

Creating Phase Envelopes by Using Equilibrium Calculations

Separation processes play an obvious key role in chemical plants across the world. Unit operations including flash distillation, where the resulting vapor-liquid equilibrium of the mixture is used to separate the species, is a basic building block in many such processes.

Consider, as an example, an equimolar liquid mixture of water and ethanol at atmospheric conditions. The pure species boiling points are 100°C for water and 78°C for ethanol. However, the resulting equimolar mixture boils at around 80°C, producing a vapor phase with a mole fraction for ethanol of 0.65. This shows that just by boiling a mixture at one composition, we can obtain an ethanol-enriched vapor mixture.

In this model, the phase envelope for a nonideal mixture of chloroform and methanol is studied. This mixture has a constant boiling point known as an azeotrope. Two diagrams will be produced, one temperature-mole fraction (T-x) diagram, and one enthalpy-mole fraction (h-x) diagram. These kind of diagrams are informative, and the latter was traditionally even used to solve distillation calculations graphically in the so called Ponchon–Savarit method. The h-x diagram shape provides information about the energy balances. If the dew and boiling lines are straight and parallel, the heat of vaporization is constant in terms of composition, which historically allowed for the more simplified McCabe—Thiele method to be used.

Model Definition

The basic principle for spontaneous separation processes, is that under equilibrium conditions the species concentration in each phase differs, but the chemical potential is equal. The number of quantities needed to determine the equilibrium state can be called the degrees of freedom of the system. The number of degrees of freedom for a closed system at equilibrium is given by Gibbs phase rule

$$F = C - P + 2$$

where C is the number of species in the system and P is the number of available phases. The number of degrees of freedom, F, corresponds to the number of independent intensive variables needed to fully determine the state of the system. For the case considered here, a binary mixture in two available phases, the number of degrees of freedom is equal to 2. This means that the system state is determined by prescribing two of the following intensive properties: the temperature, the pressure, the species compositions in the vapor phase, and the species compositions in the liquid phase.

In principle, flash calculations are straightforward and involve combining the vapor-liquidequations (VLE-equations) with the component mass balances, and in some cases the energy balance. Considering Gibbs' phase rule, some flash calculations are:

- Bubble point at given T
- Bubble point at given p
- Dew point at given T
- Dew point at given p
- Flash at given p and T
- Flash at given p and H ("standard" flash, for example, for a flash tank after a valve)
- Flash at given p and S (for example, for condensing turbine)
- Flash at given U and V (for example, for dynamic simulation of an adiabatic flash drum)

Below, equations for the bubble point are presented. At equilibrium

$$\hat{f}_i^v = \hat{f}_i^l \tag{1}$$

where $\hat{f}_{i,v}$, $\hat{f}_{i,v}$ are the partial fugacities of species i in mixture for vapor, v, and liquid phase, l, respectively. The partial fugacity of species i can be defined as

$$y_i \hat{\phi}_i P = x_i \gamma_i f_i^l \tag{2}$$

where y is the mole fraction of species i in the gas phase, $\hat{\phi}$ is the fugacity coefficients of species i in mixture, P is the total pressure, x is the mole fraction of species i in liquid phase, γ is the activity coefficient of species i, and f is the liquid fugacity of pure species at the equilibrium temperature and pressure which can be defined as

$$f_i^l = \phi_i^{\text{sat}} P_i^{\text{sat}} \exp\left(\frac{\hat{v}_i^l}{RT} (P - P_i^{\text{sat}})\right)$$
 (3)

where \hat{v}_i is the partial molar volume of species and P^{sat} is the species saturated vapor pressure. Inserting Equation 3 to Equation 2 gives

$$y_i \Phi_i P = x_i \gamma_i P_i^{\text{sat}} \tag{4}$$

where

$$\Phi_{i} = \left(\frac{\hat{\phi}_{i}}{\phi_{i}^{\text{sat}}}\right) \exp\left(\frac{\hat{v}_{i}^{l}}{RT}(P_{i}^{\text{sat}} - P)\right)$$
 (5)

The fugacity and activity coefficients are calculated based on the selected thermodynamic model. Here we use UNIFAC for the liquid-phase model and ideal gas for the gas-phase model. For more information about available thermodynamic models, see the section Thermodynamic Models in the Liquid & Gas Properties Module User's Guide.

