Created in COMSOL Multiphysics 6.0

Diffraction Patterns

Introduction

This example simulates a double-slit interference experiment with water waves or sound. The model mimics the incoming plane-wave excitation with two thin waveguides leading to slits in a screen and computes the diffraction pattern on the other side of the screen.

Model Definition

Figure 1 shows the model geometry.

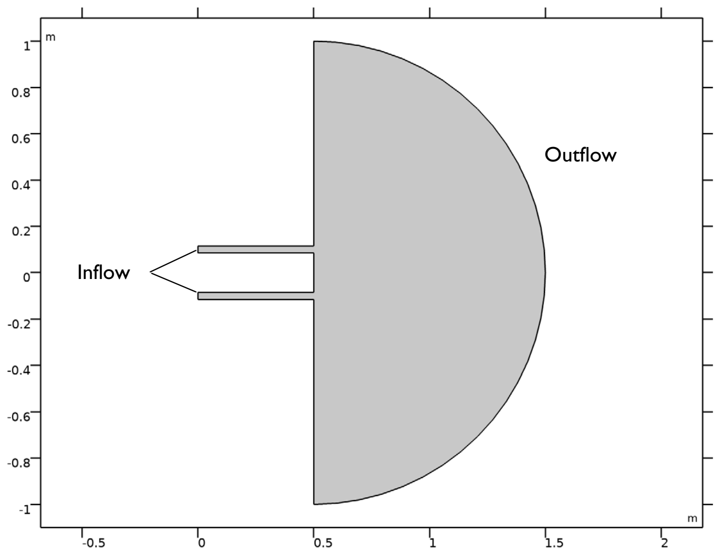


Figure 1: Model geometry with inflow and outflow boundaries indicated. On all other boundaries, a zero flux condition applies.

Theory predicts amplitude minima along rays where the difference in travel distance is an odd multiple of half the wavelength, and maxima at even multiples. For $n = 0, \pm 1, \pm 2, \ldots$:

$$\begin{cases} \text{min}, \quad \sin\theta = \left(n + \dfrac{1}{2}\right)\dfrac{\lambda}{D} \\ \text{max}, \quad \sin\theta = n\dfrac{\lambda}{D} \end{cases}$$

In this example, the distance D between the slits is 2λ. Maxima should then be found at $\theta = 0°$ and $30°$, while minima should appear at $\theta = 14.48°$ and $48.59°$.

Equation

For time-harmonic propagation, the wave equation turns into the Helmholtz equation:

$$-\nabla \cdot (\nabla u) - k^2 u = 0, \qquad k = \frac{2\pi}{\lambda}$$

Boundary Conditions

On the inflow and outflow boundaries (see Figure 1), absorbing boundary conditions apply. Let us briefly show how such conditions in their simplest form can be derived. First, assume the solution at the boundaries to be the sum of an incident plane wave, u_{in}, propagating in an arbitrary direction and a scattered wave, u_{sc}, propagating in the normal direction:

$$u = u_{\mathrm{in}} + u_{\mathrm{sc}} = u_0 e^{-i\mathbf{k} \cdot \mathbf{x}} + u_s e^{-ik\mathbf{n} \cdot \mathbf{x}} \tag{1}$$

Here $\mathbf{n}$ is the outward boundary normal vector and $k \equiv |\mathbf{k}|$. At the boundary of the modeling domain, Γ, we then have

$$\nabla u|_{\Gamma} = -i\mathbf{k}u_0 e^{-i\mathbf{k} \cdot \mathbf{x}} - ik\mathbf{n}u_s e^{-ik\mathbf{n} \cdot \mathbf{x}} = -ik\mathbf{n}u|_{\Gamma} - iu_0(\mathbf{k} - k\mathbf{n})e^{-i\mathbf{k} \cdot \mathbf{x}} \tag{2}$$

where Equation 1 was used in the second step. It follows that

$$\mathbf{n} \cdot \nabla u|_{\Gamma} = -iku|_{\Gamma} + iu_0(k - \mathbf{n} \cdot \mathbf{k})e^{-i\mathbf{k} \cdot \mathbf{x}} \tag{3}$$

There is no incident wave on the outflow boundary, which means that the second term on the right-hand side of Equation 3 vanishes. For the inflow boundary, make the further approximation that the incident wave propagates in the inward normal direction, so that $\mathbf{k} = -k\mathbf{n}$. We then arrive at the following boundary conditions:

$$\begin{aligned} \mathbf{n} \cdot \nabla u|_{\Gamma} &= -iku|_{\Gamma} + 2iku_0 e^{ik\mathbf{n} \cdot \mathbf{x}}, \quad \text{inflow} \\ \mathbf{n} \cdot \nabla u|_{\Gamma} &= -iku|_{\Gamma}, \qquad\qquad\qquad \text{outflow} \end{aligned} \tag{4}$$

In this model, these conditions are readily imposed using the Coefficient Form PDE interface’s Flux/Source condition. The default condition, Zero Flux, applies on the remaining boundaries.

