Created in COMSOL Multiphysics 6.0

Relativistic Diverging Electron Beam

Introduction

When modeling the propagation of charged particle beams at high currents and relativistic speeds, the space charge and beam current create significant electric and magnetic forces that tend to expand and focus the beam, respectively. The Charged Particle Tracing interface can use an iterative procedure to efficiently compute the strongly coupled particle trajectories and electric and magnetic fields for a beam operating at constant current. To validate the model, the change in beam radius from the waist position is compared to an analytic expression for the shape of a relativistic beam envelope.

Note: This application requires the AC/DC Module and Particle Tracing Module.

Model Definition

This model is almost identical to the Electron Beam Divergence Due to Self Potential model but with higher beam current and particle velocities. To accurately compute the relativistic particle trajectories, a correction has to be applied to the mass of the electrons,

$$m_e = \frac{m_r}{\sqrt{1 - \frac{v^2}{c^2}}} \tag{1}$$

where

- $m_r = 9.10938356 \times 10^{-31}$ kg is the rest mass of the electron,
- $c = 2.99792458 \times 10^8$ m/s is the speed of light in a vacuum, and
- v (SI unit: m/s) is the magnitude of the electron velocity.

At relativistic speeds, the electron beam generates a magnetic field that exerts a significant magnetic force on the electrons. The ratio of self-induced magnetic and electric forces is proportional to $\beta^2 = (v/c)^2$ (Ref. 1).

As in the nonrelativistic case, the shape of the beam envelope has the analytic solution

$$z = \frac{R_0 F(\chi)}{\sqrt{2K}} \tag{2}$$

where z (SI unit: m) is the distance from the beam waist, R_0 (SI unit: m) is the waist radius, K (dimensionless) is the generalized beam perveance,

$$K = \frac{eI_0}{2\pi\varepsilon_0 m_e (v_z\gamma)^3}$$

γ (dimensionless) is the relativistic factor defined as

$$\gamma = \frac{1}{\sqrt{1-\frac{v^2}{c^2}}} \tag{3}$$

χ (dimensionless) is the ratio of the beam radius to the beam waist radius, and

$$F(\chi) = \int_1^{\chi} \frac{dy}{\sqrt{\ln(y)}} \tag{4}$$

This analytical expression for the relationship between axial position and beam envelope radius is used to determine the accuracy of the solution.

Results and Discussion

The electron trajectories are plotted in Figure 1 while the electric potential distribution and magnetic flux norm are respectively shown on Figure 2, and Figure 3. The distance from the beam waist as a function of beam radius is compared to the result of Equation 2 using a **Global Evaluation**. The results agree to within a few percentage points. The sources of numerical error include discretization error of the charge density and current density, both of which use constant shape functions within each mesh element.

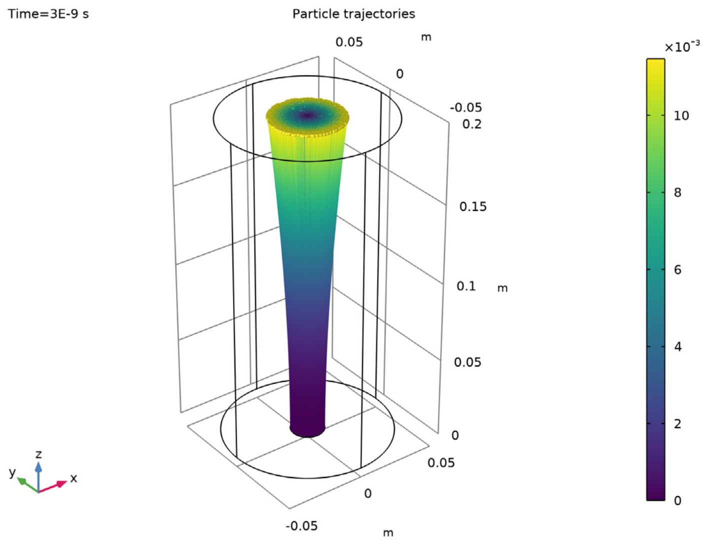

Figure 1: A beam of electrons with a waist located at z = 0 diverges due to transverse beam forces. The color represents the radial displacement of each electron from its initial position.