Equation 4 provides the starting point for all equilibrium calculations. Considering the fact that the sum of all species mole fractions in each phase is unity, it can be rewritten as

$$P = \sum \left(\frac{x_i \gamma_i P_i^{\text{sat}}}{\Phi_i}\right) = \sum \left(\frac{\gamma_i P_i^{\text{sat}}}{y_i \Phi_i}\right)$$
 (6)

It is possible to rewrite Equation 4 in terms of the K factor, defined as the distribution of species in vapor to liquid phase such as

$$\sum (K_i x_i) = \sum \left(\frac{y_i}{K_i}\right) = 1 \tag{7}$$

where

$$K_i = \frac{\gamma_i P_i^{\text{sat}}}{\Phi_i P} = \frac{y_i}{x_i} \tag{8}$$

Note that $\Phi_i = \gamma_i = 1$ represents an ideal system with Raoult's law.

BUBBLE POINT CALCULATIONS

Consider the binary system at a given pressure or temperature and feed compositions. The bubble point of a liquid is the point where the liquid just starts to evaporate (boil), that is, when the first vapor bubble is formed. Therefore, at boiling point, feed compositions correspond to liquid-phase compositions. The bubble point is estimated by adjusting the pressure or temperature until the computed sum of the vapor mole fractions is unity, which means that Equation 6 and Equation 7 are satisfied.

In a similar way, it is possible to calculate the dew point of the system. Using the Thermodynamics feature in COMSOL, you can easily create functions under **Equilibrium Calculation**, and by modifying the input arguments you can create the different flashcalculation cases mentioned above, as is shown in the Modeling Instructions section.

HEAT-MOLE FRACTION DIAGRAM

The heat-mole fraction diagram requires the following data at constant pressure:

- Heat capacity of liquid as a function of temperature, composition, and pressure
- Heat of solution as a function of temperature and composition
- Heat of vaporization as a function of composition and temperature
- Boiling point as a function of pressure, composition, and temperature

The diagram at a given pressure can be based on an arbitrary reference state.

Results and Discussion

Figure 1 shows the temperature-mole fraction phase diagram for chloroform and methanol mixture with an azeotrope at 0.66 chloroform mole fraction with a boiling point of 326 K.

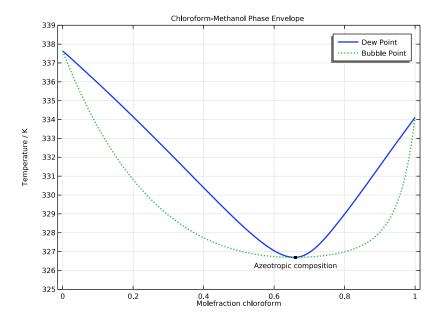


Figure 1: Temperature-mole fraction diagram for chloroform and methanol.

In the figure above, the blue line shows the dew point of the mixture. Above this line, the system consists solely of vapor. The green dashed line shows the bubble point temperature. Under this line, the system is liquid. The region between bubble-line and dew-line represents coexistence of both vapor and liquid phases.

Figure 2 shows the enthalpy-mole fraction diagram for chloroform and methanol. This diagram includes both material- and energy-balance calculations. One important observation is that the bubble and dew lines are not straight and parallel due to the nonideal behavior of the system. It suggests that the heat of vaporization of the system is not constant with respect to species composition.

