

Optimization of a Tesla Microvalve with Transient Flow

A Tesla valve inhibits backward flow on a fixed geometry by utilizing friction forces instead of moving parts, Ref. 1. This means fluid can flow freely in one direction but not in the reverse direction. Note that the objective function is different from Optimization of a Tesla Microvalve, because the objective is defined in terms of the flow rate with a oscillating pressure difference (instead of the other way around with a objective defined in terms of the pressure drop and oscillating flow rate).

Model Definition

The model solves the transient Navier–Stokes equations with an oscillating pressure drop. The Reynolds number is 500 in this example. A measure of the effectiveness of the design is the average flow rate in the x direction during the period T_{period} :

$$U_{\text{average}} = \iint_{T\Omega} \mathbf{u} \cdot \hat{\mathbf{x}} d\Omega dt / T_{\text{period}}$$

where T is the simulation time taken as two periods of pressure oscillation. The model uses an ODE to perform the time integration (see Figure 1):

$$\frac{d\Phi}{dt}t_{\text{max}} = f_{\text{step}}U_{\text{average}}/U_{\text{scale}}$$

where $t_{\text{max}} = 1.75 T_{\text{period}}$ and f_{step} is defined so that integration occurs over the last period:

$$f_{\text{step}} = \begin{cases} 0, t < t_{\text{max}} - T_{\text{period}} \\ 1, & \text{otherwise} \end{cases}$$

The fluid flow is described by the incompressible Navier-Stokes equations:

$$\rho \left[(\mathbf{u} \cdot \nabla) \mathbf{u} + \frac{d\mathbf{u}}{dt} \right] = -\nabla p + \nabla \cdot \eta (\nabla \mathbf{u} + (\nabla \mathbf{u})^T) - \alpha(\theta_f) \mathbf{u}$$
$$\nabla \cdot \mathbf{u} = 0$$

where the coefficient $\alpha(\theta)$ depends on the distribution of material which impedes the flow within the device. In this example, $\alpha(\theta)$ is given by

$$\begin{split} \alpha(\theta_p) &= \alpha_0 \theta_p, \quad \theta_p &= \frac{\alpha_0(q+\theta)}{q+\theta} \\ \theta &= \frac{(\tanh(\beta(\theta_f - \theta_\beta)) + \tanh(\beta\theta_\beta))}{(\tanh(\beta(1 - \theta_\beta)) + \tanh(\beta\theta_\beta))} \\ \theta_f &= R_{\min}^2 \nabla^2 \theta_f + \theta_c \end{split}$$

where θ_c and θ_f are the control- and filtered material volume factors. To avoid the effect of grayscale, the filtered field is projected to construct the material volume factor, θ , which is related to the damping term using a convex function, see Ref. 3..

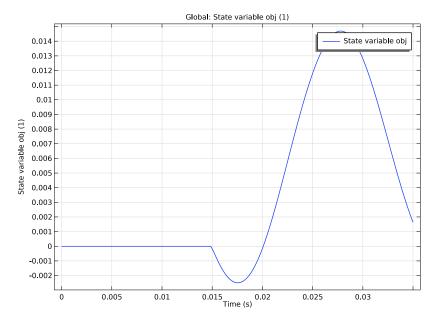


Figure 1: The ODE variable (used to perform the time integration) is plotted for the optimized design.

Results and Discussion

As expected, design has features similar to that seen in Optimization of a Tesla Microvalve, but the central obstacle is placed further to the right, as seen in Figure 2 and Figure 3. A verification simulation is perform using a body fitted mesh, see Figure 4 and Figure 5. The analysis can be used to ensure that the optimization does not rely on unphysical effects due to the approximate wall descriptions of the optimization.

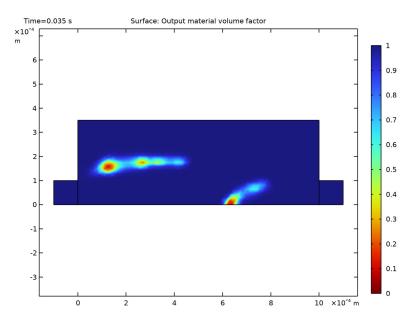


Figure 2: The filtered material volume factor is plotted for the optimized design.

