Created in COMSOL Multiphysics 6.0

Radiative Heat Transfer in Finite Cylindrical Media

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This application uses the discrete ordinates method (DOM) to solve a 3D radiative transfer problem in an emitting, absorbing, and linearly anisotropic-scattering finite cylindrical medium. Using the S6 quadrature of DOM leads to accurate results, which are needed in combined modes of heat transfer. The calculated incident radiation and heat fluxes agree well with published results obtained by transformed integral methods (see Ref. 1). In addition, The DOM formulation of the Heat Transfer Module easily handles the effects of boundary emission and reflection.

Introduction

There are numerous engineering applications of radiative transfer in absorbing, emitting, and anisotropically scattering media with variable radiation properties. Examples include, among others, coal-fired combustion systems, light-weight fibrous insulations, and heat transfer systems containing small scattering particles. Furthermore, the efficiency of radiative transfer depends on the boundary conditions, for example, the temperature and the emissivity of the surrounding walls, and the target where heat transfer is desired. Studies have shown that radiative transfer is highly sensitive to the wall emissivity. In this study you build a validation model representing a cylinder with homogeneous walls. Then you go on to consider walls with space-dependent emissivity and investigate the effects of albedo and scattering.

Model Definition

In this tutorial you validate the DOM formulation in COMSOL Multiphysics by examining three benchmark cases. You investigate the method's efficiency through parametric analyses by changing the single-scattering albedo, wall emissivity, and linear function. In particular, in the final case, you approximate the scattering phase function by a linear function reflecting highly backward, isotropic, and highly forward scattering.

The model geometry, shown in Figure 1, is a cylinder of radius $R = 0.5$ m and height $L = 1$ m.

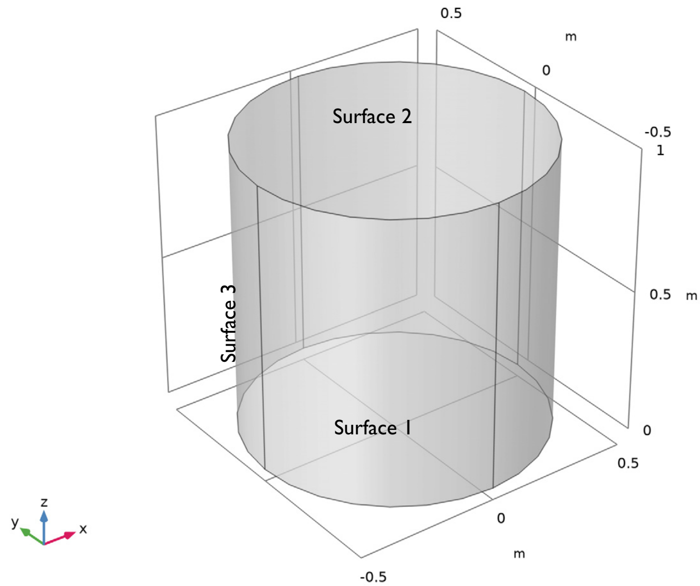


Figure 1: Schematic diagram of the physical model.

These examples use the S6 discrete ordinate method for predicting the heat flux on the enclosure side walls and the incident radiation distribution inside the domain.

VERIFICATION CASE

For the initial study, assume cold boundaries, that is, $I_i = 0$ where i = 1, 2, 3 refers to the surface index in Figure 1. Furthermore, assume that the walls diffusively reflect radiation, that is, $\varepsilon_i = 0.5$ for i = 1, 2, 3. The medium is at a uniform temperature T such that the blackbody radiation intensity in an arbitrary direction per unit area and solid angle $I_b(T) = \sigma T^4/\pi$ equals 1 W/(m^2·sr).

WALLS WITH VARIABLE EMISSIVITY

Two cases are computed for comparison purposes. These 3D cases represent opaque partial side wall diffuse emission and/or reflection. In both cases, the radiosity on the side walls (Surface 3) varies with the angular coordinate along the full height of the finite cylinder according to $\varepsilon_3 = (1 - y/R)/2$. Both cases also have cold walls at the cylinder top (Surface 1) and bottom (Surface 2).

Case 1 has isotropic scattering function and compares results for different albedos. Case 2 has constant albedo and compares results with highly forward, isotropic, and highly backward scattering function parameterized by the Legendre coefficient a_1.

TABLE 1: NONSTANDARD TEST CASES.

