Created in COMSOL Multiphysics 6.0

Organ Pipe Design

Introduction

In this model, an organ flue pipe is designed and analyzed using a pipe acoustics model. Because it is a 1D model, it is fast to solve but still retains most of the relevant physical parameters when designing an organ pipe. The model includes the elastic properties of the organ pipe walls, the end impedance properties at the open pipe end, and the possibility to add a small background airflow inside the pipe. In this model, the pipe is driven at 440 Hz which is the A4 note (or a').

A sketch of an organ pipe is shown in Figure 1. An airflow is pushed in at the bottom of the organ pipe and out via the mouth. At the mouth, an air jet strikes the sharp upper lip and this sets the air into vibration. The vibrations resonate with the organ pipe body to create the note of the pipe. In an open pipe, like the one sketched here, the fundamental tone corresponds to a half wave resonance in the pipe. The harmonics are then multiples of this frequency.

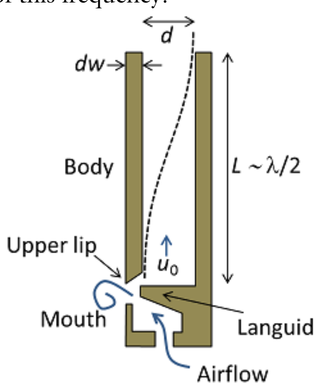

Figure 1: Sketch of an organ pipe including the mouth and the pipe body.

Note: This application requires the Acoustics Module or the Pipe Flow Module.

The timbre of the organ pipe depends on the combination of the fundamental tone and all the harmonics. This depends on the shape of the pipe (the length and diameter) as well as on the elastic properties of the pipe walls and their thickness. Moreover, a small residual airflow in the pipe u_0, may alter the natural frequencies slightly (this effect is not modeled here). Changing any of these parameters influences not only the natural frequencies of the organ pipe, but also the damping and Q value of the corresponding frequency response resonance peaks. This in turn yields a different timbre.

Model Definition

The organ pipe geometry is defined in terms of its length L, inner pipe diameter d, wall thickness dw (see Figure 1), and cross-section shape (here circular). Only the length is used when drawing the pipe geometry as a straight line segment. The inner radius, wall thickness, and pipe shape are parameters entering the governing equations. The elastic properties of the pipe wall are Young's modulus E_w and Poisson's ratio ν_w. The model parameters are given in the table below.

TABLE 1: MODEL PARAMETERS.

NAME	EXPRESSION	DESCRIPTION
f	440 Hz	Frequency of an A4 note
L_{guess}	c_0/(2 f)	Half wavelength at f
L	0.3805 m	Pipe length giving a resonance at 440 Hz
d	3 cm	Inner pipe diameter
dw	2 mm	Wall thickness
E_w	10^9 Pa	Pipe wall Young's modulus
ν_w	0.4	Pipe wall Poisson's ratio
c_0	343 m/s	Speed of sound
ka	0.12	Wave number times tube radius
h_{min}	c_0/3000 Hz/20	Mesh size at 3000 Hz
T_0	20°C	Ambient temperature
p_0	1 atm	Ambient pressure
dL	0.6a	End correction
f_{est}	c_0/(2(L+dL)	Estimated resonance frequency

The open end of the pipe is modeled by adding an end impedance property. This is an engineering relation for the case of a pipe of circular cross section ending in free space (an unflanged pipe).

Results and Discussion

The frequency response of the pipe is obtained by plotting the sound pressure level L_p at the open pipe end,

$$L_p = 10\log\left[\left(\frac{p_{rms}}{p_{ref}}\right)^2\right] \qquad p_{rms}^2 = \frac{1}{2}pp^*$$

where p_{ref} is the reference pressure for air, 20 µPa, and * is the complex conjugate.

The frequency response around the first resonance frequency is plotted in Figure 2 for several values of the pipe diameter. Changing the pipe diameter clearly shifts the resonance frequency but also changes the damping and Q value, that is, the width of the peak. Hence this is an important factor when designing organ pipes.

