clear Selection.
- 4 Select Boundaries 4–9 only.
- **5** Locate the **Secondary Emission Parameters** section. In the γ_i text field, type 0.
- **6** In the ε_i text field, type 0.

3: Ar+=>Ar

- I In the Model Builder window, under Component I (comp1)>Plasma, Time Periodic (ptp) right-click I: Ar+=>Ar and choose Duplicate.
- 2 In the Settings window for Surface Reaction, locate the Reaction Formula section.
- **3** In the **Formula** text field, type Ars=>Ar.
- 4 Locate the Reaction Parameters section. In the γ_f text field, type 1.
- 4: Ar + = > Ar
- I In the Model Builder window, under Component I (comp1)>Plasma, Time Periodic (ptp) right-click 2: Ar+=>Ar and choose Duplicate.
- 2 In the Settings window for Surface Reaction, locate the Reaction Formula section.
- **3** In the **Formula** text field, type Ars=>Ar.
- **4** Locate the **Reaction Parameters** section. In the γ_f text field, type 1.

Plasma Model I

- I In the Model Builder window, click Plasma Model I.
- 2 In the Settings window for Plasma Model, locate the Model Inputs section.
- **3** In the T text field, type 300[K].
- **4** In the p_A text field, type p0.

Wall I

- I In the **Physics** toolbar, click **Boundaries** and choose **Wall**.
- 2 In the Settings window for Wall, locate the Boundary Selection section.
- **3** From the Selection list, choose All boundaries.

Define the electric excitation of the discharge. Set a ground electrode on the top and a RF power source on bottom.

At the other boundaries use dielectric contact to ensure that the period average conduction current is zero.

Ground I

- I In the Physics toolbar, click Boundaries and choose Ground.
- **2** Select Boundary **3** only.

Dielectric Contact I

- I In the Physics toolbar, click Boundaries and choose Dielectric Contact.
- **2** Select Boundaries 4–9 only.

The most stable way of driving the electrode is to use a fixed power. Use the previously defined parameter Prf to set the **RF power**. The exact values for the power sweep are set later in the study.

Metal Contact 1

- I In the Physics toolbar, click Boundaries and choose Metal Contact.
- 2 In the Settings window for Metal Contact, locate the RF Source section.
- **3** In the $P_{\rm rf}$ text field, type Prf.
- **4** In the f_p text field, type f0.
- 5 Locate the DC Source section. Select the Compute DC self-bias check box.
- 6 Select Boundary 2 only.

The mesh needs to be refined near the electrodes where the power is absorbed. A mapped mesh can be used across the main discharge area, and a free triangular mesh everywhere else.

MESH I

Edge I

- I In the Mesh toolbar, click A Edge.
- 2 Select Boundaries 1 and 12 only.

Distribution I

- I Right-click Edge I and choose Distribution.
- 2 In the Settings window for Distribution, locate the Distribution section.
- 3 From the Distribution type list, choose Predefined.

- 4 In the Number of elements text field, type 30.
- 5 In the Element ratio text field, type 5.
- 6 Select the Symmetric distribution check box.

Edge 2

- I In the Mesh toolbar, click 🛕 Edge.
- 2 Select Boundaries 3 and 10 only.

Distribution I

- I Right-click Edge 2 and choose Distribution.
- 2 In the Settings window for Distribution, locate the Distribution section.
- 3 In the Number of elements text field, type 20.

Mapped I

- I In the Mesh toolbar, click Mapped.
- 2 In the Settings window for Mapped, locate the Domain Selection section.
- 3 From the Geometric entity level list, choose Domain.
- **4** Select Domain 1 only.
- 5 Click 🖷 Build Selected.

Free Triangular 1

- I In the Mesh toolbar, click Kree Triangular.
- 2 In the Settings window for Free Triangular, click to expand the Control Entities section.
- **3** Clear the **Smooth across removed control entities** check box.

Size 1

- I Right-click Free Triangular I and choose Size.
- 2 In the Settings window for Size, locate the Element Size section.
- 3 From the Predefined list, choose Extra fine.

Boundary Layers 1

- I In the Mesh toolbar, click Boundary Layers.
- 2 In the Settings window for Boundary Layers, click to expand the Transition section.
- **3** Clear the **Smooth transition to interior mesh** check box.
- **4** Click to expand the **Corner Settings** section. From the **Handling of sharp corners** list, choose **Trimming**.

Boundary Layer Properties

- I In the Model Builder window, click Boundary Layer Properties.
- **2** In the **Settings** window for **Boundary Layer Properties**, locate the **Boundary Selection** section.
- 3 From the Selection list, choose All boundaries.
- **4** Select Boundaries 2–11 only.
- 5 Locate the Layers section. In the Number of layers text field, type 2.
- 6 In the Stretching factor text field, type 1.4.
- 7 Click 📗 Build All.

