

Submodeling Analysis of a Shaft

Introduction

Submodeling can be used when it is not possible to resolve all details of a complex geometry in a global model.

You can cope with this type of problems with a technique known as *submodeling*. First you solve the complete model with a mesh which is sufficient to capture the stiffness of the structure. In a second analysis you create a local model (submodel) of the region around the stress concentration with a fine mesh, and solve it using the displacements from the global model as boundary conditions.

There are some underlying assumptions when using submodels:

- The global model is accurate enough to give correct displacements on the boundary to the submodel.
- The improvements introduced in the submodel are so small that they do not introduce significant changes in stiffness on the global level. Given this, it could still be possible to introduce a nonlinear material locally in the submodel.

In this example this technique is applied to perform an accurate stress evaluation in a structural mechanics model, but the same approach is applicable to many physical problems. The example geometry as such is not so complicated, so there is nothing to gain from a submodeling in this case. The purpose of the example is to show the technique.

Model Definition

The geometry consists of a shaft with a sudden change in diameter. At the location of the diameter change, there is a fillet with a small radius. In the fillet, stress concentrations will appear. There is also a central hole through the shaft. The geometry and mesh are shown in Figure 1.

The shaft is fixed at the thick end. On the thin end, a tensile force of 300 N and a shear force of 100 N are applied.



Figure 1: The full model.

As submodel, a region around the fillet at side giving the highest stress is chosen. The cuts where the boundary conditions are applied on the submodel should preferably be placed at locations where the stress field is fairly smooth.

The geometry and mesh of the submodel are shown in Figure 2. As can be seen, the fillet has a very good mesh resolution since the purpose is to obtain results with high accuracy there.



Figure 2: The submodel.

Results

The general stress distribution is shown in Figure 3. Since the load is unsymmetrical (there is both an axial and a bending component), the highest stress occurs at the side with positive Z coordinate.

The cut through the model in Figure 4 displays that the stresses are not well resolved. There are significant jumps between the neighboring elements.

In the corresponding figures from the submodel, Figure 5 and Figure 6, the stress field is smooth and well resolved.

The computed peak stress for the global model is about 9% different compared to that in the submodel, which is expected given the coarse mesh in the global model.



Figure 3: Stress distribution in the global model.



Figure 4: Stress within the full model (cut view).



Figure 5: Stress distribution in the submodel.



Figure 6: Stress within the submodel (cut view).

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As a verification, it is a good habit to always check the stress state at the cuts where the submodel has displacements prescribed by the results in the global model. Such a comparison is shown in Figure 7. The results have an almost perfect match, which strongly indicates that the submodel has been set up correctly.



Figure 7: Stress distribution over one of the cuts. Global model (left) and submodel (right) are compared.

Notes About the COMSOL Implementation

Two different components are used within the same mph file. In the global model, a general extrusion feature is introduced in order to describe the mapping of results from the global model to the submodel. The prescribed displacements on the cut boundaries in the submodel reference the displacements in the global model through this mapping.

In this example, there are no volume forces. If there were, such forces must be applied also on the submodel.

Application Library path: COMSOL_Multiphysics/Structural_Mechanics/ shaft_submodeling

Modeling Instructions

From the File menu, choose New.

NEW

In the New window, click 🔗 Model Wizard.

MODEL WIZARD

- I In the Model Wizard window, click 间 3D.
- 2 In the Select Physics tree, select Structural Mechanics>Solid Mechanics (solid).
- 3 Click Add.
- 4 Click \bigcirc Study.
- 5 In the Select Study tree, select General Studies>Stationary.
- 6 Click 🗹 Done.

GEOMETRY I

In the Model Builder window, under Component I (compl) right-click Geometry I and choose 3D Part.

FULL GEOMETRY

Create the geometry. To simplify this step, insert a prepared geometry sequence.

- I In the Settings window for Part, locate the Units section.
- 2 From the Length unit list, choose mm.
- 3 In the Geometry toolbar, click Insert Sequence and choose Insert Sequence.
- 4 Browse to the model's Application Libraries folder and double-click the file shaft_submodeling_geom_sequence.mph.
- 5 In the Geometry toolbar, click 🟢 Build All.
- 6 In the Label text field, type Full Geometry.

FULL MODEL

I In the Model Builder window, click Component I (compl).

2 In the Settings window for Component, type Full model in the Label text field.

GEOMETRY I

- I In the Model Builder window, under Full model (compl) click Geometry I.
- 2 In the Settings window for Geometry, locate the Units section.
- 3 From the Length unit list, choose mm.