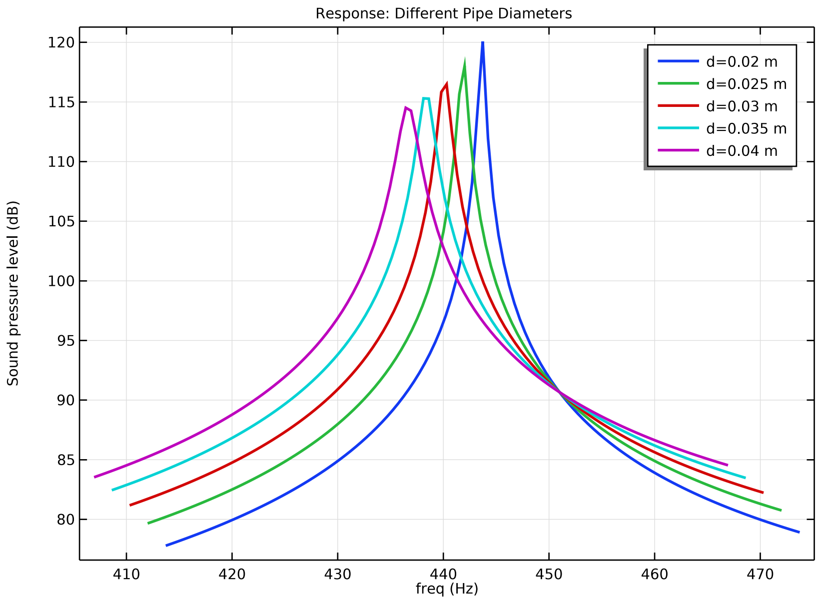

Figure 2: Resonance peak of the fundamental frequency at 440 Hz for different inner pipe diameters.

The response for different values of the pipe wall width is plotted in Figure 3. Here it is also seen that changing the pipe wall width (in general any of the pipe wall properties) changes the resonance slightly. This is because the elastic properties of the pipe wall have influence on the effective compressibility of the system in a given cross section. This in turn changes the effective speed of sound in the pipe and thus the resonance. The value of Young's modulus in this example model was chosen to demonstrate this effect. A typical circular organ pipe is however made of a tin-lead alloy, which has a much higher value of Young's modulus.

In the final plot in Figure 4, the parameter values giving a fundamental resonance at 440 Hz are selected (see the parameters list) and the response is plotted for frequencies from 100 Hz to 3000 Hz. The plot shows the fundamental resonance at 440 Hz as well as the following five resonance frequencies of the organ pipe. The shape of this curve is related to the timbre of the pipe.

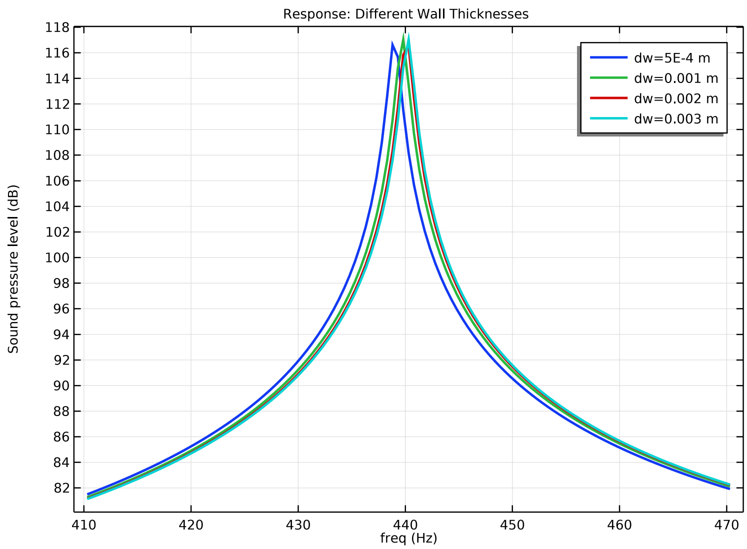

Figure 3: Resonance peak of the fundamental frequency at 440 Hz for different pipe wall thickness.