Another important observation is the change in the slope of tie-lines (isotherms) between boiling and dew points. The azeotrope point lies between the tie lines where the slope changes sign. This diagram can be used for fractional distillation column design, a process which is known as the Panchon-Savarit graphical equilibrium stage method.

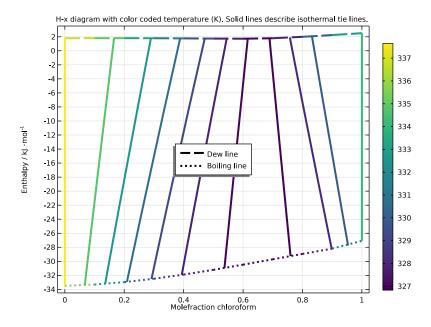


Figure 2: Enthalpy-mole fraction diagram for chloroform and methanol.

Application Library path: Liquid and Gas Properties Module/Tutorials/ phase envelope

Modeling Instructions

From the File menu, choose New.

NEW

GLOBAL DEFINITIONS

Parameters 1

- I In the Model Builder window, under Global Definitions click Parameters I.
- 2 In the Settings window for Parameters, locate the Parameters section.
- **3** In the table, enter the following settings:

Name	Expression	Value	Description
Р	1[atm]	1.0133E5 Pa	Pressure
xCh	.5	0.5	Mole fraction of chloroform

4 In the Physics toolbar, click Thermodynamics and choose Thermodynamic System.

SELECT SYSTEM

- I Go to the Select System window.
- 2 From the Phase list, choose Vapor-liquid.
- 3 Click **Next** in the window toolbar.

SELECT SPECIES

- I Go to the Select Species window.
- 2 In the Species list, select chloroform (67-66-3, CHCl3).
- 3 Click + Add Selected.
- 4 In the Species list, select methanol (67-56-1, CH40).
- 5 Click + Add Selected.
- 6 Click Next in the window toolbar.

SELECT THERMODYNAMIC MODEL

- I Go to the Select Thermodynamic Model window.
- 2 From the list, choose UNIFAC VLE.
- 3 Click Finish in the window toolbar.

Vapor-Liquid System I (pp I)

In the Model Builder window, under Global Definitions>Thermodynamics right-click Vapor-Liquid System I (pp I) and choose Equilibrium Calculation.

SELECT SPECIES

- I Go to the Select Species window.
- 2 In the list, select chloroform.
- 3 Click + Add Selected.
- 4 In the list, select methanol.
- 5 Click + Add Selected.
- 6 Click **Next** in the window toolbar.

EQUILIBRIUM SPECIFICATIONS

- I Go to the **Equilibrium Specifications** window.
- **2** From the Amount base unit list, choose mol.
- **3** Find the **Equilibrium conditions** subsection. From the **First condition** list, choose **Pressure**.
- 4 From the Second condition list, choose Phase fraction.
- 5 Click **Next** in the window toolbar.

EOUILIBRIUM FUNCTION OVERVIEW

- I Go to the Equilibrium Function Overview window.
- 2 Click Finish in the window toolbar.

GLOBAL DEFINITIONS

Variables 1

- I In the Model Builder window, right-click Global Definitions and choose Variables.
- 2 In the Settings window for Variables, locate the Variables section.
- **3** In the table, enter the following settings:

Name	Expression	Unit	Description
xM	1-xCh		Mole fraction of methanol

Name	Expression	Unit	Description
T_DewPoint	Flash1_1_Temperature(P,1,xCh,xM)	K	Dew Point
T_BubblePoint	Flash1_1_Temperature(P,0,xCh,xM)	K	Bubble Point

ADD STUDY

- I In the Home toolbar, click Add Study to open the Add Study window.
- 2 Go to the Add Study window.
- 3 Find the Studies subsection. In the Select Study tree, select Preset Studies for Selected Physics Interfaces>Stationary.
- 4 Click Add Study in the window toolbar.
- 5 In the Home toolbar, click Add Study to close the Add Study window.