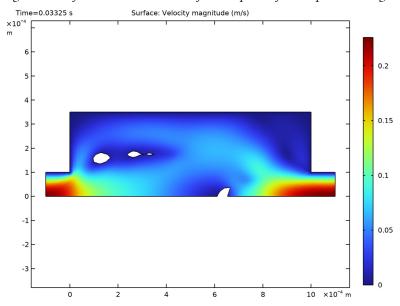


Figure 3: The flow velocity is plotted, while the pressure drop is driving the fluid backwards.

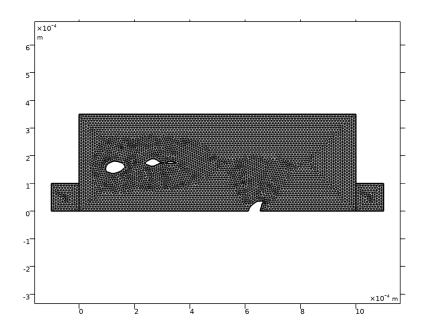


Figure 4: The body fitted mesh used for the verification allows for no-slip boundary conditions.

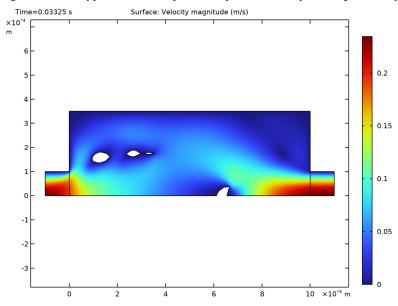


Figure 5: The flow velocity is plotted, while the pressure drop is driving the fluid backwards.

A quantitative comparison with the optimization results reveals that objective is slightly worse for the verification, as opposed to the behavior in Optimization of a Tesla Microvalve.

Notes About the COMSOL Implementation

This example uses an objective function defined as a time integral, but the evaluation of transient objectives is always performed at the last time step. Therefore the time integration is implemented as an ODE.

A fixed time step is combined with a segregated solver to ensure robustness of the adjoint problem.

References

- 1. S. Lin, "Topology Optimization of Micro Tesla Valve in low and moderate Reynolds number," Chinese Academy of Sciences, China, September 27, 2011 http:// senlin.weebly.com/uploads/6/6/1/4/6614199/ sen_lin_topology_optimization_of_micro_tesla_valve.pdf.
- 2. L. Højgaard Olesen, F. Okkels, and H. Bruus, "A high-level programming-language implementation of topology optimization applied to steady-state Navier-Stokes flow," Int. J. Numer. Meth. Engng, vol. 65, pp. 975-1001, 2006.
- 3. T. Borrvall and J. Petersson, "Topology optimization of fluids in Stokes flow," Int. J. Numer. Meth. Fluid, vol. 41, pp. 77-107, 2003.

Notes About the COMSOL Implementation

The model uses an ODE for the time integration, and the original MMA optimization solver (which is not globally convergent) is used. Finally, fixed time steps and a segregated solver is used to ensure robustness of the adjoint problem.

Application Library path: Optimization Module/Topology Optimization/ tesla_microvalve_transient_optimization

From the File menu, choose New.

NEW

In the New window, click Model Wizard.

MODEL WIZARD

- I In the Model Wizard window, click **2** 2D.
- 2 In the Select Physics tree, select Fluid Flow>Single-Phase Flow>Laminar Flow (spf).
- 3 Click Add.
- 4 In the Select Physics tree, select Mathematics>ODE and DAE Interfaces> Global ODEs and DAEs (ge).
- 5 Click Add.
- 6 Click Study.
- 7 In the Select Study tree, select General Studies>Time Dependent.
- 8 Click M Done.

GLOBAL DEFINITIONS

Parameters 1

- I In the Model Builder window, under Global Definitions click Parameters I.
- 2 In the Settings window for Parameters, locate the Parameters section.
- 3 Click Load from File.
- **4** Browse to the model's Application Libraries folder and double-click the file tesla_microvalve_transient_optimization_parameters.txt.