Results and Discussion

The plot in Figure 2 shows the diffraction pattern clearly. The effect of discretization is that the numerical wavelength differs from λ, which results in a shift of the angles. You can correct for this effect by adjusting the value of k in the Helmholtz equation to the element size. These practices are important for modeling the interference effects of monochromatic waves.

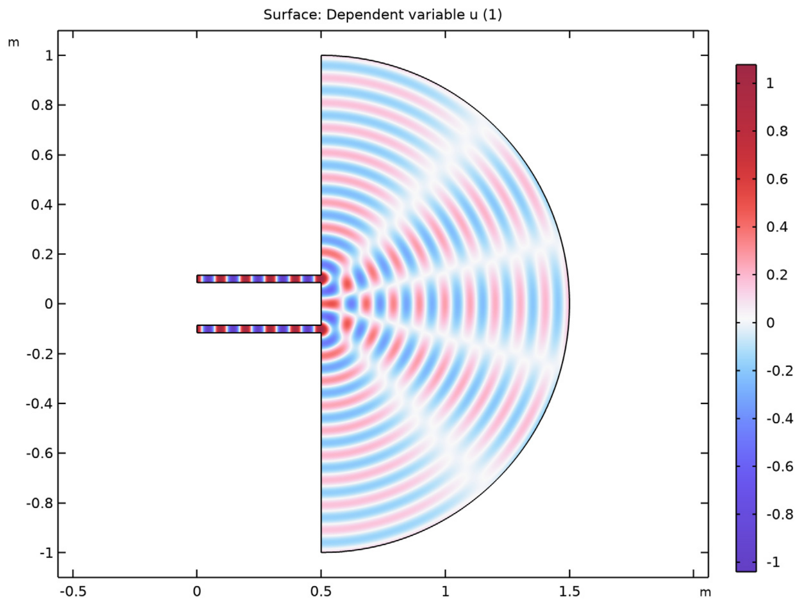


Figure 2: Diffraction pattern in the simulated double-slit experiment.

Application Library path: `COMSOL_Multiphysics/Equation_Based/diffraction_patterns`

Modeling Instructions

From the **File** menu, choose **New**.

NEW

In the **New** window, click **Model Wizard**.

MODEL WIZARD

1 In the **Model Wizard** window, click **2D**.

2 In the **Select Physics** tree, select **Mathematics>PDE Interfaces>Coefficient Form PDE (c)**.

3 Click **Add**.

4 Click **Study**.

5 In the **Select Study** tree, select **General Studies>Stationary**.

6 Click **Done**.

GLOBAL DEFINITIONS

Parameters 1

1 In the **Model Builder** window, under **Global Definitions** click **Parameters 1**.

2 In the **Settings** window for **Parameters**, locate the **Parameters** section.

3 In the table, enter the following settings:

Name	Expression	Value	Description
l	0.1[m]	0.1 m	Wavelength
k	2*pi[rad]/l	62.832 rad/m	Wave number
u0	1	1	Incident wave amplitude at inflow boundaries

GEOMETRY 1

Circle 1 (c1)

1 In the **Geometry** toolbar, click **Circle**.

2 In the **Settings** window for **Circle**, locate the **Size and Shape** section.

3 In the **Sector angle** text field, type 180.

4 Locate the **Position** section. In the **x** text field, type 0.5.

5 Locate the **Rotation Angle** section. In the **Rotation** text field, type -90.

6 Click **Build Selected**.

Rectangle 1 (r1)

1 In the **Geometry** toolbar, click **Rectangle**.

2 In the **Settings** window for **Rectangle**, locate the **Size and Shape** section.

3 In the **Width** text field, type 0.5.

4 In the **Height** text field, type 0.03.

5 Locate the **Position** section. In the **y** text field, type -0.015-0.1.

6 Click **Build Selected**.

Copy 1 (copy1)

1 In the **Geometry** toolbar, click **Transforms** and choose **Copy**.

2 Select the object **r1** only.

3 In the **Settings** window for **Copy**, locate the **Displacement** section.

4 In the **y** text field, type `0.2`.

5 Click **Build Selected**.

Union 1 (uni1)

1 In the **Geometry** toolbar, click **Booleans and Partitions** and choose **Union**.

2 Click in the **Graphics** window and then press Ctrl+A to select all objects.

3 In the **Settings** window for **Union**, locate the **Union** section.

4 Clear the **Keep interior boundaries** check box.

5 In the **Geometry** toolbar, click **Build All**.

6 Click the **Zoom Extents** button in the **Graphics** toolbar.

The model geometry is now complete. Next, turn to the physics settings.