STUDY I - DEW POINT AND BOILING POINT CURVES

- I In the Model Builder window, click Study I.
- 2 In the Settings window for Study, type Study 1 Dew Point and Boiling Point Curves in the Label text field.

Step 1: Stationary

- I In the Model Builder window, under Study I Dew Point and Boiling Point Curves click Step 1: Stationary.
- 2 In the Settings window for Stationary, click to expand the Study Extensions section.
- 3 Select the Auxiliary sweep check box.
- 4 From the Sweep type list, choose All combinations.
- 5 Click + Add.
- **6** In the table, click to select the cell at row number 1 and column number 1.
- 7 In the table, enter the following settings:

Parameter name	Parameter value list	Parameter unit
xCh (Mole fraction of chloroform)	range(0,0.01,1)	

8 In the Home toolbar, click **Compute**.

RESULTS

T-x Diagram

- I In the Model Builder window, expand the Results node.
- 2 Right-click Results and choose ID Plot Group.
- 3 In the Settings window for ID Plot Group, type T-x Diagram in the Label text field.
- **4** Locate the **Plot Settings** section. Select the **x-axis label** check box.
- **5** In the associated text field, type Molefraction chloroform.
- **6** Select the **y-axis label** check box.
- 7 In the associated text field, type Temperature / K.

Global I

- I Right-click T-x Diagram and choose Global.
- 2 In the Settings window for Global, locate the y-Axis Data section.
- **3** In the table, enter the following settings:

Expression	Unit	Description
T_DewPoint	K	Dew Point
T_BubblePoint	K	Bubble Point

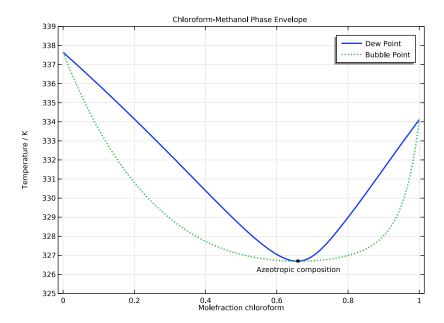
- 4 Click to expand the **Title** section. From the **Title type** list, choose **Manual**.
- 5 In the Title text area, type Chloroform-Methanol Phase Envelope.
- 6 Click to expand the Coloring and Style section. Find the Line style subsection. From the Line list, choose Cycle.
- 7 Set the Width value to 2.
- 8 In the T-x Diagram toolbar, click Plot.

Annotation I

- I In the Model Builder window, right-click T-x Diagram and choose Annotation.
- 2 In the Settings window for Annotation, locate the Annotation section.
- 3 In the **Text** text field, type Azeotropic composition.
- **4** Locate the **Position** section. In the **y** text field, type 326.7.
- 5 In the x text field, type 0.66.
- 6 Locate the Coloring and Style section. From the Anchor point list, choose Upper middle.
- 7 In the T-x Diagram toolbar, click Plot.

T-x Diagram

- I In the Model Builder window, click T-x Diagram.
- 2 In the Settings window for ID Plot Group, locate the Axis section.
- 3 Select the Manual axis limits check box.
- 4 In the y minimum text field, type 325.
- 5 In the y maximum text field, type 339.
- 6 In the T-x Diagram toolbar, click Plot.



GLOBAL DEFINITIONS

Parameters 1

- I In the Model Builder window, under Global Definitions click Parameters I.
- 2 In the Settings window for Parameters, locate the Parameters section.
- 3 In the table, enter the following settings:

Name	Expression	Value	Description
n	0	0	Vapor phase fraction

Variables I

I In the Model Builder window, click Variables I.

- 2 In the Settings window for Variables, locate the Variables section.
- **3** In the table, enter the following settings:

Name	Expression	Unit	Description
T_iso	0.5*(T_DewPoint+ T_BubblePoint)	К	Isothermal tie-line

ADD COMPONENT

In the **Home** toolbar, click **Add Component** and choose **OD**.