Full Geometry 1 (pil)

- I In the Geometry toolbar, click 🔶 Parts and choose Full Geometry.
- 2 In the Settings window for Part Instance, click 🟢 Build All Objects.
- **3** Click to expand the **Boundary Selections** section. In the table, enter the following settings:

Name	Кеер	Physics	Contribute to
Boundary Load	\checkmark	\checkmark	None
Fixed Constrained	\checkmark	\checkmark	None

The geometry sequence contains two selections, these will be used later to apply loads and constraints.

ADD MATERIAL

- I In the Home toolbar, click 🙀 Add Material to open the Add Material window.
- 2 Go to the Add Material window.
- 3 In the tree, select Built-in>Structural steel.
- 4 Right-click and choose Add to Global Materials.
- 5 In the Home toolbar, click 🙀 Add Material to close the Add Material window.

The material is now defined as global material so that both the full model and the submodel can use the same material definition, defined at one location only in the model tree.

MATERIALS

Material Link I (matlnk I)

In the Model Builder window, under Full model (compl) right-click Materials and choose More Materials>Material Link.

SOLID MECHANICS (SOLID)

Fixed Constraint I

- I In the Model Builder window, under Full model (compl) right-click Solid Mechanics (solid) and choose Fixed Constraint.
- 2 In the Settings window for Fixed Constraint, locate the Boundary Selection section.
- 3 From the Selection list, choose Fixed Constrained (Full Geometry I).

Boundary Load 1

- I In the Physics toolbar, click 🔚 Boundaries and choose Boundary Load.
- 2 In the Settings window for Boundary Load, locate the Boundary Selection section.
- **3** From the Selection list, choose Boundary Load (Full Geometry I).
- 4 Locate the Force section. From the Load type list, choose Total force.
- **5** Specify the **F**_{tot} vector as

-300	x
0	у
-100	z

MESH I

I In the Model Builder window, under Full model (compl) click Mesh I.

2 In the Settings window for Mesh, locate the Physics-Controlled Mesh section.

- **3** From the **Element size** list, choose **Coarse**.
- 4 Click 📗 Build All.
- **5** Click the \longleftrightarrow **Zoom Extents** button in the **Graphics** toolbar.

FULL MODEL

- I In the Model Builder window, click Study I.
- 2 In the Settings window for Study, type Full model in the Label text field.
- **3** In the **Home** toolbar, click **= Compute**.

RESULTS

Stress - Full model

I In the Settings window for 3D Plot Group, type Stress - Full model in the Label text field.

2 Locate the **Color Legend** section. Select the **Show maximum and minimum values** check box.

Volume 1

- I In the Model Builder window, expand the Stress Full model node, then click Volume I.
- 2 In the Settings window for Volume, click to expand the Quality section.
- 3 From the Resolution list, choose No refinement.
- 4 From the **Smoothing** list, choose **None**.
- 5 In the Stress Full model toolbar, click **9** Plot.
- 6 Click the \leftrightarrow Zoom Extents button in the Graphics toolbar.

Cut Plane I

- I In the Model Builder window, expand the Results>Datasets node.
- 2 Right-click **Results>Datasets** and choose **Cut Plane**.
- 3 In the Settings window for Cut Plane, locate the Plane Data section.
- 4 From the Plane list, choose XZ-planes.

Cut Plane Stress -Full model

- I In the **Results** toolbar, click **2D Plot Group**.
- 2 In the Settings window for 2D Plot Group, type Cut Plane Stress -Full model in the Label text field.
- **3** Locate the **Color Legend** section. Select the **Show maximum and minimum values** check box.

Surface 1

- I Right-click Cut Plane Stress -Full model and choose Surface.
- 2 In the Settings window for Surface, locate the Expression section.
- 3 In the **Expression** text field, type solid.mises.
- 4 Click to expand the Quality section. From the Resolution list, choose No refinement.
- 5 From the Smoothing list, choose None.
- 6 In the Cut Plane Stress -Full model toolbar, click 🗿 Plot.
- 7 In the Model Builder window, expand the Results>Views node.

Axis

- I In the Model Builder window, expand the Results>Views>View 2D 4 node, then click Axis.
- 2 In the Settings window for Axis, locate the Axis section.