GEOMETRY I

Rectangle I (rI)

- I In the Geometry toolbar, click Rectangle.
- 2 In the Settings window for Rectangle, locate the Size and Shape section.
- 3 In the Width text field, type L.
- 4 In the **Height** text field, type H.
- 5 Locate the Selections of Resulting Entities section. Select the Resulting objects selection check box.

Square I (sql)

- I In the Geometry toolbar, click Square.
- 2 In the Settings window for Square, locate the Size section.
- 3 In the Side length text field, type L1.
- 4 Locate the **Position** section. In the **x** text field, type -L1.
- 5 Locate the Selections of Resulting Entities section. Select the Resulting objects selection check box.

Move I (movI)

- I In the Geometry toolbar, click Transforms and choose Move.
- 2 In the Settings window for Move, locate the Input section.
- 3 From the Input objects list, choose Square 1.
- 4 Select the **Keep input objects** check box.
- **5** Locate the **Displacement** section. In the **x** text field, type L+L1.

Symmetry

- I In the Geometry toolbar, click \(\frac{1}{2} \) Selections and choose Box Selection.
- 2 In the Settings window for Box Selection, locate the Geometric Entity Level section.
- 3 From the Level list, choose Boundary.
- 4 In the Label text field, type Symmetry.
- 5 Locate the Box Limits section. In the y maximum text field, type eps.
- 6 Locate the Output Entities section. From the Include entity if list, choose Entity inside box.

Outlet

- I Right-click Symmetry and choose Duplicate.
- 2 In the Settings window for Box Selection, type Outlet in the Label text field.
- 3 Locate the Box Limits section. In the x maximum text field, type -L1*0.999.
- 4 In the y maximum text field, type Inf.

Inlet

- I Right-click **Outlet** and choose **Duplicate**.
- 2 In the Settings window for Box Selection, type Inlet in the Label text field.
- 3 Locate the **Box Limits** section. In the x minimum text field, type L+L1*0.999.
- 4 In the x maximum text field, type Inf.

Inlet/Outlet

- I In the Geometry toolbar, click \(\frac{1}{2} \) Selections and choose Union Selection.
- 2 In the Settings window for Union Selection, type Inlet/Outlet in the Label text field.
- 3 Locate the Geometric Entity Level section. From the Level list, choose Boundary.
- **4** Locate the **Input Entities** section. Click + **Add**.
- 5 In the Add dialog box, in the Selections to add list, choose Outlet and Inlet.
- 6 Click OK.

Pressure Reference Point

- I In the Geometry toolbar, click \(\frac{1}{2} \) Selections and choose Disk Selection.
- 2 In the Settings window for Disk Selection, type Pressure Reference Point in the Label text field.
- 3 Locate the Geometric Entity Level section. From the Level list, choose Point.
- 4 Locate the Size and Shape section. In the Outer radius text field, type L/1000.

MATERIALS

Water

- I In the Model Builder window, under Component I (compl) right-click Materials and choose Blank Material.
- 2 In the Settings window for Material, type Water in the Label text field.
- **3** Locate the **Material Contents** section. In the table, enter the following settings:

Property	Variable	Value	Unit	Property group
Density	rho	rho0	kg/m³	Basic
Dynamic viscosity	mu	mu0	Pa·s	Basic

COMPONENT I (COMPI)

In the Definitions toolbar, click ? Optimization and choose Topology Optimization> Density Model.

TOPOLOGY OPTIMIZATION

Density Model I (dtopol)

- I In the Settings window for Density Model, locate the Geometric Entity Selection section.
- 2 From the Selection list, choose Rectangle 1.
- 3 Locate the Filtering section. From the R_{\min} list, choose User defined.

- 4 In the text field, type meshsz.
- **5** Locate the **Projection** section. From the **Projection type** list, choose Hyperbolic tangent projection.
- **6** In the β text field, type beta.
- 7 Locate the Interpolation section. From the Interpolation type list, choose Darcy.
- **8** In the q_{Darcy} text field, type q.
- 9 Locate the Control Variable Discretization section. From the Element order list, choose Constant.
- **10** Locate the **Control Variable Initial Value** section. In the θ_0 text field, type 1.