CASE	MEDIUM PROPERTIES)
1	albedo = 0.1, 0.5, 0.9 $a_1 = 0$
2	albedo = 0.5 $a_1 = -0.99, 0, 0.99$

THERMAL ANALYSIS

The discrete ordinates method relies on the discrete representation of the directional dependence of the radiation intensity. It involves solving the radiative equation for a set of directions that span the full solid angle range of 4π around a point in space.

The radiation transport equation (RTE) for this type of configuration can be written as

$$\Omega \cdot \nabla I(\Omega, s) = \kappa I_b(T) - \beta I(\Omega, s) + \frac{\sigma_s}{4\pi}\int_0^{4\pi} I(\Omega, \Omega')\phi(\Omega, \Omega')\partial\Omega'$$

where

- $I(\Omega, s)$ is the radiation intensity at a given position s in the direction Ω
- T is the temperature
- κ, β, and σ_s are absorption, extinction, and scattering coefficients, respectively
- $I_b(T)$ is the blackbody radiation intensity
- $\phi(\Omega, \Omega') = 1 + a_1\mu_0$ where $\mu_0 = \Omega \cdot \Omega'$ is the cosine of the scattering angle.

The boundary intensities at the cylinder walls are given by the sum of the effective emitted intensity and the reflected incident intensities in the given direction:

$$I_{bnd}(\Omega) = \varepsilon_w I_b(T) + \frac{\rho_d}{\pi}q_{out} \qquad \text{for all } \Omega \text{ such that } \mathbf{n} \cdot \Omega < 0$$

where

- ε_w is the surface emissivity, which is in the range $[0, 1]$
- $\rho_d = 1 - \varepsilon_w$ is the diffusive reflectivity
- $\mathbf{n}$ is the outward normal vector

- q_{out} is the heat flux striking the wall:

$$q_{\text{out}} = \int_{\mathbf{n} \cdot \Omega' > 0} (\mathbf{n} \cdot \Omega') I(\Omega', s) \partial\Omega'$$

The above equations can be discretized in Cartesian coordinates for monochromatic or gray radiation as

$$\Omega_i \cdot \nabla I_i = \kappa I_b(T) - \beta I_i + \frac{\sigma_s}{4\pi} \int_0^{4\pi} I(\Omega, \Omega') \phi(\Omega, \Omega') \partial\Omega'$$

The Sn approximation of the RTE in the direction i can be expressed as

$$\Omega_i \cdot \nabla I_i = \kappa I_b(T) - \beta I_i + \frac{\sigma_s}{4\pi} \sum_{j=1}^{n} w_j I_j \phi(\Omega_j, \Omega_i)$$

For a discrete direction, Ω_i, the values of $\Omega_{i,1}$, $\Omega_{i,2}$, and $\Omega_{i,3}$ define the direction cosines of Ω_i obeying the condition ${\Omega_{i,1}}^2 + {\Omega_{i,2}}^2 + {\Omega_{i,3}}^2 = 1$. The index j in the above equation denotes the direction of incoming radiation contributing to the direction Ω_i.

For a diffuse reflecting surface on a wall boundary, the boundary condition equation is transformed as

$$I_{i,\text{bnd}} = \varepsilon_w I_b(T) + \frac{\rho_d}{\pi} \sum_{\mathbf{n} \cdot \Omega_j > 0} w_j I_j \mathbf{n} \cdot \Omega_j \qquad \text{for all } \Omega_i \text{ such that } \mathbf{n} \cdot \Omega_i < 0$$

Results and Discussion

The results demonstrate that the DOM procedure for the prediction of radiation is an elegant and accurate method for modeling multidimensional radiative heat transfer in cylindrical geometries.

VERIFICATION CASE

This case treats the effects of the scattering albedo on the incident radiation and heat fluxes. Figure 2 shows the distribution of the net heat flux $q_{r,\text{net}}(R, 0, z)$ versus axial optical thickness. There is good overall agreement of the present work with published literature results (Ref. 2, Ref. 3, and Ref. 4).