- 3 In the x minimum text field, type 25.8.
- 4 In the **x maximum** text field, type 34.2.
- 5 In the y minimum text field, type 1.5.
- 6 In the **y maximum** text field, type 7.5.
- 7 Click 🚺 Update.

ADD COMPONENT

In the Model Builder window, right-click the root node and choose Add Component>3D.

SUBMODEL

In the Settings window for Component, type Submodel in the Label text field.

SUBMODEL CUT

- I In the Model Builder window, under Global Definitions right-click Geometry Parts and choose 3D Part.
- 2 In the Settings window for Part, type Submodel Cut in the Label text field.
- 3 Locate the Units section. From the Length unit list, choose mm.

Cylinder I (cyl1)

- I In the Geometry toolbar, click 🔲 Cylinder.
- 2 In the Settings window for Cylinder, locate the Size and Shape section.
- 3 In the Radius text field, type 7.
- 4 In the Height text field, type 8.
- 5 Locate the **Position** section. In the **x** text field, type 26.
- 6 Locate the Axis section. From the Axis type list, choose x-axis.

Block I (blkI)

- I In the **Geometry** toolbar, click **[]** Block.
- 2 In the Settings window for Block, locate the Size and Shape section.
- 3 In the Width text field, type 10.
- 4 In the **Depth** text field, type 14.
- 5 In the **Height** text field, type 10.
- 6 Locate the Position section. In the x text field, type 26.
- 7 In the y text field, type -7.

Submodel Cut

- I In the Geometry toolbar, click 🔲 Booleans and Partitions and choose Intersection.
- 2 In the Settings window for Intersection, type Submodel Cut in the Label text field.
- 3 Click in the Graphics window and then press Ctrl+A to select both objects.
- **4** Locate the **Selections of Resulting Entities** section. Select the **Resulting objects selection** check box.
- 5 From the Show in instances list, choose Boundary selection.

GEOMETRY 2

- I In the Model Builder window, under Submodel (comp2) click Geometry 2.
- 2 In the Settings window for Geometry, locate the Units section.
- 3 From the Length unit list, choose mm.

Full Geometry I (pil)

In the **Geometry** toolbar, click \bigwedge **Parts** and choose **Full Geometry**.

Submodel Cut 1 (pi2)

- I In the Geometry toolbar, click A Parts and choose Submodel Cut.
- 2 In the Settings window for Part Instance, click 틤 Build Selected.
- 3 Locate the Boundary Selections section. In the table, enter the following settings:

Name	Кеер	Physics	Contribute to
Submodel Cut	\checkmark		None

Intersection 1 (int1)

- I In the Geometry toolbar, click 💻 Booleans and Partitions and choose Intersection.
- 2 Click in the Graphics window and then press Ctrl+A to select both objects.
- 3 In the Settings window for Intersection, click 🟢 Build All Objects.

The submodel geometry is now built. In order to create a structured mesh, you need to partition the geometry following the xz-plane.

Work Plane I (wp1)

- I In the Geometry toolbar, click Work Plane.
- 2 In the Settings window for Work Plane, locate the Plane Definition section.
- 3 From the Plane list, choose zx-plane.

Partition Objects 1 (parl)

I In the Geometry toolbar, click 💻 Booleans and Partitions and choose Partition Objects.

- 2 Select the object intl only.
- 3 In the Settings window for Partition Objects, locate the Partition Objects section.
- 4 From the Partition with list, choose Work plane.

Mesh Control Domains 1 (mcd1)

- I In the Geometry toolbar, click 🏷 Virtual Operations and choose Mesh Control Domains.
- 2 On the object fin, select Domains 1 and 2 only.
- 3 In the Geometry toolbar, click 🟢 Build All.
- **4** Click the \leftarrow **Zoom Extents** button in the **Graphics** toolbar.



MATERIALS

Material Link 2 (matlnk2)

In the Model Builder window, under Submodel (comp2) right-click Materials and choose More Materials>Material Link.

ADD PHYSICS

- I In the Home toolbar, click 🙀 Add Physics to open the Add Physics window.
- 2 Go to the Add Physics window.
- 3 In the tree, select Structural Mechanics>Solid Mechanics (solid).
- **4** Find the **Physics interfaces in study** subsection. In the table, select the **Solve** check box for **Full model**.

- 5 Click Add to Submodel in the window toolbar.
- 6 In the Home toolbar, click 🖄 Add Physics to close the Add Physics window.

DEFINITIONS (COMPI)

In the Model Builder window, under Full model (compl) click Definitions.