Prescribed Material I

- I In the Definitions toolbar, click ?? Optimization and choose Topology Optimization> Prescribed Material.
- 2 In the Settings window for Prescribed Material, locate the Geometric Entity Selection section.
- 3 From the Selection list, choose Square 1.

DEFINITIONS

Velocity Average

- I In the Definitions toolbar, click Probes and choose Domain Probe.
- 2 In the Settings window for Domain Probe, type Velocity Average in the Label text field.
- 3 In the Variable name text field, type uAvg.
- 4 Locate the Source Selection section. From the Selection list, choose Rectangle 1.
- **5** Locate the **Probe Type** section. From the **Type** list, choose **Integral**.
- **6** Locate the **Expression** section. In the **Expression** text field, type u/L/H.

LAMINAR FLOW (SPF)

Symmetry I

- I In the Model Builder window, under Component I (compl) right-click Laminar Flow (spf) and choose Symmetry.
- 2 In the Settings window for Symmetry, locate the Boundary Selection section.
- **3** From the **Selection** list, choose **Symmetry**.

Periodic Flow Condition I

- In the Physics toolbar, click Boundaries and choose Periodic Flow Condition.
- 2 In the Settings window for Periodic Flow Condition, locate the Boundary Selection section.
- 3 From the Selection list, choose Inlet/Outlet.
- **4** Locate the **Flow Condition** section. In the Δp text field, type dp*cos(2*pi*t/Tperiod).

Pressure Point Constraint I

- I In the Physics toolbar, click Points and choose Pressure Point Constraint.
- 2 In the Settings window for Pressure Point Constraint, locate the Point Selection section.
- 3 From the Selection list, choose Pressure Reference Point.

Volume Force 1

- I In the Physics toolbar, click **Domains** and choose **Volume Force**.
- 2 In the Settings window for Volume Force, locate the Domain Selection section.
- 3 From the Selection list, choose Rectangle 1.
- **4** Locate the **Volume Force** section. Specify the \mathbf{F} vector as

-alpha*dtopo1.theta_p*u	x
-alpha*dtopo1.theta_p*v	у

GLOBAL ODES AND DAES (GE)

Global Equations 1

- I In the Model Builder window, under Component I (compl) > Global ODEs and DAEs (ge) click Global Equations 1.
- 2 In the Settings window for Global Equations, locate the Global Equations section.
- **3** In the table, enter the following settings:

Name	f(u,ut,utt,t) (1)	Initial value (u_0) (1)	Initial value (u_t0) (1/s)	Description
obj	objt* tmax- (tmax- Tperiod < t)*uAvg/ U0	0	0	

The boolean expression filters out initialization effects in the first period.

MESH I

Free Triangular 1

In the Mesh toolbar, click Free Triangular.

Size

- I In the Model Builder window, click Size.
- 2 In the Settings window for Size, locate the Element Size section.
- **3** Click the **Custom** button.
- 4 Locate the Element Size Parameters section. In the Maximum element size text field, type meshsz.
- 5 Click Build All.

OPTIMIZATION

- I In the Model Builder window, click Study I.
- 2 In the Settings window for Study, type Optimization in the Label text field.

Topology Optimization

- I In the Study toolbar, click optimization and choose Topology Optimization.
- 2 In the Settings window for Topology Optimization, locate the Optimization Solver section.
- 3 In the Maximum number of iterations text field, type 25.
- 4 Select the Move limits check box.
- 5 In the associated text field, type 0.2.
- 6 Click Add Expression in the upper-right corner of the Objective Function section. From the menu, choose Component I (compl)>Global ODEs and DAEs>compl.obj -State variable obj.
- 7 Locate the **Objective Function** section. From the **Type** list, choose **Maximization**.
- 8 From the Objective scaling list, choose Manual.
- 9 In the Scale text field, type 1.5e-3.
- 10 Locate the Output While Solving section. From the Probes list, choose None.

Step 1: Time Dependent

- I In the Model Builder window, click Step I: Time Dependent.
- 2 In the Settings window for Time Dependent, locate the Study Settings section.
- 3 In the Output times text field, type range (0, tmax/20, tmax).
- 4 In the Study toolbar, click $t_{=0}^{U}$ Get Initial Value.