ADD PHYSICS

- I In the Home toolbar, click Add Physics to open the Add Physics window.
- 2 Go to the Add Physics window.
- 3 In the tree, select Mathematics>ODE and DAE Interfaces>Global ODEs and DAEs (ge).
- 4 Find the Physics interfaces in study subsection. In the table, clear the Solve check box for Study I - Dew Point and Boiling Point Curves.
- 5 Click Add to Component I in the window toolbar.
- 6 In the Home toolbar, click Add Physics to close the Add Physics window.

GLOBAL ODES AND DAES (GE)

xCh at T_iso for Phase n

- I In the Model Builder window, expand the Component I (compl)> Global ODEs and DAEs (ge)>Global Equations I node, then click Global Equations I.
- 2 In the Settings window for Global Equations, locate the Global Equations section.
- **3** In the table, enter the following settings:

Name	f(u,ut,utt,t) (1)	Initial value (u_0) (1)	Initial value (u_t0) (1/s)	Description
xCh_at_n	T_iso- Flash1_1_Tempera ture(P, n, xCh_at_n[mol], (1- xCh_at_n)[mol])	xCh	0	

- 4 Locate the Units section. Click

 Define Source Term Unit.
- 5 Click Select Source Term Quantity.
- 6 In the Physical Quantity dialog box, type temperature in the text field.

- 7 In the tree, select General>Temperature (K).
- 8 Click OK.
- 9 In the Settings window for Global Equations, type xCh at T iso for Phase n in the Label text field.

Vapor-Liquid System I (pp I)

In the Model Builder window, under Global Definitions>Thermodynamics>Vapor-Liquid System I (ppI) right-click Mixture and choose Mixture Property.

SELECT PROPERTIES

- I Go to the Select Properties window.
- 2 From the Amount base unit list, choose mol.
- 3 In the list, select Enthalpy (J/mol).
- 4 Click + Add Selected.
- 5 Click **Next** in the window toolbar.

SELECT PHASE

- I Go to the Select Phase window.
- 2 Click Next in the window toolbar.

SELECT SPECIES

- I Go to the Select Species window.
- 2 Click the Mole fraction button.
- 3 Click Add All.
- 4 Click **Next** in the window toolbar.

MIXTURE PROPERTY OVERVIEW

- I Go to the Mixture Property Overview window.
- 2 Click Finish in the window toolbar.

Enthalpy I (Enthalpy_chloroform_methanol_Vapor I I, Enthalby chloroform methanol Vapor I I Dtemperature, Enthalby chloroform methanol Vapor II Doressure)

I In the Model Builder window, under Global Definitions>Thermodynamics>Vapor-

Liquid System I (ppI)>Mixture>Vapor click

Enthalpy I (Enthalpy_chloroform_methanol_Vapor I I,

Enthalpy chloroform methanol Vapor II Dtemperature,

Enthalpy_chloroform_methanol_Vapor I I_Dpressure).

2 In the Settings window for Mixture Property, type hv in the Function name text field.

Vapor-Liquid System I (pp I)

In the Model Builder window, under Global Definitions>Thermodynamics>Vapor-Liquid System I (ppI) right-click Mixture and choose Mixture Property.

SELECT PROPERTIES

- I Go to the Select Properties window.
- 2 From the Amount base unit list, choose mol.
- 3 In the list, select Enthalpy (I/mol).
- 4 Click + Add Selected.
- 5 Click Next in the window toolbar.

SELECT PHASE

- I Go to the Select Phase window.
- 2 From the list, choose Liquid.
- 3 Click **Next** in the window toolbar.

SELECT SPECIES

- I Go to the Select Species window.
- 2 Click the Mole fraction button.
- 3 Click Add All.
- 4 Click **Next** in the window toolbar.