General Extrusion 1 (genext1)

- I In the Definitions toolbar, click 🖉 Nonlocal Couplings and choose General Extrusion.
- **2** Select Domain 1 only.
- 3 In the Settings window for General Extrusion, locate the Destination Map section.
- 4 In the x-expression text field, type X.
- 5 In the y-expression text field, type Y.
- 6 In the **z-expression** text field, type Z.
- 7 Locate the Source section. From the Source frame list, choose Material (X, Y, Z).

You have now created the nonlocal coupling which will be used for mapping the solution from the full model to the submodel.

SOLID MECHANICS 2 (SOLID2)

In the Model Builder window, under Submodel (comp2) click Solid Mechanics 2 (solid2).

Prescribed Displacement 1

- I In the Physics toolbar, click 🔚 Boundaries and choose Prescribed Displacement.
- 2 In the Settings window for Prescribed Displacement, locate the Boundary Selection section.
- 3 From the Selection list, choose Submodel Cut (Submodel Cut I).
- **4** Locate the **Prescribed Displacement** section. Select the **Prescribed in x direction** check box.
- **5** In the u_{0x} text field, type compl.genextl(compl.u).
- 6 Select the Prescribed in y direction check box.
- 7 In the u_{0v} text field, type compl.genext1(compl.v).
- 8 Select the Prescribed in z direction check box.
- **9** In the u_{0z} text field, type compl.genext1(compl.w).

MESH 2

Mapped I

- I In the Mesh toolbar, click A Boundary and choose Mapped.
- **2** Select Boundary 2 only.

Distribution I

- I Right-click Mapped I and choose Distribution.
- 2 Select Edges 1 and 3 only.
- 3 In the Settings window for Distribution, locate the Distribution section.
- **4** From the **Distribution type** list, choose **Predefined**.
- 5 In the Number of elements text field, type 6.
- 6 In the **Element ratio** text field, type 3.
- 7 Select the **Reverse direction** check box.

Distribution 2

- I In the Model Builder window, right-click Mapped I and choose Distribution.
- 2 Select Edge 13 only.
- 3 In the Settings window for Distribution, locate the Distribution section.
- **4** From the **Distribution type** list, choose **Predefined**.
- 5 In the Number of elements text field, type 6.
- 6 In the Element ratio text field, type 3.

Distribution 3

- I Right-click Mapped I and choose Distribution.
- **2** Select Edge 15 only.
- 3 In the Settings window for Distribution, locate the Distribution section.
- 4 In the Number of elements text field, type 6.

Distribution 4

- I Right-click Mapped I and choose Distribution.
- **2** Select Edge 10 only.
- **3** In the **Settings** window for **Distribution**, locate the **Distribution** section.
- 4 From the **Distribution type** list, choose **Predefined**.
- 5 In the Number of elements text field, type 10.
- 6 In the **Element ratio** text field, type 2.

Swept I

In the Mesh toolbar, click A Swept.

Distribution I

- I Right-click Swept I and choose Distribution.
- 2 In the Settings window for Distribution, locate the Distribution section.
- **3** In the **Number of elements** text field, type **12**.
- 4 Click 📗 Build All.
- 5 In the Model Builder window, right-click Mesh 2 and choose Build All.
- 6 Click the 4 Zoom Extents button in the Graphics toolbar.

ADD STUDY

- I In the Home toolbar, click \sim°_{1} Add Study to open the Add Study window.
- 2 Go to the Add Study window.
- 3 Find the Studies subsection. In the Select Study tree, select General Studies>Stationary.
- 4 Find the Physics interfaces in study subsection. In the table, clear the Solve check box for Solid Mechanics (solid).
- 5 Click Add Study in the window toolbar.
- 6 In the Model Builder window, click the root node.
- 7 In the Home toolbar, click $\stackrel{\text{res}}{\longrightarrow}$ Add Study to close the Add Study window.

SUBMODEL

In the Settings window for Study, type Submodel in the Label text field.

Step 1: Stationary

- I In the Model Builder window, under Submodel click Step I: Stationary.
- **2** In the **Settings** window for **Stationary**, click to expand the **Values of Dependent Variables** section.
- **3** Find the **Values of variables not solved for** subsection. From the **Settings** list, choose **User controlled**.
- 4 From the Method list, choose Solution.
- 5 From the Study list, choose Full model, Stationary.
- 6 In the Home toolbar, click **=** Compute.