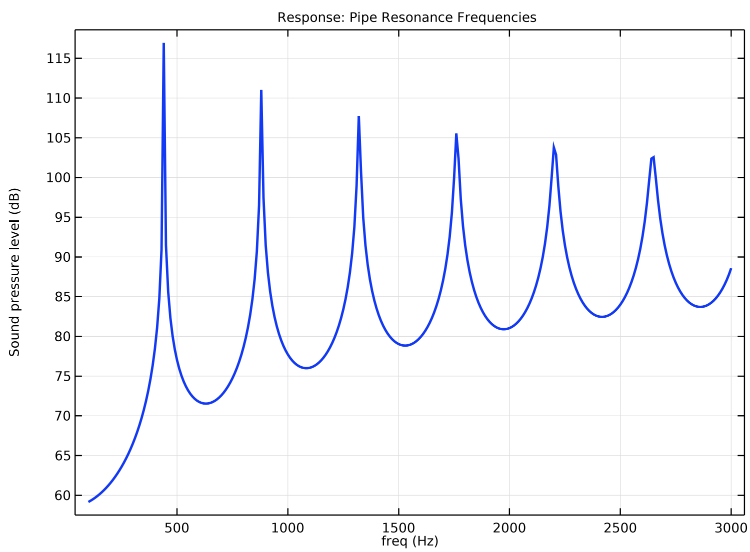

Figure 4: Resonance peaks of the six first natural frequencies of the pipe from 100 to 3000 Hz.

Application Library path: `Pipe_Flow_Module/Pipe_Acoustics/organ_pipe_design`

Modeling Instructions

From the **File** menu, choose **New**.

NEW

In the **New** window, click **Model Wizard**.

MODEL WIZARD

1 In the **Model Wizard** window, click **3D**.

2 In the **Select Physics** tree, select **Acoustics>Pipe Acoustics>Pipe Acoustics, Frequency Domain (pafd)**.

3 Click **Add**.

4 Click **Study**.

5 In the **Select Study** tree, select **General Studies>Frequency Domain**.

6 Click **Done**.

GLOBAL DEFINITIONS

Parameters 1

1 In the **Model Builder** window, under **Global Definitions** click **Parameters 1**.

2 In the **Settings** window for **Parameters**, locate the **Parameters** section.

3 Click **Load from File**.

4 Browse to the model's Application Libraries folder and double-click the file `organ_pipe_design_parameters.txt`.

GEOMETRY 1

Polygon 1 (pol1)

1 In the **Geometry** toolbar, click **More Primitives** and choose **Polygon**.

2 In the **Settings** window for **Polygon**, locate the **Coordinates** section.

3 In the table, enter the following settings:

x (m)	y (m)	z (m)
0	0	0
0	0	L

4 Click **Build All Objects**.

ADD MATERIAL

1 In the **Home** toolbar, click **Add Material** to open the **Add Material** window.

2 Go to the **Add Material** window.

3 In the tree, select **Built-in>Air**.

4 Click **Add to Component** in the window toolbar.

5 In the **Home** toolbar, click **Add Material** to close the **Add Material** window.

PIPE ACOUSTICS, FREQUENCY DOMAIN (PAFD)

Set the cross-section shape of the organ pipe and the elastic properties of the pipe walls.

Fluid Properties 1

1 In the **Model Builder** window, under **Component 1 (comp1)>Pipe Acoustics, Frequency Domain (pafd)** click **Fluid Properties 1**.

2 In the **Settings** window for **Fluid Properties**, locate the **Model Input** section.

3 In the T_0 text field, type `T0`.

4 In the p_0 text field, type `p0`.

Pipe Properties 1

1 In the **Model Builder** window, click **Pipe Properties 1**.

2 In the **Settings** window for **Pipe Properties**, locate the **Pipe Shape** section.

3 From the list, choose **Circular**.

4 In the d_i text field, type `d`.

5 Locate the **Pipe Model** section. From the **Pipe model** list, choose **Anchored at one end**.

6 From the E list, choose **User defined**. In the associated text field, type `Ew`.

7 From the ν list, choose **User defined**. In the associated text field, type `nuw`.

8 In the Δw text field, type `dw`.

End Impedance 1

At the open end, the organ pipe is sitting in free air. Use the **Unflanged pipe, circular** end impedance to get the correct acoustic behavior here. Note that a low ka limit version also exists but this one is only valid for $k \cdot a \leq \ \leq 1$. In this model $k \cdot a \geq 0.12$ as seen in the parameters list.