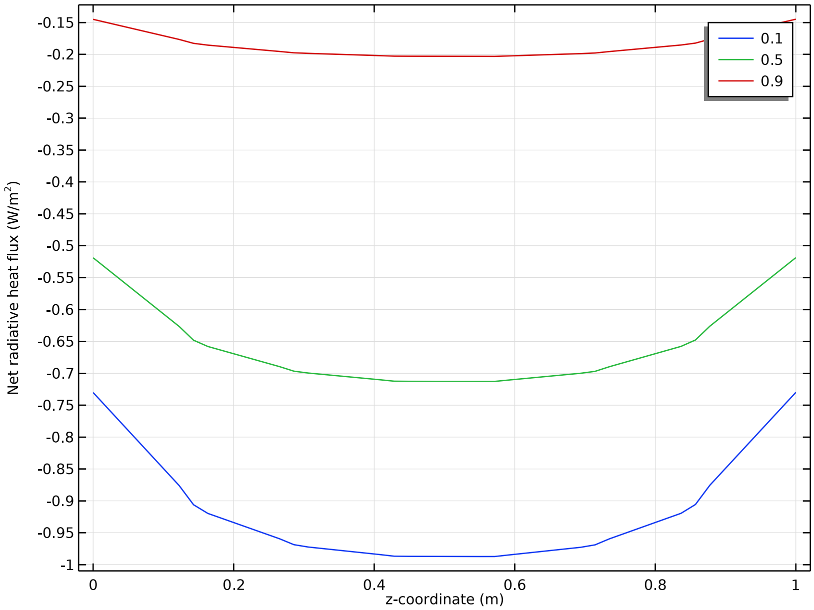


Figure 2: The effects of the scattering albedo on the radial heat flux $q_{\mathrm{r,\,net}}(R, 0, z)$*; for a hot cylindrical medium enclosed by cold walls,* $\varepsilon_1 = \varepsilon_2 = \varepsilon_3 = 0.5$.

The effects of albedo on the distribution of centerline incident radiation in axial direction are shown in Figure 3. The incident radiation is symmetric with respect to $z = L/2$ plane and decrease with increasing scattering albedo. Furthermore, results become more uniform with larger scattering albedo.

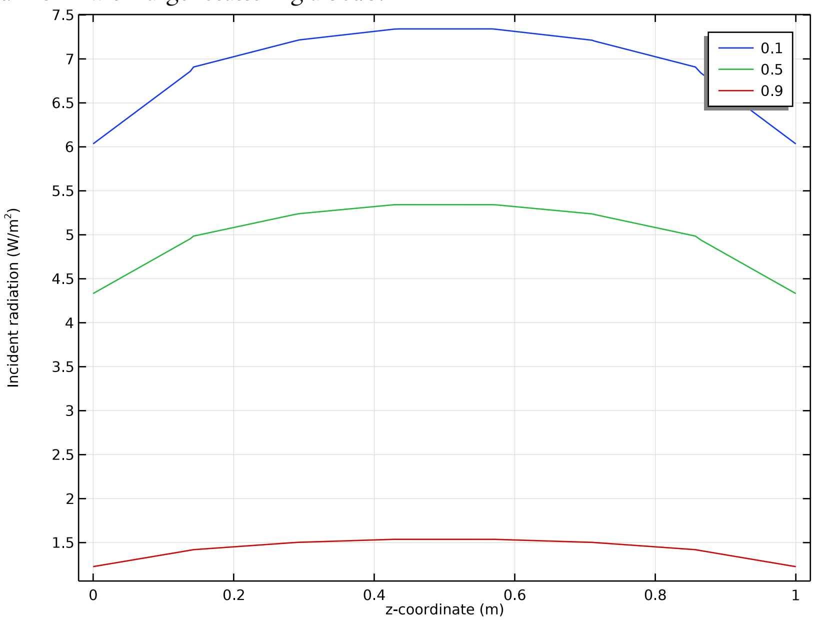


Figure 3: The effects of the scattering albedo on the distribution of centerline incident radiation $G(0, 0, z)$ *for a hot cylindrical medium enclosed by cold walls,* $\varepsilon_1 = \varepsilon_2 = \varepsilon_3 = 0.5$.

Figure 4 displays the distributions of the incident radiation across the midplane radius $G(x, 0, L/2)$ with respect to normalized optical thickness x/R.

