Created in COMSOL Multiphysics 6.0

Eigenmodes of a Room

Introduction

Resonance can at times be a problem in everyday life. The low bass notes from the music system or home theater in the living room can shake the windows and make the floor vibrate. This happens only for certain frequencies — the eigenfrequencies of the room. In addition to impairing the listening experience, resonances can greatly increase the transmission of sound to the neighboring rooms.

It is only in the low-frequency range that the eigenfrequencies are well separated. In the mid- and high-frequency ranges, the eigenfrequencies are packed so closely, with less than a halftone between them, that the individual resonances are insignificant for music and other natural sounds. Nevertheless, the music experience is affected by the acoustics of the room.

When designing a listening room, it is extremely important to take the resonances into account. For a clear and neutral sound, the eigenfrequencies should be controlled and evenly spaced. For the home theater or music system owner, who cannot change the shape of the living room, another question is more relevant: where should the speakers be located for the best sound?

Model Definition

For example, let us consider a room with the dimensions 5 m by 4 m by 2.6 m equipped with a flat-screen TV, a sideboard, two speakers, and a couch with a coffee table. To illustrate the effects on the sound field, a few resonance frequencies in the vicinity of 90 Hz are computed together with the corresponding eigenmodes. The eigenmode shows the sound intensity pattern for its associated eigenfrequency. From the characteristics of the eigenmodes, we can draw some conclusions as to where the speakers should be placed.

DOMAIN EQUATIONS

Sound propagating in free air is described by the wave equation:

$$-\Delta p + \frac{1}{c^2}\frac{\partial^2 p}{\partial t^2} = 0$$

where p is the pressure, and c is the speed of sound. If the air is brought into motion by a harmonically oscillating source, for example, a loudspeaker, only one frequency f exists in the room. For that reason it makes sense to look for a time-harmonic solution of the form

$$p = \hat{p}e^{i\omega t}$$

The wave equation then simplifies to the Helmholtz equation for p, the amplitude of the acoustic disturbances:

$$\Delta\hat{p} + \frac{\omega^2}{c^2}\hat{p} = 0$$

BOUNDARY CONDITIONS

This model assumes that all boundaries — walls, floor, ceiling, and furniture — are perfectly rigid (sound hard boundaries). This means that it returns no information of the damping properties of the room, but the distribution of the pressure should still be reasonably correct.

ANALYTIC COMPARISON

It is possible to solve the simpler case of an empty rectangular room analytically. Each eigenfrequency corresponds to an integer triplet (i, l, m):

$$f_{i,l,m} = \frac{c}{2}\sqrt{\left(\frac{i}{L_x}\right)^2 + \left(\frac{l}{L_y}\right)^2 + \left(\frac{m}{L_z}\right)^2}$$

The eigenmodes can be divided into three distinct classes:

- Eigenfrequencies with only one index different from zero give rise to axial modes, that is, plane standing waves between two opposite walls.
- If one index is zero, the mode is tangential.
- If all indices are different from zero, the mode is oblique.

Theoretical resonance frequencies below 100 Hz for the room at hand without furniture are found in the following table. All modes have local maxima in the corners of an empty room so speakers in the corners excite all eigenfrequencies.

MODE INDEX	FREQUENCY	MODE INDEX	FREQUENCY
0,0,0	0	0,1,1	78.7
1,0,0	34.3	2,1,0	80.9
0,1,0	42.9	0,2,0	85.8
1,1,0	54.9	1,1,1	85.8
0,0,1	66.0	1,2,0	92.4
2,0,0	68.6	2,0,1	95.2
1,0,1	74.3	3,0,0	103

Results and Discussion

The relevant quantity when it comes to placing the loudspeakers is the amplitude of the standing pressure wave. A sound source excites an eigenmode the most if it is placed in one of the pressure antinodes for the mode. Conversely, with the source in a pressure node, the eigenmode remains silent. The loudspeakers should therefore be placed away from both the pressure nodes and antinodes to avoid canceling or boosting any frequency.

This simulation predicts eigenmodes that strongly resemble those of the corresponding empty room. However, the higher the frequency, the larger the influence of the furniture. For instance, some of the higher-frequency eigenmodes are located behind the couch.

In the strictest sense, the results of this simulation only apply to a room with perfectly rigid walls and nonabsorbing furniture. The prediction that speakers placed in the corners of the room excite all eigenmodes and give rise to resonances, however, holds for real-life rooms.