RESULTS

Output material volume factor

In the Model Builder window, expand the Results>Topology Optimization node.

Surface 1

- I In the Model Builder window, expand the Output material volume factor node, then click Surface 1.
- 2 In the Settings window for Surface, locate the Coloring and Style section.
- 3 From the Color table transformation list, choose Reverse.

OPTIMIZATION

Solver Configurations

In the Model Builder window, expand the Optimization>Solver Configurations node.

Solution I (soll)

In this case the original MMA (1987) converges to the correct topology with fewer model evaluations.

- I In the Model Builder window, expand the Optimization>Solver Configurations> Solution I (soll) node, then click Optimization Solver I.
- 2 In the Settings window for Optimization Solver, locate the Optimization Solver section.
- 3 Clear the Globally Convergent MMA check box. We need to fix the time stepping to ensure robustness of the adjoint problem.
- 4 In the Model Builder window, expand the Optimization>Solver Configurations> Solution I (soll)>Optimization Solver I node, then click Time-Dependent Solver I.
- 5 In the Settings window for Time-Dependent Solver, click to expand the Time Stepping section.
- 6 From the Steps taken by solver list, choose Manual.
- 7 In the Time step text field, type tstep.
- 8 Right-click Time-Dependent Solver I and choose Segregated. Switch to a **segregated solver** for increased robustness of the adjoint problem.
- 9 Right-click Segregated I and choose Segregated Steptwice.
- 10 In the Settings window for Segregated Step, type Optimization in the Label text field.
- II Locate the General section. In the Variables list, choose Pressure (compl.p), Velocity field (compl.u), and State variable obj (compl.ODEI).

- 12 Under Variables, click Delete.
- 13 In the Model Builder window, under Optimization>Solver Configurations> Solution I (soll)>Optimization Solver I>Time-Dependent Solver I>Segregated I click Segregated Step 1.
- 14 In the Settings window for Segregated Step, type Fluid Flow in the Label text field.
- 15 Locate the General section. Under Variables, click + Add.
- 16 In the Add dialog box, in the Variables list, choose Control material volume factor (compl.dtopol.theta_c), Pressure (compl.p), and Velocity field (compl.u).

17 Click OK.

- 18 In the Model Builder window, under Optimization>Solver Configurations> Solution I (soll)>Optimization Solver I>Time-Dependent Solver I>Segregated I click Segregated Step 2.
- 19 In the Settings window for Segregated Step, type Objective in the Label text field.
- 20 Locate the General section. Under Variables, click + Add.
- 21 In the Add dialog box, in the Variables list, choose Control material volume factor (compl.dtopol.theta_c) and State variable obj (compl.ODEI).
- 22 Click OK.
- 23 In the Model Builder window, click Direct 1.
- 24 In the Settings window for Direct, locate the General section.
- **25** From the **Solver** list, choose **PARDISO**to reduce the computational time.

Topology Optimization

- I In the Model Builder window, under Optimization click Topology Optimization.
- 2 In the Settings window for Topology Optimization, locate the Output While Solving section.
- **3** Select the **Plot** check box.
- 4 From the Plot group list, choose Output material volume factor.
- 5 In the Study toolbar, click **Compute**.

RESULTS

Objective

I In the Model Builder window, under Results click ID Plot Group 3.

- 2 In the Settings window for ID Plot Group, type Objective in the Label text field.
- 3 In the **Objective** toolbar, click **Plot**.

Output material volume factor

- I In the Model Builder window, under Results>Topology Optimization click Output material volume factor.
- 2 In the Output material volume factor toolbar, click **1** Plot.
- 3 Click the Zoom Extents button in the Graphics toolbar.

Velocity (spf)

- I In the Model Builder window, under Results click Velocity (spf).
- 2 In the Settings window for 2D Plot Group, locate the Data section.
- 3 From the Dataset list, choose Filter.
- 4 From the Time (s) list, choose 0.03325.

It is obvious to see that there are significant approximation errors for the no-slip boundary condition in the optimization.