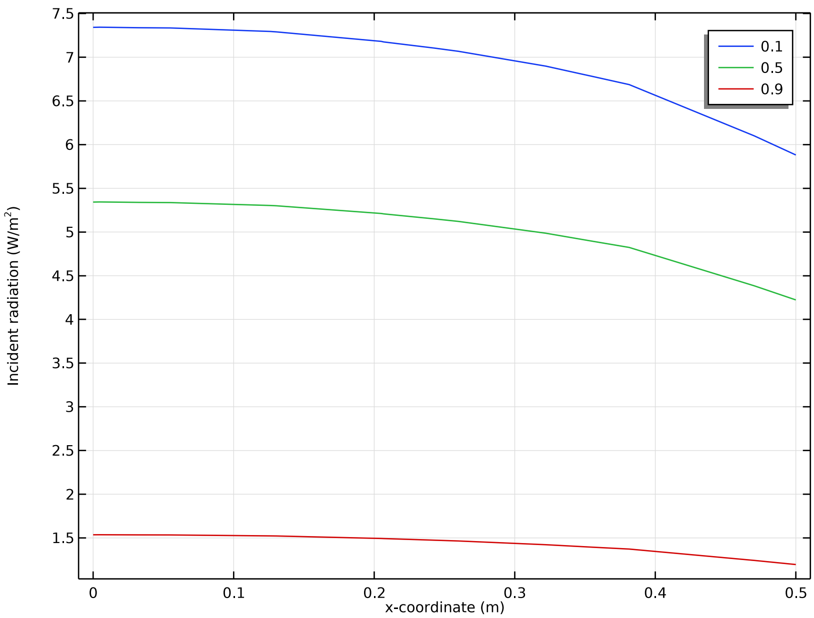


Figure 4: The effects of the scattering albedo on the distributions of the incident radiation $G(x, 0, L/2)$ *with respect to normalized optical thickness* x/R *for a hot cylindrical medium enclosed by cold walls,* $\varepsilon_1 = \varepsilon_2 = \varepsilon_3 = 0.5$.

WALLS WITH VARIABLE EMISSIVITY

The incident radiation at the midplane $z = L/2$ at the radial position $R/2$ is shown in Figure 5. At the side surface $R/2$ distance from the cylinder axis, $G(R, 0, L/2)$ changes with the azimuthal angle. The changes in scattering albedo are also illustrated. The smallest albedo makes the biggest change around the azimuthal angle.

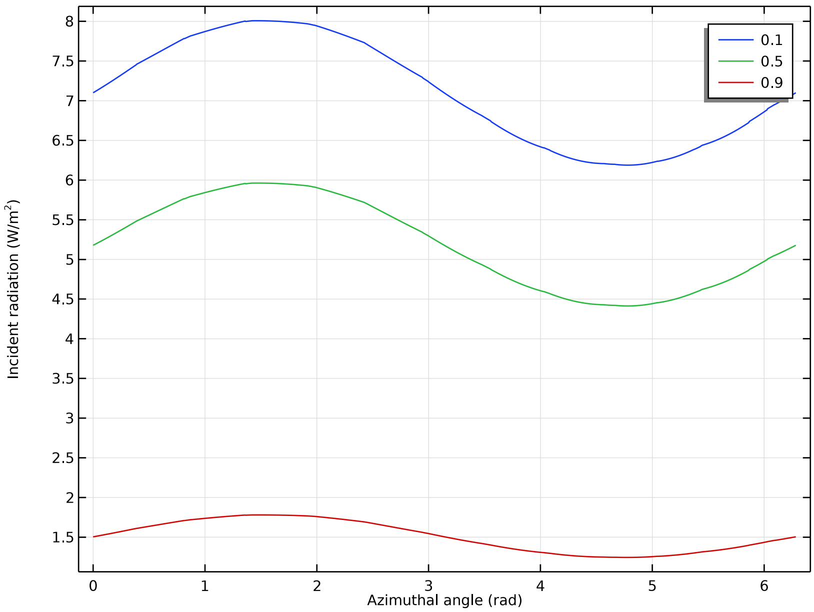


Figure 5: The effects of scattering albedo on the distribution of incident radiation at midplane $z = L/2$ *at* $R/2$ *distance from the cylinder axis with respect to an azimuthal coordinate for Case 1.*

Figure 6 shows the effect of a nonzero linear anisotropic scattering coefficient a_1. As expected, the differences between isotropic scattering, forward scattering, and backward scattering are most accentuated far from the boundary.