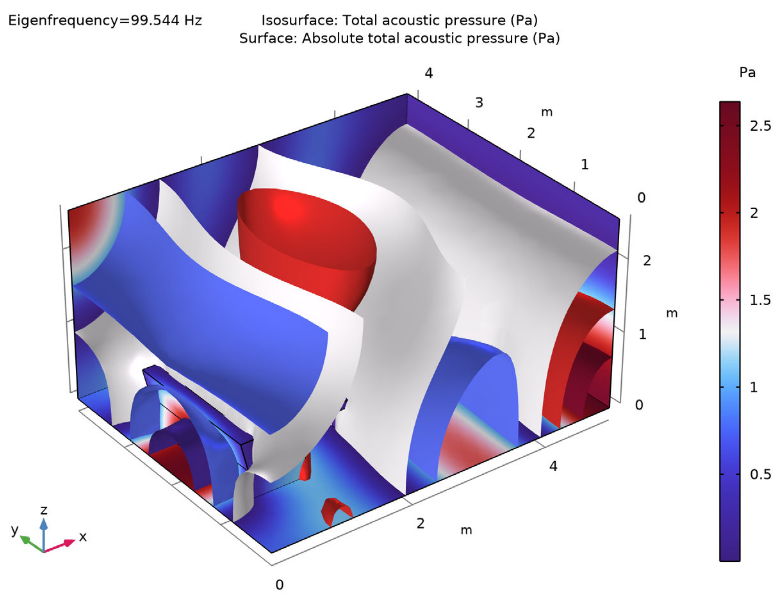


Figure 1: The sound pressure distribution for an eigenfrequency of 99.5 Hz. The real part of the pressure is visualized as an isosurface plot, and the absolute value of the pressure as a boundary plot. Note that this mode does not correspond to any of the analytical modes listed above.

Notes About the COMSOL Implementation

When simulating acoustics, or in general any wave phenomenon, it is important to resolve the expected wavelength properly. As a rule of thumb, 6 second-order element per wavelength is a reasonable tradeoff between computational effort and accuracy. For this example, where wavelengths of interest are below 100 Hz, this implies a maximum element size of

$$h_{\max} = \frac{\lambda}{6} = \frac{c}{6f} = \frac{343}{6 \cdot 100}$$

Therefore, an element size of 57 cm or less, corresponding to at least five elements between floor and ceiling, provides sufficient accuracy. The default mesh for this geometry meets the requirement. However, in general it is advisable to specify a maximum element size explicitly in the mesh settings.

Application Library path: `COMSOL_Multiphysics/Acoustics/eigenmodes_of_room`

Modeling Instructions

From the **File** menu, choose **New**.

NEW

In the **New** window, click **Model Wizard**.

MODEL WIZARD

1 In the **Model Wizard** window, click **3D**.

2 In the **Select Physics** tree, select **Acoustics>Pressure Acoustics>Pressure Acoustics, Frequency Domain (acpr)**.

3 Click **Add**.

4 Click **Study**.

5 In the **Select Study** tree, select **General Studies>Eigenfrequency**.

6 Click **Done**.

GEOMETRY 1

The geometry in this model can be created within COMSOL Multiphysics. Here it is imported for convenience.

Import 1 (imp1)

1 In the **Home** toolbar, click **Import**.
2 In the **Settings** window for **Import**, locate the **Import** section.
3 Click **Browse**.
4 Browse to the model's Application Libraries folder and double-click the file `eigenmodes_of_room.mphbin`.
5 Click **Import**.
6 Click the **Wireframe Rendering** button in the **Graphics** toolbar.

MATERIALS

Air

1 In the **Model Builder** window, under **Component 1 (comp1)** right-click **Materials** and choose **Blank Material**.
2 In the **Settings** window for **Material**, type `Air` in the **Label** text field.
3 Select Domain 1 only.
4 Locate the **Material Contents** section. In the table, enter the following settings:

Property	Variable	Value	Unit	Property group
Density	rho	1.25	kg/m³	Basic
Speed of sound	c	343	m/s	Basic

PRESSURE ACOUSTICS, FREQUENCY DOMAIN (ACPR)

The boundary conditions are all **Sound Hard Boundary (Wall)** by default.

STUDY 1

Step 1: Eigenfrequency

1 In the **Model Builder** window, under **Study 1** click **Step 1: Eigenfrequency**.
2 In the **Settings** window for **Eigenfrequency**, locate the **Study Settings** section.
3 In the **Search for eigenfrequencies around** text field, type `90`.

   This setting will get you at least 6 solutions with eigenfrequencies in the vicinity of 90 Hz. For this model you typically get additional eigensolutions that the eigenvalue solver finds.
4 In the **Home** toolbar, click **Compute**.

RESULTS

Acoustic Pressure (acpr)

The first default plot shows the pressure distribution on the exterior boundaries of the geometry. To see what goes on inside the room, you need to suppress some of the boundaries. This is most conveniently done by first selecting all boundaries and then removing a few of them (number 1, 2, and 4) from the selection.

Study 1/Solution 1 (sol1)

In the **Model Builder** window, expand the **Results>Datasets** node, then click **Study 1/ Solution 1 (sol1)**.