STUDY I

In the Study toolbar, click $\underset{t=0}{\bigcup}$ Get Initial Value.

The objective is to obtain a solution at 30 W. To obtain a high power solution it is easier to start at a low power and then sweep over power until the desired power is reached.

In this study a solution of 1 W is used as initial condition. The power is then increased by 1 W until a value of 30 W is reached. Setting a maximum power increment for the parametric solver makes convergence smoother as the power is increased.

The maximum increase in the damping factor between nonlinear iterations is modified from its default value of 0.05 to 0.03. This makes it easier for the nonlinear solver to converge for the first power value. Lower values for the maximum increase makes convergence easier, although possibly at the expense of more nonlinear iterations and thus longer computation times.

Step 1: Time Periodic

- I In the Model Builder window, under Study I click Step I: Time Periodic.
- 2 In the Settings window for Time Periodic, click to expand the Study Extensions section.
- 3 Select the Auxiliary sweep check box.
- 4 Click + Add.
- **5** In the table, enter the following settings:

Parameter name	Parameter value list	Parameter unit	
Prf	1 30	W	

Solver Configurations

In the Model Builder window, expand the Study I>Solver Configurations node.

Solution 1 (soll)

- I In the Model Builder window, expand the Study I>Solver Configurations> Solution I (soll)>Stationary Solver I node, then click Fully Coupled I.
- **2** In the **Settings** window for **Fully Coupled**, click to expand the **Method and Termination** section.
- 3 In the Restriction for step-size increase text field, type 0.03.
- 4 Click to expand the **Results While Solving** section. Select the **Plot** check box.
- 5 In the Model Builder window, click Parametric.
- 6 In the Settings window for Parametric, click to expand the Continuation section.
- 7 Select the **Tuning of step size** check box.
- 8 In the Initial step size text field, type 0.25.
- 9 In the Maximum step size text field, type 0.5.
- **IO** In the **Study** toolbar, click **= Compute**.

So far, we have computed the periodic steady-state solution only. In order to see the timedependent behavior of the plasma, we need to convert the solution to the time domain. To do this, use the **Time Periodic to Time Dependent** study.

ADD STUDY

- I In the Study toolbar, click \sim Add Study to open the Add Study window.
- 2 Go to the Add Study window.
- 3 Find the Studies subsection. In the Select Study tree, select Preset Studies for Selected Physics Interfaces>Time Periodic to Time Dependent.
- 4 Click Add Study in the window toolbar.
- 5 In the Study toolbar, click \sim Add Study to close the Add Study window.

STUDY 2

Step 1: Time Periodic to Time Dependent

The final output time should correspond to 1 RF cycle. The number of output times should typically be around 100. When computing the time-periodic solution, only 30 points were used in the (hidden) time axis. When converting to the time domain, COMSOL Multiphysics uses linear interpolation of the solution between these points.

- I In the Settings window for Time Periodic to Time Dependent, locate the Study Settings section.
- 2 Click Range.

- 3 In the Range dialog box, type (1/f0)/101 in the Step text field.
- 4 In the **Stop** text field, type 1/f0.
- 5 Click Replace.
- 6 In the Settings window for Time Periodic to Time Dependent, click to expand the Values of Dependent Variables section.
- 7 Find the Values of variables not solved for subsection. From the Settings list, choose User controlled.
- 8 From the Method list, choose Solution.
- 9 From the Study list, choose Study I, Time Periodic.
- 10 From the Parameter value (Prf (W)) list, choose Last.
- II In the **Study** toolbar, click **= Compute**.

Add the Charged Particle Tracing (cpt) interface to compute the IEDF.

ADD PHYSICS

- I In the Home toolbar, click 🙀 Add Physics to open the Add Physics window.
- 2 Go to the Add Physics window.
- 3 In the tree, select AC/DC>Particle Tracing>Charged Particle Tracing (cpt).
- 4 Find the Physics interfaces in study subsection. In the table, clear the Solve check boxes for Study 1 and Study 2.
- 5 Click Add to Component I in the window toolbar.
- 6 In the Home toolbar, click 🙀 Add Physics to close the Add Physics window.

DEFINITIONS (COMPI)

Add analytic functions for the ion resonant charge exchange and elastic cross sections.