1 In the **Physics** toolbar, click **Points** and choose **End Impedance**.

2 Select Point 2 only.

3 In the **Settings** window for **End Impedance**, locate the **End Impedance** section.

4 From the **Impedance model** list, choose **Unflanged pipe, circular**.

Pressure 1

1 In the **Physics** toolbar, click **Points** and choose **Pressure**.

2 Select Point 1 only.

3 In the **Settings** window for **Pressure**, locate the **Pressure** section.

4 In the p_{in} text field, type `1`.

MESH 1

In this model, the mesh is set up manually. Proceed by directly adding the desired mesh component. In general, 5 to 6 second-order elements per wavelength are needed to resolve the waves. For more details, see *Meshing (Resolving the Waves)* in the *Acoustics Module User's Guide*. Here, we use 20 elements per wavelength at the maximum frequency.

Edge 1

1 In the **Mesh** toolbar, click **Boundary** and choose **Edge**.

2 Select Edge 1 only.

Size

1 In the **Model Builder** window, click **Size**.

2 In the **Settings** window for **Size**, locate the **Element Size** section.

3 Click the **Custom** button.

4 Locate the **Element Size Parameters** section. In the **Maximum element size** text field, type `h_min`.

5 In the **Minimum element size** text field, type `h_min/2`.

6 Click **Build All**.

STUDY 1

Step 1: Frequency Domain

1 In the **Model Builder** window, under **Study 1** click **Step 1: Frequency Domain**.

2 In the **Settings** window for **Frequency Domain**, locate the **Study Settings** section.

3 In the **Frequencies** text field, type `range(f_est-30,0.5,f_est+30)`.

Here the variable `f_est` corresponds to the estimated resonance frequency of the pipe taking the end correction into account. For narrow pipes the calculated resonance `f_est` is close to the ideal A4 (or a') note of $f = 440$ Hz.

Parametric Sweep

The sweep over the pipe diameter is done using a Parametric Sweep.

1 In the **Study** toolbar, click **Parametric Sweep**.

2 In the **Settings** window for **Parametric Sweep**, locate the **Study Settings** section.

3 Click **Add**.

4 In the table, enter the following settings:

Parameter name	Parameter value list	Parameter unit
d (Inner pipe diameter)	2[cm] 2.5[cm] 3[cm] 3.5[cm] 4[cm]	m

5 In the **Model Builder** window, click **Study 1**.

6 In the **Settings** window for **Study**, type `Study 1 - Inner Pipe Diameter Sweep` in the **Label** text field.

7 In the **Study** toolbar, click **Compute**.

ADD STUDY

1 In the **Study** toolbar, click **Add Study** to open the **Add Study** window.

2 Go to the **Add Study** window.

3 Find the **Studies** subsection. In the **Select Study** tree, select **General Studies> Frequency Domain**.

4 Click **Add Study** in the window toolbar.

5 In the **Study** toolbar, click **Add Study** to close the **Add Study** window.

STUDY 2

Step 1: Frequency Domain

1 In the **Settings** window for **Frequency Domain**, locate the **Study Settings** section.

2 In the **Frequencies** text field, type `range(f_est-30,0.5,f_est+30)`.

3 Click to expand the **Study Extensions** section. Select the **Auxiliary sweep** check box.

4 Click + **Add**.

5 In the table, enter the following settings:

Parameter name	Parameter value list	Parameter unit
dw (Pipe wall thickness)	0.5[mm] 1[mm] 2[mm] 3[mm]	m

6 In the **Model Builder** window, click **Study 2**.

7 In the **Settings** window for **Study**, type `Study 2 - Pipe Wall Thickness Sweep` in the **Label** text field.

8 Locate the **Study Settings** section. Clear the **Generate default plots** check box.

9 In the **Study** toolbar, click **Compute**.

ADD STUDY

1 In the **Study** toolbar, click **Add Study** to open the **Add Study** window.

2 Go to the **Add Study** window.

3 Find the **Studies** subsection. In the **Select Study** tree, select **General Studies> Frequency Domain**.

4 Click **Add Study** in the window toolbar.

5 In the **Study** toolbar, click **Add Study** to close the **Add Study** window.

STUDY 3

Step 1: Frequency Domain

1 In the **Settings** window for **Frequency Domain**, locate the **Study Settings** section.

2 In the **Frequencies** text field, type `range(100,10,3000)`.

3 In the **Model Builder** window, click **Study 3**.

4 In the **Settings** window for **Study**, type `Study 3 - Extended Frequency Sweep` in the **Label** text field.

5 Locate the **Study Settings** section. Clear the **Generate default plots** check box.

6 In the **Study** toolbar, click **Compute**.

RESULTS

Acoustic Pressure (pafd)

The first two figures show the pressure distribution and the velocity field in the pipe section as line plots in 3D. You can select different parameter values and frequencies and evaluate the plots to study the pressure distribution in the pipe.