COEFFICIENT FORM PDE (C)

Coefficient Form PDE 1

1 In the **Model Builder** window, under **Component 1 (comp1)>Coefficient Form PDE (c)** click **Coefficient Form PDE 1**.

2 In the **Settings** window for **Coefficient Form PDE**, locate the **Absorption Coefficient** section.

3 In the a text field, type `-k^2`.

4 Locate the **Source Term** section. In the f text field, type `0`.

Proceed to apply the boundary conditions Equation 4 at the inflow and outflow boundaries.

Flux/Source 1

1 In the **Physics** toolbar, click **Boundaries** and choose **Flux/Source**.

2 Select Boundaries 1 and 4 only.

3 In the **Settings** window for **Flux/Source**, locate the **Boundary Flux/Source** section.

4 In the g text field, type `-i*k*u+2*u0*i*k*exp(-i*k*x)`.

Note that the phase factor in this expression evaluates to 1 because x = 0. It has been included nevertheless for completeness.

Flux/Source 2

1 In the **Physics** toolbar, click **Boundaries** and choose **Flux/Source**.

2 Select Boundaries 11 and 12 only.

3 In the **Settings** window for **Flux/Source**, locate the **Boundary Flux/Source** section.

4 In the *g* text field, type `-i*k*u`.

MESH 1

Create a mesh with a maximum element size determined by the wavelength. As a rule of thumb, you need 5 elements per wavelength for quadratic elements (the default for the PDE interface) to fully resolve the wave.

Free Triangular 1

In the **Mesh** toolbar, click **Free Triangular**.

Size

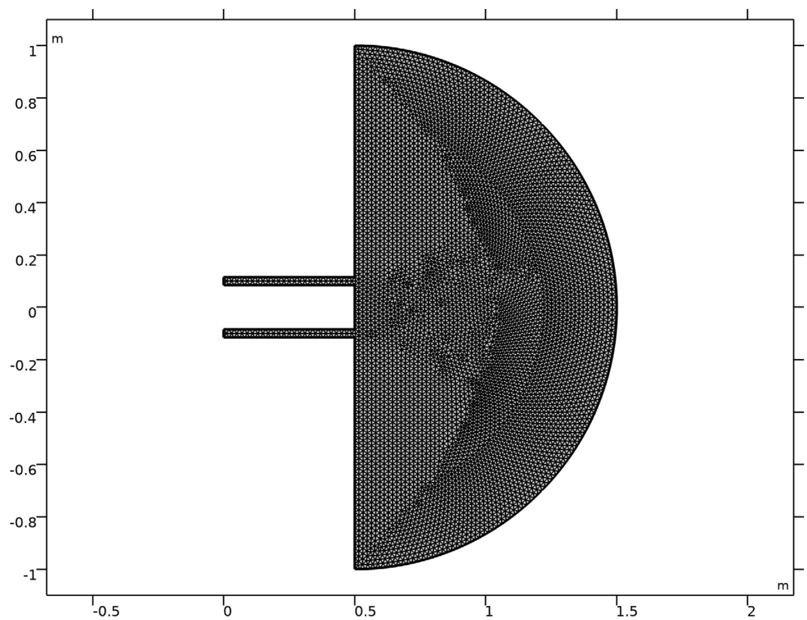
1 In the **Model Builder** window, click **Size**.

2 In the **Settings** window for **Size**, locate the **Element Size** section.

3 Click the **Custom** button.

4 Locate the **Element Size Parameters** section. In the **Maximum element size** text field, type `l/5`.

5 Click **Build All**.



STUDY 1

In the **Home** toolbar, click **Compute**.

RESULTS

2D Plot Group 1

To reproduce the plot shown in Figure 2, just change the color table.

Surface 1

1 In the **Model Builder** window, expand the **2D Plot Group 1** node, then click **Surface 1**.
2 In the **Settings** window for **Surface**, locate the **Coloring and Style** section.
3 From the **Color table** list, choose **WaveLight**.
4 In the **2D Plot Group 1** toolbar, click **Plot**.