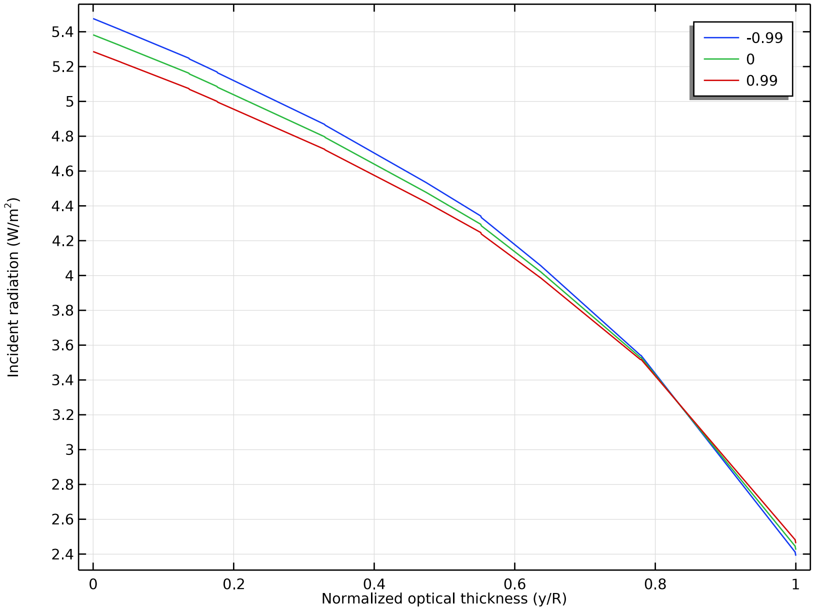


Figure 6: The effects of linear anisotropic scattering coefficient a_1 on the distribution of incident radiation $G(0, y, L/2)$ with respect to normalized optical thickness $-y/R$ for Case 2.

References

1. X.L. Chen and W.H. Sutton, "Radiative Transfer in Finite Cylindrical Media Using Transformed Integral Equations," *J. Quantitative Spectroscopy and Radiative Transfer*, vol. 77, pp. 233–271, 2003.

2. X. Chen, *Transformed Integral Equations of Radiative Transfer and Combined Convection-radiation Heat Transfer Enhancement with Porous Insert*, Doctoral Thesis, University of Oklahoma, 2003.

3. S.T. Thynell and M.N. Ozisik, "Radiation Transfer in Absorbing, Emitting, Isotropically Scattering, Homogeneous Cylindrical Media," *J. Quantitative Spectroscopy and Radiative Transfer*, vol. 38, no. 6, pp. 413–426, 1987.

4. H.Y. Li, M.N. Ozisik, and J.R. Tsai, “Two-dimensional radiation in a cylinder with spatially varying albedo,” *J. Thermophysics and Heat Transfer*, vol. 6, no. 1, pp. 180–182, 1992.

Application Library path: `Heat_Transfer_Module/Verification_Examples/cylinder_participating_media`

Modeling Instructions — Validation Case

From the **File** menu, choose **New**.

NEW

In the **New** window, click **Model Wizard**.

MODEL WIZARD

1. In the **Model Wizard** window, click **3D**.
2. In the **Select Physics** tree, select **Heat Transfer>Radiation>Radiation in Participating Media (rpm)**.
3. Click **Add**.
4. Click **Study**.
5. In the **Select Study** tree, select **General Studies>Stationary**.
6. Click **Done**.

GLOBAL DEFINITIONS

Parameters 1

1. In the **Model Builder** window, under **Global Definitions** click **Parameters 1**.
2. In the **Settings** window for **Parameters**, locate the **Parameters** section.
3. In the table, enter the following settings:

Name	Expression	Value	Description
T0	(1[W/m^2]*pi/sigma_const)^(1/4)	86.275 K	Body temperature
Tw	0[K]	0 K	Wall temperature
ew	0.5	0.5	Wall emissivity

Name	Expression	Value	Description
omega	0.9	0.9	Single-scattering albedo
sigma_s	omega	0.9	Scattering coefficient
kappa	sigma_s*(1/omega-1)	0.1	Absorption coefficient
R	0.5[m]	0.5 m	Cylinder radius
L	1[m]	1 m	Cylinder length

GEOMETRY 1

Cylinder 1 (cyl1)

1 In the **Geometry** toolbar, click **Cylinder**.

2 In the **Settings** window for **Cylinder**, locate the **Size and Shape** section.

3 In the **Radius** text field, type R.

4 In the **Height** text field, type L.

5 Click **Build All Objects**.

MATERIALS

Add a material to specify the absorption and scattering coefficients inside the cylinder.

Domain

1 In the **Materials** toolbar, click **Blank Material**.

2 In the **Settings** window for **Material**, type Domain in the **Label** text field.

3 Locate the **Material Contents** section. In the table, enter the following settings:

Property	Variable	Value	Unit	Property group
Absorption coefficient	kappaR	kappa	1/m	Basic
Scattering coefficient	sigmaS	sigma_s	1/m	Basic

Analogously, specify the emissivity of the walls using a material.

Walls

1 In the **Materials** toolbar, click **Blank Material**.

2 In the **Settings** window for **Material**, type Walls in the **Label** text field.

3 Locate the **Geometric Entity Selection** section. From the **Geometric entity level** list, choose **Boundary**.

4 From the **Selection** list, choose **All boundaries**.

5 Locate the **Material Contents** section. In the table, enter the following settings:

Property	Variable	Value	Unit	Property group
Surface emissivity	epsilon_rad	ew	1	Basic

RADIATION IN PARTICIPATING MEDIA (RPM)

1 In the **Model Builder** window, under **Component 1 (comp1)** click **Radiation in Participating Media (rpm)**.

2 In the **Settings** window for **Radiation in Participating Media**, locate the **Participating Media Settings** section.