- 5 In the **Velocity (spf)** toolbar, click **Plot**.
- **6** Click the **Zoom Extents** button in the **Graphics** toolbar.

Perform a verification in a new component to ensure that the optimization results does not rely on unphysical effects.

Filter

- I In the Model Builder window, expand the Results>Datasets node, then click Filter.
- 2 In the Settings window for Filter, click Replace Expression in the upper-right corner of the Expression section. From the menu, choose Component I (compl)>Definitions> Density Model I>Auxiliary variables>dtopol.theta_f - Filtered material volume factor.
- 3 Right-click Results>Datasets>Filter and choose Create Mesh Part.

MESH PART I

Imbort I

- I In the Settings window for Import, locate the Import section.
- 2 Click Import.
- 3 In the Model Builder window, right-click Mesh Part I and choose Create Geometry.

Copy/paste the physics, materials, and probe from the 1st component and correct the selections.

GLOBAL ODES AND DAES (GE), LAMINAR FLOW (SPF)

- I In the Model Builder window, under Component I (compl), Ctrl-click to select Laminar Flow (spf) and Global ODEs and DAEs (ge).
- 2 Right-click and choose Copy.

LAMINAR FLOW (SPF2)

In the Model Builder window, right-click Component 2 (comp2) and choose Paste Multiple Items.

GLOBAL ODES AND DAES (GE2), LAMINAR FLOW (SPF2)

- I In the Model Builder window, under Component 2 (comp2), Ctrl-click to select Laminar Flow (spf2) and Global ODEs and DAEs (ge2).
- 2 In the Messages from Paste dialog box, click OK.

MATERIALS

Water (mat I)

In the Model Builder window, under Component I (compl)>Materials right-click Water (mat I) and choose Copy.

MATERIALS

Water (mat2)

In the Model Builder window, under Component 2 (comp2) right-click Materials and choose Paste Material

DEFINITIONS (COMPI)

Velocity Average (uAvg)

In the Model Builder window, under Component I (compl)>Definitions right-click Velocity Average (uAvg) and choose Copy.

DEFINITIONS (COMP2)

Velocity Average (dom2)

- I In the Model Builder window, under Component 2 (comp2) right-click Definitions and choose Paste Domain Probe.
- 2 In the Settings window for Domain Probe, type uAvg in the Variable name text field.
- 3 Locate the Source Selection section. From the Selection list, choose Rectangle I (Import I).

LAMINAR FLOW (SPF2)

Symmetry I

- I In the Model Builder window, expand the Component 2 (comp2)>Laminar Flow (spf2) node, then click Symmetry 1.
- 2 In the Settings window for Symmetry, locate the Boundary Selection section.
- 3 From the Selection list, choose Symmetry (Import 1).

Periodic Flow Condition 1

- I In the Model Builder window, click Periodic Flow Condition I.
- 2 In the Settings window for Periodic Flow Condition, locate the Boundary Selection section.
- 3 From the Selection list, choose Inlet/Outlet (Import I).

Pressure Point Constraint I

- I In the Model Builder window, click Pressure Point Constraint I.
- 2 In the Settings window for Pressure Point Constraint, locate the Point Selection section.
- 3 From the Selection list, choose Pressure Reference Point (Import 1).

Volume Force 1

In the Model Builder window, under Component 2 (comp2)>Laminar Flow (spf2) right-click Volume Force I and choose Delete.

GLOBAL ODES AND DAES (GE2)

Global Equations 1

- I In the Model Builder window, expand the Component 2 (comp2)> Global ODEs and DAEs (ge2) node, then click Global Equations 1.
- 2 In the Settings window for Global Equations, locate the Global Equations section.
- **3** In the table, enter the following settings:

Name	f(u,ut,utt,t) (1)	Initial value (u_0) (1)	Initial value (u_t0) (1/s)	Description
obj	objt* tmax- (tmax- Tperiod < t)*uAvg/ U0	0	0	

MESH 2

Free Triangular I

In the Mesh toolbar, click Free Triangular.

Size

Change the settings to give a uniform mesh with an element size half of that used for the optimization.