MIXTURE PROPERTY OVERVIEW

- I Go to the Mixture Property Overview window.
- 2 Click Finish in the window toolbar.

Enthalpy 2 (Enthalpy_chloroform_methanol_Liquid I 2, Enthalby chloroform methanol Liquid 12 Dtemperature, Enthalby chloroform methanol Liquid 12 Dpressure)

I In the Model Builder window, under Global Definitions>Thermodynamics>Vapor-

Liquid System I (ppI)>Mixture>Liquid click

Enthalpy 2 (Enthalpy_chloroform_methanol_Liquid12,

Enthalpy_chloroform_methanol_Liquid I2_Dtemperature,

Enthalpy_chloroform_methanol_Liquid I2_Dpressure).

2 In the Settings window for Mixture Property, type hl in the Function name text field.

Parameters 1

- I In the Model Builder window, under Global Definitions click Parameters I.
- 2 In the Settings window for Parameters, locate the Parameters section.
- **3** In the table, enter the following settings:

Name	Expression	Value	Description
Tref	273.15[K]	273.15 K	Reference temperature

DEFINITIONS

Variables 2

- I In the Model Builder window, under Component I (compl) right-click Definitions and choose Variables.
- 2 In the Settings window for Variables, locate the Variables section.
- **3** In the table, enter the following settings:

Name	Expression	Unit	Description
xM_at_n	1-comp1.xCh_at_n		
hVapor	hv(T_iso,P, comp1.xCh_at_n, xM_at_n)	J/mol	Enthalpy of Vapor
hLiquid	hl(T_iso,P, comp1.xCh_at_n, xM_at_n)	J/mol	Enthalpy of Liquid
hMix	n*hVapor+(1-n)*hLiquid	J/mol	Enthalpy of Vapor/ Liquid mixture

ADD STUDY

- I In the Home toolbar, click Add Study to open the Add Study window.
- 2 Go to the Add Study window.
- 3 Find the Studies subsection. In the Select Study tree, select General Studies>Stationary.
- 4 Right-click and choose Add Study.
- 5 In the Home toolbar, click Add Study to close the Add Study window.

STUDY 2

Parametric Sweep

- I In the Study toolbar, click Parametric Sweep.
- 2 In the Settings window for Parametric Sweep, locate the Study Settings section.
- 3 Click + Add.
- **4** In the table, enter the following settings:

Parameter name	Parameter value list	Parameter unit
xCh (Mole fraction of chloroform)	1e-6 range(0.1,0.1,0.9) 1-1e-6	

Step 1: Stationary

- I In the Model Builder window, click Step I: Stationary.
- 2 In the Settings window for Stationary, locate the Study Extensions section.
- 3 Select the Auxiliary sweep check box.
- 4 Click + Add.
- **5** In the table, click to select the cell at row number 1 and column number 3.
- **6** In the table, enter the following settings:

Parameter name	Parameter value list	Parameter unit
n (Vapor phase fraction)	range(0,.1,1)	

- 7 In the **Study** toolbar, click **Compute**.
- 8 In the Model Builder window, click Study 2.
- 9 In the Settings window for Study, type Study 2 Isotherm Curves in the Label text field.

RESULTS

I In the Model Builder window, click Results.

- 2 In the Settings window for Results, locate the Update of Results section.
- 3 Select the Only plot when requested check box.

ID Plot Group 2

In the Home toolbar, click 連 Add Plot Group and choose ID Plot Group.

Global I

Right-click ID Plot Group 2 and choose Global.

H-x Diagram

- I In the Settings window for ID Plot Group, type H-x Diagram in the Label text field.
- 2 Locate the Legend section. From the Position list, choose Center.