Analytic I (an I)

- I In the Home toolbar, click f(X) Functions and choose Global>Analytic.
- 2 In the Settings window for Analytic, locate the Definition section.
- **3** In the **Expression** text field, type (7*1e-10-0.6*1e-10*log(x))^2.
- 4 Locate the **Units** section. In the table, enter the following settings:

Argument	Unit
x	eV

5 In the Function text field, type m^2.

6 In the Function name text field, type Qex.

Analytic 2 (an2)

- I In the Home toolbar, click f(X) Functions and choose Global>Analytic.
- 2 In the Settings window for Analytic, locate the Definition section.
- 3 In the Expression text field, type $2e \frac{19}{(x^{(0.5)*(1+x)})+3e \frac{19*x}{(1+x/3)^{(2.3)}}$.
- 4 Locate the **Units** section. In the table, enter the following settings:

Argument	Unit
x	eV

- **5** In the **Function** text field, type m².
- 6 In the Function name text field, type Qele.

Variables 1

- I In the Model Builder window, right-click Definitions and choose Variables.
- 2 In the Settings window for Variables, locate the Variables section.
- **3** In the table, enter the following settings:

Name	Expression	Unit	Description
Mion	0.04[kg/mol]/N_A_const	kg	

CHARGED PARTICLE TRACING (CPT)

Particle Properties 1

- I In the Model Builder window, under Component I (compl)>Charged Particle Tracing (cpt) click Particle Properties I.
- 2 In the Settings window for Particle Properties, locate the Particle Mass section.
- **3** In the $m_{\rm p}$ text field, type Mion.
- **4** Locate the **Charge Number** section. In the Z text field, type 1.

The electric force is computed with the time dependent potential from the previous study.

Electric Force 1

- I In the **Physics** toolbar, click **Domains** and choose **Electric Force**.
- **2** Select Domain 1 only.
- 3 In the Settings window for Electric Force, locate the Electric Force section.
- **4** From the **Specify force using** list, choose **Electric potential**.

- **5** From the *V* list, choose **Electric potential (ptp/pes1)**.
- 6 Locate the Advanced Settings section. From the Time dependence of field list, choose Periodic.
- 7 Select the Use piecewise polynomial recovery on field check box.

Allow for collision events that ions undergo along their trajectory. This is done by setting the background gas density and defining cross sections for resonant charge exchange and elastic collisions.

Collisions I

- I In the Physics toolbar, click 🔵 Domains and choose Collisions.
- 2 In the Settings window for Collisions, locate the Fluid Properties section.
- **3** In the N_d text field, type ptp.Nn.
- **4** In the *T* text field, type 300[K].
- **5** Select Domain 1 only.

Resonant Charge Exchange 1

- I In the Physics toolbar, click Attributes and choose Resonant Charge Exchange.
- 2 In the Settings window for Resonant Charge Exchange, locate the Collision Frequency section.
- **3** In the σ text field, type Qex(cpt.Ep).
- 4 In the χ text field, type 5[deg].

Collisions I

In the Model Builder window, click Collisions I.

Elastic I

- I In the Physics toolbar, click Attributes and choose Elastic.
- 2 In the Settings window for Elastic, locate the Collision Frequency section.
- 3 In the σ text field, type Qele(cpt.Ep).

Define where and when the ions are released. The ions should be released at different times along the period, and they should be released from a point outside the plasma sheath.

Release from Grid 1

- I In the Physics toolbar, click 🖗 Global and choose Release from Grid.
- 2 In the Settings window for Release from Grid, locate the Release Times section.
- 3 Click Range.

- 4 In the **Range** dialog box, type 0 in the **Start** text field.
- 5 In the Step text field, type (1/f0)/30.
- 6 In the **Stop** text field, type 1/f0.
- 7 Click Replace.
- 8 In the Settings window for Release from Grid, locate the Initial Coordinates section.
- **9** In the $q_{r,0}$ text field, type 40.
- **IO** In the $q_{z,0}$ text field, type 8.
- II Locate the Initial Velocity section. From the Initial velocity list, choose Maxwellian.
- 12 From the Sampling from distribution list, choose Random.
- **I3** In the $N_{\mathbf{v}}$ text field, type **30**.
- I4 In the T_0 text field, type 300[K].

ADD STUDY

- I In the Home toolbar, click \sim_1° Add Study to open the Add Study window.
- 2 Go to the Add Study window.
- **3** Find the **Physics interfaces in study** subsection. In the table, clear the **Solve** check box for **Plasma, Time Periodic (ptp)**.
- 4 Find the Studies subsection. In the Select Study tree, select General Studies> Time Dependent.
- 5 Click Add Study in the window toolbar.
- 6 In the Home toolbar, click ~ 2 Add Study to close the Add Study window.