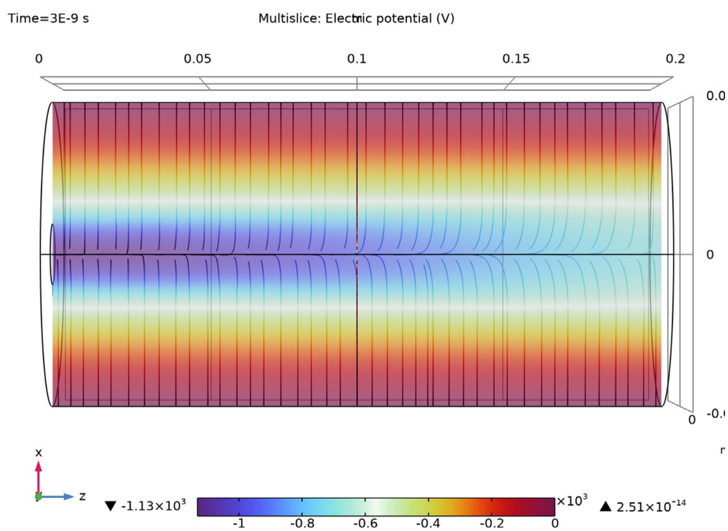

Figure 2: Electric potential in the relativistic beam. The magnitude of the potential is greatest at the beam waist.

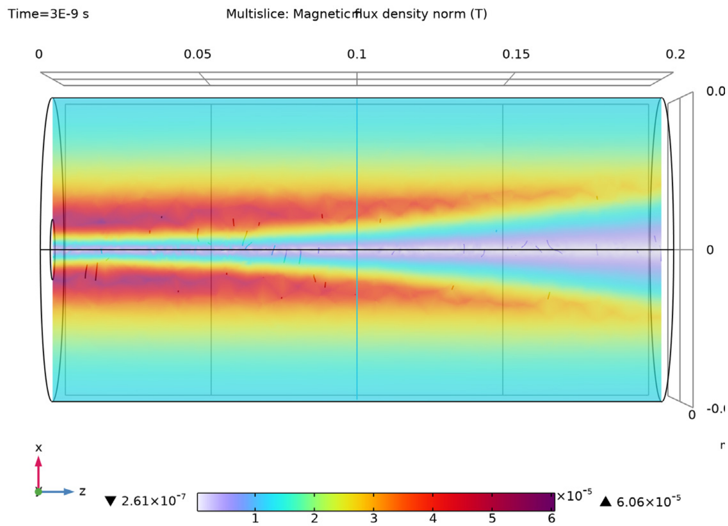

Figure 3: Magnetic flux density norm in the beam.

Reference

1. S. Humphries, *Charged Particle Beams*, Dover Publications, New York, 2013.

Application Library path: `Particle_Tracing_Module/Charged_Particle_Tracing/electron_beam_divergence_relativistic`

Model Instructions

From the **File** menu, choose **New**.

NEW

In the **New** window, click **Model Wizard**.

MODEL WIZARD

1 In the **Model Wizard** window, click **3D**.

2 In the **Select Physics** tree, select **AC/DC>Particle Tracing>Particle Field Interaction, Relativistic**.

3 Click **Add**.

4 Click **Study**.

5 In the **Select Study** tree, select **Preset Studies for Selected Physics Interfaces> Charged Particle Tracing>Bidirectionally Coupled Particle Tracing**.

6 Click **Done**.

GLOBAL DEFINITIONS

To save time, the parameters can be loaded from a file.

Parameters 1

1 In the **Model Builder** window, under **Global Definitions** click **Parameters 1**.

2 In the **Settings** window for **Parameters**, locate the **Parameters** section.

3 Click **Load from File**.

4 Browse to the model's Application Libraries folder and double-click the file `electron_beam_divergence_relativistic_parameters.txt`.