RESULTS

Stress - Submodel

- I In the Settings window for 3D Plot Group, type Stress Submodel in the Label text field.
- 2 Locate the Color Legend section. Select the Show maximum and minimum values check box.

Volume 1

- I In the Model Builder window, expand the Stress Submodel node, then click Volume I.
- 2 In the Settings window for Volume, locate the Quality section.
- **3** From the **Resolution** list, choose **No refinement**.
- 4 From the **Smoothing** list, choose **None**.
- 5 In the Stress Submodel toolbar, click **I** Plot.
- 6 Click the 🕂 Zoom Extents button in the Graphics toolbar.

Cut Plane 2

- I In the **Results** toolbar, click **Cut Plane**.
- 2 In the Settings window for Cut Plane, locate the Plane Data section.
- 3 From the Plane list, choose XZ-planes.
- 4 Locate the Data section. From the Dataset list, choose Submodel/Solution 2 (3) (sol2).

Cut Plane Stress - Submodel

- I In the **Results** toolbar, click **2D Plot Group**.
- 2 In the Settings window for 2D Plot Group, type Cut Plane Stress Submodel in the Label text field.
- **3** Locate the **Color Legend** section. Select the **Show maximum and minimum values** check box.
- 4 Locate the Data section. From the Dataset list, choose Cut Plane 2.

Surface 1

- I Right-click Cut Plane Stress Submodel and choose Surface.
- 2 In the Settings window for Surface, locate the Expression section.
- 3 In the **Expression** text field, type solid2.mises.
- 4 Locate the Quality section. From the Resolution list, choose No refinement.
- 5 From the Smoothing list, choose None.
- 6 In the Cut Plane Stress Submodel toolbar, click 🗿 Plot.

7 Click the + Zoom Extents button in the Graphics toolbar.

Now verify that the differences in stress between the full model and the submodel are small where the displacements are mapped.

Cut Plane 3

- I In the **Results** toolbar, click **Cut Plane**.
- 2 In the Settings window for Cut Plane, locate the Plane Data section.
- 3 In the X-coordinate text field, type 26.

Stress comparison

- I In the **Results** toolbar, click **2D Plot Group**.
- 2 In the Settings window for 2D Plot Group, type Stress comparison in the Label text field.
- 3 Locate the Data section. From the Dataset list, choose None.

Surface 1

- I Right-click Stress comparison and choose Surface.
- 2 In the Settings window for Surface, locate the Data section.
- 3 From the Dataset list, choose Cut Plane 3.
- 4 Locate the Expression section. In the Expression text field, type solid.mises.
- 5 Locate the Quality section. From the Resolution list, choose No refinement.
- 6 From the Smoothing list, choose None.
- 7 In the Stress comparison toolbar, click **O** Plot.
- 8 In the **Results** toolbar, click **More Datasets** and choose **Surface**.

Surface 1

- I In the Settings window for Surface, locate the Data section.
- 2 From the Dataset list, choose Submodel/Solution 2 (3) (sol2).
- **3** Select Boundary 1 only.
- 4 Locate the Parameterization section. From the x- and y-axes list, choose YZ-plane.

Surface 2

- I In the Model Builder window, right-click Stress comparison and choose Surface.
- 2 In the Settings window for Surface, locate the Data section.
- 3 From the Dataset list, choose Surface I.
- 4 Locate the Expression section. In the Expression text field, type solid2.mises.

- 5 Locate the Quality section. From the Resolution list, choose No refinement.
- 6 From the Smoothing list, choose None.
- 7 Click to expand the Inherit Style section. From the Plot list, choose Surface I.

Deformation I

- I Right-click Surface 2 and choose Deformation.
- 2 In the Settings window for Deformation, locate the Expression section.
- 3 In the x component text field, type 11.
- **4** In the **y component** text field, type **0**.
- 5 Locate the Scale section. Select the Scale factor check box.
- 6 In the associated text field, type 1.
- 7 In the Stress comparison toolbar, click 💿 Plot.
- 8 Click the 🕂 Zoom Extents button in the Graphics toolbar.

The solution is computed, follow the steps below if you want to save and reuse the model in the future.

FULL MODEL

Step 1: Stationary

- I In the Model Builder window, under Full model click Step I: Stationary.
- 2 In the Settings window for Stationary, locate the Physics and Variables Selection section.
- 3 In the table, clear the Solve for check box for Solid Mechanics 2 (solid2).