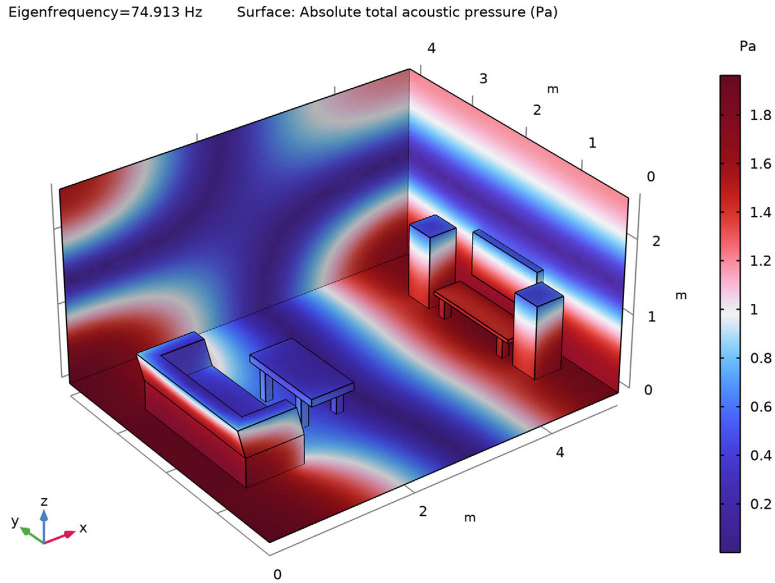
Selection

1. In the **Results** toolbar, click **Attributes** and choose **Selection**.
2. In the **Settings** window for **Selection**, locate the **Geometric Entity Selection** section.
3. From the **Geometric entity level** list, choose **Boundary**.
4. From the **Selection** list, choose **All boundaries**.
5. Select Boundaries 3 and 5–79 only.

Surface 1

1. In the **Model Builder** window, expand the **Results>Acoustic Pressure (acpr)** node, then click **Surface 1**.
2. In the **Settings** window for **Surface**, click **Replace Expression** in the upper-right corner of the **Expression** section. From the menu, choose **Component 1 (comp1)> Pressure Acoustics, Frequency Domain>Pressure and sound pressure level>acpr.absp_t - Absolute total acoustic pressure - Pa**.
3. Locate the **Coloring and Style** section. Clear the **Symmetrize color range** check box.
4. In the **Acoustic Pressure (acpr)** toolbar, click **Plot**.

5 Click the  **Zoom Extents** button in the **Graphics** toolbar.



The first plot group now shows the distribution of the absolute value of the pressure for the lowest one of the eigenfrequencies, 74.9 Hz. This appears to be the (1,0,1) mode.

Acoustic Pressure, Isosurfaces (acpr)

The third default plot shows the isosurfaces for the same frequency.

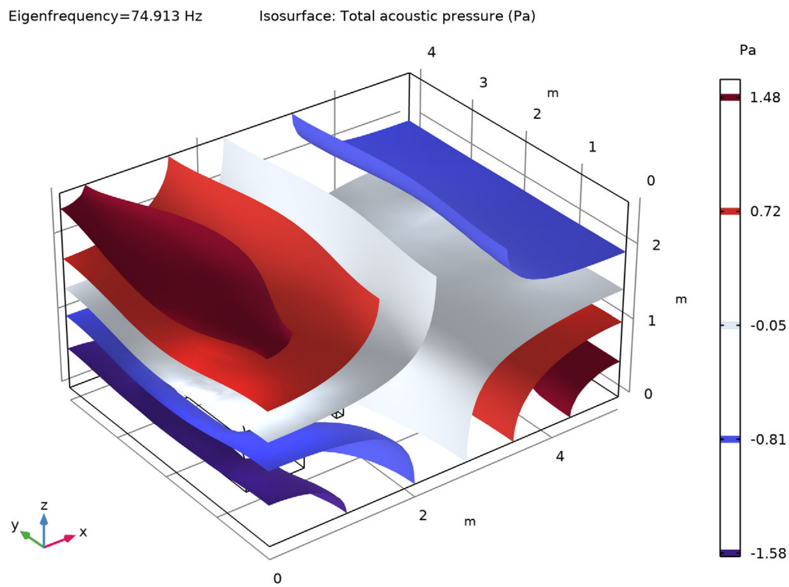
Isosurface 1

1 In the **Model Builder** window, expand the **Acoustic Pressure, Isosurfaces (acpr)** node, then click **Isosurface 1**.

2 In the **Settings** window for **Isosurface**, locate the **Levels** section.

3 In the **Total levels** text field, type 5.

4 In the **Acoustic Pressure, Isosurfaces (acpr)** toolbar, click **Plot**.



Modify the third default plot to show the isosurfaces and pressure distribution at 85 Hz; this is the (1,1,1) mode.