- I In the Model Builder window, click Size.
- 2 In the Settings window for Size, locate the Element Size section.
- **3** Click the **Custom** button.
- 4 Locate the Element Size Parameters section. In the Maximum element size text field, type meshsz/2.
- 5 In the Maximum element growth rate text field, type Inf.
- 6 In the Curvature factor text field, type Inf.
- 7 Click Build All.
- **8** Click the **Zoom Extents** button in the **Graphics** toolbar.

ADD STUDY

- I In the Home toolbar, click Add Study to open the Add Study window.
- 2 Go to the Add Study window.
- 3 Find the Studies subsection. In the Select Study tree, select Empty Study.
- 4 Click Add Study in the window toolbar.
- 5 In the Home toolbar, click Add Study to close the Add Study window.

OPTIMIZATION

Step 1: Time Dependent

In the Model Builder window, under Optimization right-click Step 1: Time Dependent and choose Copy.

STUDY 2

In the Model Builder window, right-click Study 2 and choose Paste Time Dependent.

Step 1: Time Dependent

I In the Settings window for Time Dependent, locate the Physics and Variables Selection section.

2 In the table, enter the following settings:

Physics interface	Solve for	Equation form
Laminar Flow (spf)		Automatic (Stationary)
Global ODEs and DAEs (ge)		Automatic (Stationary)
Topology Optimization (Component I)		

3 Click to expand the **Results While Solving** section. From the **Probes** list, choose **None**.

OPTIMIZATION

Step 1: Time Dependent

- I In the Model Builder window, under Optimization click Step 1: Time Dependent.
- 2 In the Settings window for Time Dependent, locate the Physics and Variables Selection section.
- **3** In the table, enter the following settings:

Physics interface	Solve for	Equation form
Laminar Flow (spf2)		Automatic (Stationary)
Global ODEs and DAEs (ge2)		Automatic (Stationary)

VERIFICATION

- I In the Model Builder window, click Study 2.
- 2 In the Settings window for Study, type Verification in the Label text field.
- 3 In the Home toolbar, click **Compute**.

RESULTS

Objective, Pressure (spf), Velocity (spf)

In the Model Builder window, under Results, Ctrl-click to select Velocity (spf), Pressure (spf), and Objective.

Objective, Pressure (spf), Velocity (spf)

In the Model Builder window, under Results, Ctrl-click to select Velocity (spf), Pressure (spf), and **Objective**.

Objective, Pressure (spf), Velocity (spf)

In the Model Builder window, under Results, Ctrl-click to select Velocity (spf), Pressure (spf), and Objective.

Velocity (spf)

Drag and drop above Topology Optimization

Output material volume factor.

Topology Optimization I

In the Model Builder window, under Results right-click Topology Optimization 1 and choose Delete.

Objective (Verification)

- I In the Model Builder window, under Results click ID Plot Group 8.
- 2 In the Settings window for ID Plot Group, type Objective (Verification) in the Label text field.

Objective (Verification), Pressure (spf2), Velocity (spf2)

- I In the Model Builder window, under Results, Ctrl-click to select Velocity (spf2), Pressure (spf2), and Objective (Verification).
- 2 Right-click and choose **Group**.

Verification

In the **Settings** window for **Group**, type Verification in the **Label** text field.

Use an **Evaluation Group** to compare the objective in the two components.

Evaluation Group 1

- I In the Results toolbar, click Evaluation Group.
- 2 In the Settings window for Evaluation Group, locate the Data section.
- **3** From the **Time selection** list, choose **Last**.

Global Evaluation 1

Right-click Evaluation Group I and choose Global Evaluation.

Global Evaluation 2

- I In the Model Builder window, right-click Global Evaluation I and choose Duplicate.
- 2 In the Settings window for Global Evaluation, locate the Data section.
- 3 From the Dataset list, choose Verification/Solution 2 (4) (sol2).
- 4 From the Time selection list, choose Last.
- 5 In the Evaluation Group I toolbar, click **= Evaluate**.

Velocity (sbf2)

I In the Model Builder window, under Results>Verification click Velocity (spf2).

- 2 In the Settings window for 2D Plot Group, click ← Plot Previous.
- **3** Click the **Zoom Extents** button in the **Graphics** toolbar.