GEOMETRY 1

Cylinder 1 (cyl1)

1 In the **Geometry** toolbar, click **Cylinder**.

2 In the **Settings** window for **Cylinder**, locate the **Size and Shape** section.

3 In the **Radius** text field, type `r0`.

4 In the **Height** text field, type `L`.

5 Click **Build Selected**.

Work Plane 1 (wp1)

1 In the **Geometry** toolbar, click **Work Plane**.

2 In the **Settings** window for **Work Plane**, locate the **Plane Definition** section.

3 From the **Plane type** list, choose **Face parallel**.

4 On the object **cyl1**, select Boundary 3 only.

It might be easier to select the correct boundary by using the **Selection List** window. To open this window, in the **Home** toolbar click **Windows** and choose **Selection List**. (If you are running the cross-platform desktop, you find **Windows** in the main menu.)

5 Click **Show Work Plane**.

Work Plane 1 (wp1)>Circle 1 (c1)

1 In the **Work Plane** toolbar, click **Circle**.

2 In the **Settings** window for **Circle**, locate the **Size and Shape** section.

3 In the **Radius** text field, type `r0beam`.

4 Click **Build Selected**.

MATERIALS

Material 1 (mat1)

1 In the **Model Builder** window, under **Component 1 (comp1)** right-click **Materials** and choose **Blank Material**.

2 In the **Settings** window for **Material**, locate the **Material Contents** section.

3 In the table, enter the following settings:

Property	Variable	Value	Unit	Property group
Relative permittivity	epsilonr_iso ; epsilonrii = epsilonr_iso, epsilonrij = 0	1	1	Basic
Relative permeability	mur_iso ; murii = mur_iso, murij = 0	1	1	Basic
Electrical conductivity	sigma_iso ; sigmaii = sigma_iso, sigmaij = 0	0	S/m	Basic

DEFINITIONS

Variables 1

1 In the **Home** toolbar, click **Variables** and choose **Local Variables**.

2 In the **Settings** window for **Variables**, locate the **Variables** section.

3 In the table, enter the following settings:

Name	Expression	Unit	Description
`qr`	`sqrt(qx^2+qy^2)`	m	`Radial distance from beam axis`
`qrmax`	`cpt.cptmaxop1(qr)`	m	`Beam radius`

Name	Expression	Unit	Description
z_avg	cpt.cptaveop1(qz)	m	Average z-coordinate
chi	qrmax/at(0,qrmax)		Ratio of beam radius to waist radius

ELECTROSTATICS (ES)

Ground 1

1 In the **Model Builder** window, under **Component 1 (comp1)** right-click **Electrostatics (es)** and choose **Ground**.

2 Select Boundaries 1, 2, 6, and 7 only.

CHARGED PARTICLE TRACING (CPT)

Particle Properties 1

1 In the **Model Builder** window, under **Component 1 (comp1)>Charged Particle Tracing (cpt)** click **Particle Properties 1**.