Next, create three plots that show the frequency response of the organ pipe. This is here the sound pressure level evaluated at the open end of the of the pipe for three different cases. The first shows the fundamental resonance and how it depends on the inner tube diameter. The next shows the dependency on the tube wall thickness. The final plot shows the solution for a larger frequency range including the six first resonance frequencies.

Response: Different Pipe Diameters

1 In the **Home** toolbar, click **Add Plot Group** and choose **1D Plot Group**.
2 In the **Settings** window for **1D Plot Group**, locate the **Data** section.
3 From the **Dataset** list, choose **Study 1 - Inner Pipe Diameter Sweep/ Parametric Solutions 1 (sol2)**.
4 In the **Label** text field, type `Response: Different Pipe Diameters`.
5 Click to expand the **Title** section. From the **Title type** list, choose **Label**.

Point Graph 1

1 Right-click **Response: Different Pipe Diameters** and choose **Point Graph**.
2 Select Point 2 only.
3 In the **Settings** window for **Point Graph**, click **Replace Expression** in the upper-right corner of the **y-Axis Data** section. From the menu, choose **Component 1 (comp1)> Pipe Acoustics, Frequency Domain>Intensity and sound pressure level>pafd.Lp - Sound pressure level - dB**.
4 Click to expand the **Coloring and Style** section. In the **Width** text field, type `2`.
5 Click to expand the **Legends** section. Select the **Show legends** check box.
6 Find the **Include** subsection. Clear the **Point** check box.
7 In the **Response: Different Pipe Diameters** toolbar, click **Plot**.

   The figure should look like the one in Figure 2.

Response: Different Wall Thicknesses

1 In the **Home** toolbar, click **Add Plot Group** and choose **1D Plot Group**.
2 In the **Settings** window for **1D Plot Group**, locate the **Data** section.
3 From the **Dataset** list, choose **Study 2 - Pipe Wall Thickness Sweep/Solution 8 (sol8)**.

4 In the **Label** text field, type `Response: Different Wall Thicknesses`.

5 Locate the **Title** section. From the **Title type** list, choose **Label**.

Point Graph 1

1 Right-click **Response: Different Wall Thicknesses** and choose **Point Graph**.

2 Select Point 2 only.

3 In the **Settings** window for **Point Graph**, locate the **y-Axis Data** section.

4 In the **Expression** text field, type `pafd.Lp`.

5 Locate the **x-Axis Data** section. From the **Axis source data** list, choose **freq**.

6 Locate the **Coloring and Style** section. In the **Width** text field, type `2`.

7 Locate the **Legends** section. Select the **Show legends** check box.

8 Find the **Include** subsection. Clear the **Point** check box.

9 In the **Response: Different Wall Thicknesses** toolbar, click **Plot**.

The figure should look like the one in Figure 3.

Response: Pipe Resonance Frequencies

1 In the **Home** toolbar, click **Add Plot Group** and choose **1D Plot Group**.

2 In the **Settings** window for **1D Plot Group**, locate the **Data** section.

3 From the **Dataset** list, choose **Study 3 - Extended Frequency Sweep/Solution 9 (sol9)**.

4 In the **Label** text field, type `Response: Pipe Resonance Frequencies`.

5 Locate the **Title** section. From the **Title type** list, choose **Label**.

Point Graph 1

1 Right-click **Response: Pipe Resonance Frequencies** and choose **Point Graph**.

2 Select Point 2 only.

3 In the **Settings** window for **Point Graph**, locate the **y-Axis Data** section.

4 In the **Expression** text field, type `pafd.Lp`.

5 Locate the **Coloring and Style** section. In the **Width** text field, type `2`.

6 In the **Response: Pipe Resonance Frequencies** toolbar, click **Plot**.

The figure should look like the one in Figure 4.