3 Find the **Radiation settings** subsection. From the P_{index} list, choose **0.3**.

With this performance index value, solving the model requires roughly 2 GB of RAM. If your computer has less available memory than that, try a value between 0.5 and 1.

4 From the **Discrete ordinates method** list, choose **S6 (48 directions)**.

Participating Medium 1

1 In the **Model Builder** window, expand the **Radiation in Participating Media (rpm)** node, then click **Participating Medium 1**.

2 In the **Settings** window for **Participating Medium**, locate the **Model Input** section.

3 In the T text field, type `T0`.

Opaque Surface 1

1 In the **Model Builder** window, click **Opaque Surface 1**.

2 In the **Settings** window for **Opaque Surface**, locate the **Model Input** section.

3 In the T text field, type `Tw`.

MESH 1

1 In the **Model Builder** window, under **Component 1 (comp1)** click **Mesh 1**.

2 In the **Settings** window for **Mesh**, locate the **Physics-Controlled Mesh** section.

3 From the **Element size** list, choose **Coarse**.

Free Triangular 1

1 In the **Mesh** toolbar, click **Boundary** and choose **Free Triangular**.

2 Select Boundary 4 only.

Swept 1

1 In the **Mesh** toolbar, click **Swept**.

2 In the **Settings** window for **Swept**, click **Build All**.

STUDY 1

Parametric Sweep

1 In the **Study** toolbar, click **Parametric Sweep**.

2 In the **Settings** window for **Parametric Sweep**, locate the **Study Settings** section.

3 Click **Add**.

4 In the table, enter the following settings:

Parameter name	**Parameter value list**	**Parameter unit**
omega (Single-scattering albedo)	`0.1 0.5 0.9`	

If your computer has 6GB of RAM or more, it is possible to greatly decrease the solution time by using a fully coupled solver.

Solution 1 (sol1)

1 In the **Study** toolbar, click **Show Default Solver**.

The tolerance is decreased to reach better accuracy.

2 In the **Model Builder** window, expand the **Solution 1 (sol1)** node, then click **Stationary Solver 1**.

3 In the **Settings** window for **Stationary Solver**, locate the **General** section.

4 In the **Relative tolerance** text field, type `1e-4`.

5 Right-click **Study 1>Solver Configurations>Solution 1 (sol1)>Stationary Solver 1** and choose **Fully Coupled**.

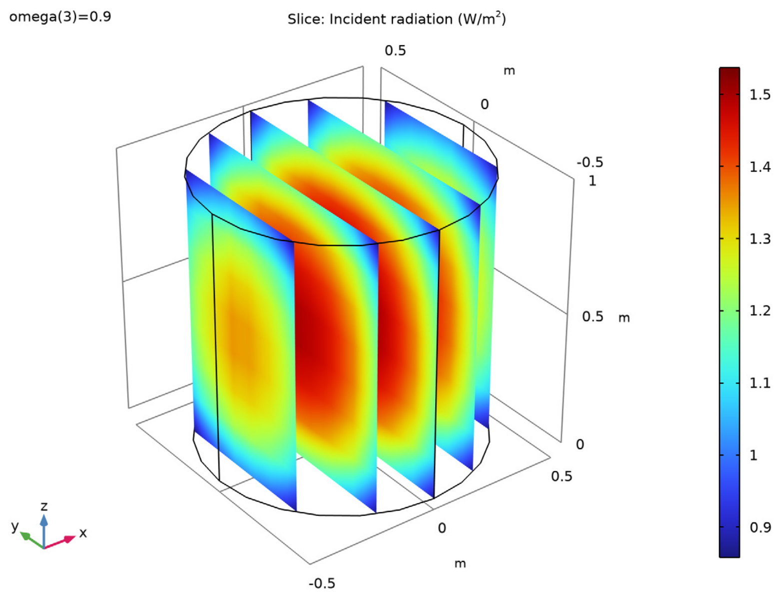
6 Right-click **Study 1>Solver Configurations>Solution 1 (sol1)>Stationary Solver 1>AMG, radiation variables** and choose **Enable**.

7 In the **Study** toolbar, click **Compute**.

RESULTS

Incident Radiation (rpm)

The first default plot shows the incident radiation distribution in 3D and the second default plot represents the net radiative heat flux distribution in 3D, see figure below.