Dew Line

- I In the Model Builder window, under Results>H-x Diagram click Global I.
- 2 In the Settings window for Global, type Dew Line in the Label text field.
- 3 Locate the Data section. From the Dataset list, choose Study 2 Isotherm Curves/ Parametric Solutions I (sol3).
- 4 From the Parameter selection (n) list, choose Last.
- **5** Locate the **y-Axis Data** section. In the table, enter the following settings:

Expression	Unit	Description
<pre>hv(T_DewPoint,P,xCh,xM)</pre>	kJ/mol	

- 6 Locate the x-Axis Data section. From the Axis source data list, choose xCh.
- 7 Click to expand the Title section. From the Title type list, choose None.
- 8 Locate the Coloring and Style section. Find the Line style subsection. From the Line list, choose Dashed.
- **9** Set the **Width** value to **3**.
- 10 Click to expand the Legends section. From the Legends list, choose Manual.
- II In the table, enter the following settings:

Legends Dew line

Boiling Line

- I In the Model Builder window, right-click H-x Diagram and choose Global.
- 2 In the Settings window for Global, type Boiling Line in the Label text field.

- 3 Locate the Data section. From the Dataset list, choose Study 2 Isotherm Curves/ Parametric Solutions I (sol3).
- 4 From the Parameter selection (n) list, choose First.
- **5** Locate the **y-Axis Data** section. In the table, enter the following settings:

Expression	Unit	Description
hl(T_BubblePoint,P,xCh,1-xCh)	kJ/mol	

- 6 Locate the x-Axis Data section. From the Axis source data list, choose xCh.
- 7 Locate the Title section. From the Title type list, choose None.
- 8 Locate the Coloring and Style section. Find the Line style subsection. From the Line list, choose **Dotted**.
- 9 Set the Width value to 3.
- 10 Locate the Legends section. From the Legends list, choose Manual.
- II In the table, enter the following settings:

Legends	
Boiling	line

H-x Diagram

- I In the Model Builder window, click H-x Diagram.
- 2 In the Settings window for ID Plot Group, locate the Plot Settings section.
- 3 Select the x-axis label check box.
- **4** In the associated text field, type Molefraction chloroform.
- **5** Select the **y-axis label** check box.
- 6 In the associated text field, type Enthalpy / kJ \cdot mol⁻¹.

Isotherms

- I Right-click H-x Diagram and choose Global.
- 2 In the Settings window for Global, type Isotherms in the Label text field.
- 3 Locate the Data section. From the Dataset list, choose Study 2 Isotherm Curves/ Parametric Solutions I (sol3).
- 4 Locate the y-Axis Data section. In the table, enter the following settings:

Expression	Unit	Description
hMix	kJ/mol	

- 5 Locate the Title section. From the Title type list, choose Manual.
- 6 In the Title text area, type H-x diagram with color coded temperature (K). Solid lines describe isothermal tie lines..
- 7 Locate the x-Axis Data section. From the Parameter list, choose Expression.
- **8** In the **Expression** text field, type xCh_at_n.
- 9 Locate the Coloring and Style section. Set the Width value to 3.
- 10 Locate the Legends section. Clear the Show legends check box.

Color Expression 1

- I In the Model Builder window, right-click Dew Line and choose Color Expression.
- 2 In the Settings window for Color Expression, locate the Expression section.
- 3 In the Expression text field, type T DewPoint.
- 4 Locate the Coloring and Style section. From the Color table list, choose Viridis.
- **5** Clear the **Color legend** check box.
- 6 Right-click Color Expression I and choose Copy.

Color Expression 1

- I In the Model Builder window, right-click Boiling Line and choose Paste Color Expression.
- 2 In the Settings window for Color Expression, locate the Expression section.
- **3** In the **Expression** text field, type T_BubblePoint.

Color Expression 1

- I In the Model Builder window, right-click Isotherms and choose Paste Color Expression.
- 2 In the Settings window for Color Expression, locate the Expression section.
- **3** In the **Expression** text field, type T_iso.
- 4 Click to expand the Title section. Locate the Coloring and Style section. Select the Color legend check box.

5 In the H-x Diagram toolbar, click Plot.