STUDY 3

Step 1: Time Dependent

- I In the Settings window for Time Dependent, locate the Study Settings section.
- 2 Click Range.
- **3** In the **Range** dialog box, The study should run for a number of periods that allows for all released ions to reach the surface.
- 4 type (1/f0) in the Step text field.
- 5 In the **Stop** text field, type (1/f0)*40.
- 6 Click Replace.
- 7 In the Settings window for Time Dependent, click to expand the Values of Dependent Variables section.

- 8 Find the Values of variables not solved for subsection. From the Settings list, choose User controlled.
- 9 From the Method list, choose Solution.
- 10 From the Study list, choose Study 2, Time Periodic to Time Dependent.

Solution 3 (sol3)

- I In the Study toolbar, click The Show Default Solver.
- 2 In the Model Builder window, expand the Solution 3 (sol3) node, then click Time-Dependent Solver 1.
- **3** In the **Settings** window for **Time-Dependent Solver**, click to expand the **Time Stepping** section.
- 4 From the Maximum step constraint list, choose Constant.
- 5 In the Maximum step text field, type 1E-9.
- 6 In the Study toolbar, click **=** Compute.

Create a plot to show the IEDF at the electrode surface.

RESULTS

Ion Energy Distribution Function (IEDF)

- I In the Home toolbar, click 🚛 Add Plot Group and choose ID Plot Group.
- 2 In the Settings window for ID Plot Group, type Ion Energy Distribution Function (IEDF) in the Label text field.
- 3 Locate the Data section. From the Dataset list, choose Particle I.
- 4 From the Time selection list, choose Last.

Histogram 1

- I Right-click Ion Energy Distribution Function (IEDF) and choose Histogram.
- 2 In the Settings window for Histogram, locate the Expression section.
- **3** In the **Expression** text field, type cpt.Ep.
- 4 From the Unit list, choose eV.
- 5 Locate the Bins section. From the Entry method list, choose Limits.
- 6 Click Range.
- 7 In the Range dialog box, type 0 in the Start text field.
- 8 In the Step text field, type 2.
- 9 In the **Stop** text field, type 400.

IO Click Replace.

II In the Ion Energy Distribution Function (IEDF) toolbar, click 💿 Plot.

Create a plot to show the IEDF and the angular distribution at the electrode surface.

Ion Angular Energy Distribution Function (IAEDF)

- I In the Home toolbar, click 🚛 Add Plot Group and choose 2D Plot Group.
- 2 In the Settings window for 2D Plot Group, type Ion Angular Energy Distribution Function (IAEDF) in the Label text field.
- 3 Locate the Data section. From the Dataset list, choose Particle I.
- 4 Click → Plot Last.

Histogram 1

- I In the Ion Angular Energy Distribution Function (IAEDF) toolbar, click More Plots and choose Histogram.
- 2 In the Settings window for Histogram, locate the x-Expression section.
- **3** In the **Expression** text field, type atan(cpt.vr/cpt.vz).
- 4 From the **Unit** list, choose °.
- **5** Select the **Description** check box.
- 6 In the associated text field, type Angle.
- 7 Locate the y-Expression section. In the Expression text field, type cpt.Ep.
- 8 From the **Unit** list, choose eV.
- 9 Locate the Bins section. Find the x bins subsection. From the Entry method list, choose Limits.
- IO Click Range.
- II In the Range dialog box, choose Number of values from the Entry method list.
- **12** In the **Start** text field, type -10.
- **I3** In the **Stop** text field, type 10.
- 14 In the Number of values text field, type 50.
- **I5** Click **Replace**.
- 16 In the Settings window for Histogram, locate the Bins section.
- 17 Find the y bins subsection. From the Entry method list, choose Limits.
- 18 Click Range.
- 19 In the Range dialog box, type 0 in the Start text field.

- **20** In the **Step** text field, type **2**.
- **2I** In the **Stop** text field, type 400.
- 22 Click Replace.
- **23** In the **Ion Angular Energy Distribution Function (IAEDF)** toolbar, click **OM Plot**.

24 Click the \longleftrightarrow **Zoom Extents** button in the **Graphics** toolbar.

Obtain the voltage amplitude and the DC bias.

Global Evaluation 1

- I In the Results toolbar, click (8.5) Global Evaluation.
- In the Settings window for Global Evaluation, click Replace Expression in the upper-right corner of the Expressions section. From the menu, choose Component I (compl)>
 Plasma, Time Periodic>Metal Contact I>ptp.mctl.Va_per Voltage amplitude V.
- 3 Click Add Expression in the upper-right corner of the Expressions section. From the menu, choose Component I (compl)>Plasma, Time Periodic>Metal Contact l> ptp.mctl.Vdcb_per DC bias voltage V.
- 4 Click **= Evaluate**.