Add a new 1D Plot to represent the net radiative heat flux along the z-coordinate and compare with Figure 2.

Net Radiative Heat Flux vs. z, 1D

1 In the **Home** toolbar, click **Add Plot Group** and choose **1D Plot Group**.

2 In the **Settings** window for **1D Plot Group**, type `Net Radiative Heat Flux vs. z, 1D` in the **Label** text field.

Line Graph 1

1 In the **Net Radiative Heat Flux vs. z, 1D** toolbar, click **Line Graph**.

2 Click the **Transparency** button in the **Graphics** toolbar.

3 Select Edge 12 only.

4 Click the **Transparency** button in the **Graphics** toolbar again to return to the original state.

5 In the **Settings** window for **Line Graph**, click **Replace Expression** in the upper-right corner of the **y-Axis Data** section. From the menu, choose **Component 1 (comp1)>Radiation in Participating Media>Boundary fluxes>rpm.qr_net - Net radiative heat flux - W/m²**.

6 Click **Replace Expression** in the upper-right corner of the **x-Axis Data** section. From the menu, choose **Component 1 (comp1)>Geometry>Coordinate>z - z-coordinate**.

7 Click to expand the **Legends** section. Select the **Show legends** check box.

Finish the plot by adjusting the title and axis labels.

Net Radiative Heat Flux vs. z, 1D

1 In the **Model Builder** window, click **Net Radiative Heat Flux vs. z, 1D**.

2 In the **Settings** window for **1D Plot Group**, click to expand the **Title** section.

3 From the **Title type** list, choose **None**.

4 Locate the **Plot Settings** section. Select the **x-axis label** check box.

5 Select the **y-axis label** check box.

6 In the **Net Radiative Heat Flux vs. z, 1D** toolbar, click **Plot**.

Cut Line 3D 1

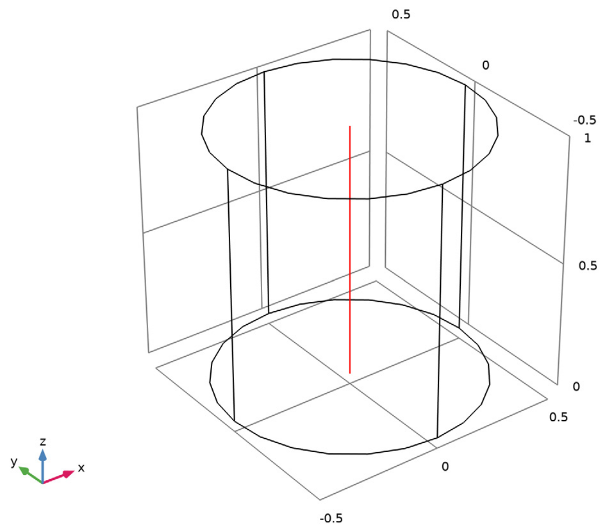
1 In the **Results** toolbar, click **Cut Line 3D**.

2 In the **Settings** window for **Cut Line 3D**, locate the **Line Data** section.

3 In row **Point 2**, set **X** to 0, and **z** to L.

4 Click **Plot**.

The Graphics window shows the location of the line in the model geometry.



Add a new 1D Plot to represent the incident radiation along the z-coordinate and compare with Figure 3.

Incident Radiation vs. z, 1D

1 In the **Results** toolbar, click **1D Plot Group**.

2 In the **Settings** window for **1D Plot Group**, type `Incident Radiation vs. z, 1D` in the **Label** text field.

3 Locate the **Data** section. From the **Dataset** list, choose **Cut Line 3D 1**.

Line Graph 1

1 In the **Incident Radiation vs. z, 1D** toolbar, click **Line Graph**.

2 In the **Settings** window for **Line Graph**, click **Replace Expression** in the upper-right corner of the **x-Axis Data** section. From the menu, choose **Component 1 (comp1)>Geometry>Coordinate>z - z-coordinate**.