2 In the **Settings** window for **Particle Properties**, locate the **Particle Species** section.

3 From the **Particle species** list, choose **Electron**.

Inlet 1

1 In the **Physics** toolbar, click **Boundaries** and choose **Inlet**.

2 Select Boundary 5 only.

3 In the **Settings** window for **Inlet**, locate the **Release Current Magnitude** section.

4 In the I text field, type `Ibeam`.

5 Locate the **Initial Position** section. From the **Initial position** list, choose **Density**.

6 In the N text field, type `1000`.

7 Locate the **Initial Velocity** section. Specify the $\mathbf{v}_0$ vector as

0	x
0	y
v0beam	z

Electric Force 1

1 In the **Model Builder** window, click **Electric Force 1**.

2 In the **Settings** window for **Electric Force**, locate the **Electric Force** section.

3 From the $\mathbf{E}$ list, choose **Electric field (es/ccn1)**.

4 Locate the **Advanced Settings** section. Select the **Use piecewise polynomial recovery on field** check box.

Magnetic Force 1

1 In the **Model Builder** window, click **Magnetic Force 1**.

2 In the **Settings** window for **Magnetic Force**, locate the **Magnetic Force** section.

3 From the $\mathbf{B}$ list, choose **Magnetic flux density (mf/al1)**.

4 Locate the **Advanced Settings** section. Select the **Use piecewise polynomial recovery on field** check box.

MESH 1

1 In the **Model Builder** window, under **Component 1 (comp1)** click **Mesh 1**.

2 In the **Settings** window for **Mesh**, locate the **Mesh Settings** section.

3 From the **Sequence type** list, choose **User-controlled mesh**.

Size

1 In the **Model Builder** window, under **Component 1 (comp1)>Mesh 1** click **Size**.

2 In the **Settings** window for **Size**, click to expand the **Element Size Parameters** section.

3 In the **Maximum element size** text field, type `hmax`.

4 Click **Build All**.

STUDY 1

Step 1: Bidirectionally Coupled Particle Tracing

1 In the **Model Builder** window, under **Study 1** click **Step 1: Bidirectionally Coupled Particle Tracing**.

2 In the **Settings** window for **Bidirectionally Coupled Particle Tracing**, locate the **Study Settings** section.

3 In the **Output times** text field, type `range(0,1.0e-10,3e-9)`.

4 From the **Tolerance** list, choose **User controlled**.

5 In the **Relative tolerance** text field, type `1.0E-5`.

6 Locate the **Iterations** section. From the **Termination method** list, choose **Convergence of global variable**.

7 In the **Global variable** text field, type `qrmax`.

8 In the **Relative tolerance** text field, type `1E-5`.

9 In the **Relative tolerance threshold** text field, type `0.015`.

10 In the **Maximum number of iterations** text field, type 8.

Solution 1 (sol1)

1. In the **Study** toolbar, click **Show Default Solver**.
2. In the **Model Builder** window, expand the **Solution 1 (sol1)** node, then click **Compile Equations: Bidirectionally Coupled Particle Tracing (2)**.
3. In the **Settings** window for **Compile Equations**, locate the **Study and Step** section.
4. Select the **Split complex variables in real and imaginary parts** check box.
5. In the **Study** toolbar, click **Compute**.

RESULTS

Plot the trajectories of the electrons, using a **Color Expression** to observe their radial displacement over time.

Particle Trajectories 1

1. In the **Model Builder** window, expand the **Results>Particle Trajectories (cpt)** node, then click **Particle Trajectories 1**.
2. In the **Settings** window for **Particle Trajectories**, locate the **Coloring and Style** section.
3. Find the **Line style** subsection. From the **Type** list, choose **Line**.

Color Expression 1

1. In the **Model Builder** window, expand the **Particle Trajectories 1** node, then click **Color Expression 1**.
2. In the **Settings** window for **Color Expression**, locate the **Expression** section.
3. In the **Expression** text field, type `qr-at(0,qr)`.
4. In the **Particle Trajectories (cpt)** toolbar, click **Plot**.
5. Click the **Zoom Extents** button in the **Graphics** toolbar. This plot should look like Figure 1.

Electric Potential (es)

1. In the **Model Builder** window, click **Electric Potential (es)**.
2. In the **Settings** window for **3D Plot Group**, locate the **Color Legend** section.
3. From the **Position** list, choose **Bottom**.
4. Click the **Go to ZX View** button in the **Graphics** toolbar. This plot should look like Figure 2.

Magnetic Flux Density Norm (mf)

1. In the **Model Builder** window, click **Magnetic Flux Density Norm (mf)**.

2 In the **Settings** window for **3D Plot Group**, locate the **Color Legend** section.

3 From the **Position** list, choose **Bottom**. This plot should look like Figure 3.

Global Evaluation 1

1 In the **Results** toolbar, click 8.5 **Global Evaluation**.

2 In the **Settings** window for **Global Evaluation**, locate the **Data** section.

3 From the **Time selection** list, choose **Last**.

4 Locate the **Expressions** section. In the table, enter the following settings:

Expression	Unit	Description
`r0beam/sqrt(2*K)*integrate(1/sqrt(log(s)),s,1+eps,chi)`	`m`	`Expected z-coordinate for beam width`
`z_avg`	`m`	`Average z-coordinate`

5 Click **Evaluate**.