Acoustic Pressure, Isosurfaces (acpr)

1 In the **Model Builder** window, click **Acoustic Pressure, Isosurfaces (acpr)**.

2 In the **Settings** window for **3D Plot Group**, locate the **Data** section.

3 From the **Eigenfrequency (Hz)** list, choose **85.096**.

Surface 1

1 Right-click **Acoustic Pressure, Isosurfaces (acpr)** and choose **Surface**.

2 In the **Settings** window for **Surface**, click **Replace Expression** in the upper-right corner of the **Expression** section. From the menu, choose **Component 1 (comp1)> Pressure Acoustics, Frequency Domain>Pressure and sound pressure level>acpr.absp_t - Absolute total acoustic pressure - Pa**.

3 Locate the **Coloring and Style** section. From the **Color table** list, choose **Wave**.

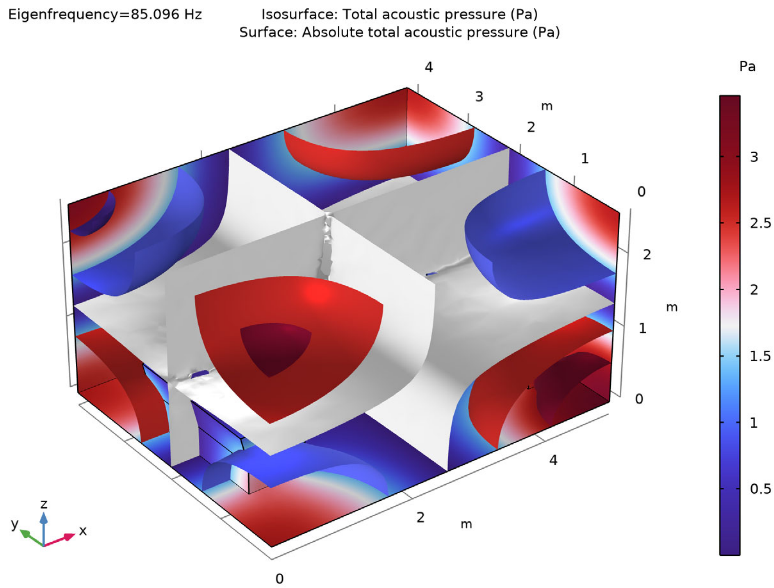
4 In the **Acoustic Pressure, Isosurfaces (acpr)** toolbar, click **Plot**.

Isosurface 1

1 In the **Model Builder** window, click **Isosurface 1**.

2 In the **Settings** window for **Isosurface**, locate the **Coloring and Style** section.

3 Clear the **Color legend** check box.



Acoustic Pressure, Isosurfaces (acpr)

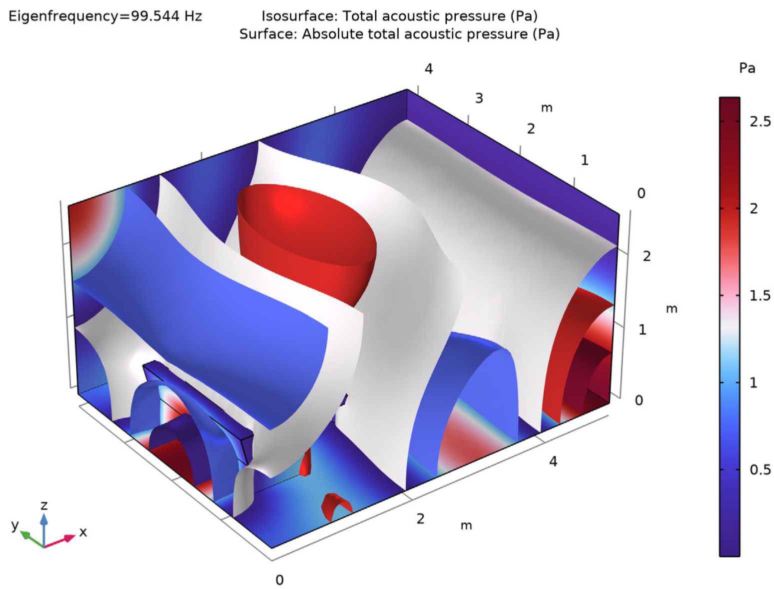
Finally, reproduce Figure 1.

1 In the **Model Builder** window, click **Acoustic Pressure, Isosurfaces (acpr)**.

2 In the **Settings** window for **3D Plot Group**, locate the **Data** section.

3 From the **Eigenfrequency (Hz)** list, choose **99.544**.

4 In the **Acoustic Pressure, Isosurfaces (acpr)** toolbar, click **Plot**.



This mode looks a little different from what you would find in an empty room. It is focused behind the couch.