3 Locate the **Legends** section. Select the **Show legends** check box.

Incident Radiation vs. z, 1D

1 In the **Model Builder** window, click **Incident Radiation vs. z, 1D**.

2 In the **Settings** window for **1D Plot Group**, locate the **Title** section.

3 From the **Title type** list, choose **None**.

4 Locate the **Plot Settings** section. Select the **x-axis label** check box.

5 Select the **y-axis label** check box.

6 In the **Incident Radiation vs. z, 1D** toolbar, click **Plot**.

Cut Line 3D 2

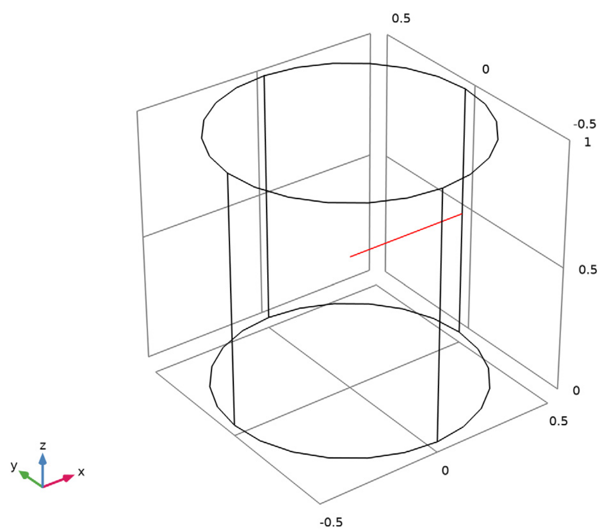
1 In the **Results** toolbar, click **Cut Line 3D**.

2 In the **Settings** window for **Cut Line 3D**, locate the **Line Data** section.

3 In row **Point 1**, set **Z** to `L/2`.

4 In row **Point 2**, set **X** to `R`, and **z** to `L/2`.

5 Click **Plot**.



Add a new 1D Plot to represent the incident radiation along the x-coordinate and compare with Figure 4.

Incident Radiation vs. x, 1D

1 Right-click **Incident Radiation vs. z, 1D** and choose **Duplicate**.

2 In the **Settings** window for **1D Plot Group**, type `Incident Radiation vs. x, 1D` in the **Label** text field.

3 Locate the **Data** section. From the **Dataset** list, choose **Cut Line 3D 2**.

4 Locate the **Plot Settings** section. In the **x-axis label** text field, type `x-coordinate (m)`.

Line Graph 1

1 In the **Model Builder** window, expand the **Incident Radiation vs. x, 1D** node, then click **Line Graph 1**.

2 In the **Settings** window for **Line Graph**, click **Replace Expression** in the upper-right corner of the **x-Axis Data** section. From the menu, choose **Component 1 (comp1)>Geometry>Coordinate>x - x-coordinate**.

3 In the **Incident Radiation vs. x, 1D** toolbar, click **Plot**.

Modeling Instructions — Case 1

RADIATION IN PARTICIPATING MEDIA (RPM)

Opaque Surface 2

1 In the **Physics** toolbar, click **Boundaries** and choose **Opaque Surface**.

2 In the **Settings** window for **Opaque Surface**, locate the **Model Input** section.

3 In the T text field, type `Tw`.

4 Select Boundaries 1, 2, 5, and 6 only.

These are the vertical wall boundaries. To reach all of them, you can rotate the geometry or click either the **Transparency** button or the **Wireframe Rendering** button in the **Graphics** toolbar.

5 Locate the **Surface Radiative Properties** section. From the ε list, choose **User defined**. In the associated text field, type `ew*(1-y/R)`.

Now, disable **Opaque Surface 2** in **Study 1** to be able to re run the same **Study 1** configuration.

STUDY 1

Step 1: Stationary

1 In the **Model Builder** window, expand the **Study 1** node, then click **Step 1: Stationary**.

2 In the **Settings** window for **Stationary**, locate the **Physics and Variables Selection** section.

3 Select the **Modify model configuration for study step** check box.

4 In the tree, select **Component 1 (Comp1)>Radiation in Participating Media (Rpm)>Opaque Surface 2**.

5 Click **Disable**.

ADD STUDY

1 In the **Home** toolbar, click **Add Study** to open the **Add Study** window.
2 Go to the **Add Study** window.
3 Find the **Studies** subsection. In the **Select Study** tree, select **General Studies>Stationary**.
4 Click **Add Study** in the window toolbar.

STUDY 2

Step 1: Stationary

In the **Home** toolbar, click **Add Study** to close the **Add Study** window.

Parametric Sweep

1 In the **Study** toolbar, click **Parametric Sweep**.
2 In the **Settings** window for **Parametric Sweep**, locate the **Study Settings** section.
3 Click **Add**.
4 In the table, enter the following settings:

Parameter name	Parameter value list	Parameter unit
omega (Single-scattering albedo)	0.1 0.5 0.9	

Solution 2 (sol2)

In the **Study** toolbar, click **Show Default Solver**.

Solver Configurations

In the **Model Builder** window, expand the **Study 2>Solver Configurations** node.

Solution 2 (sol2)

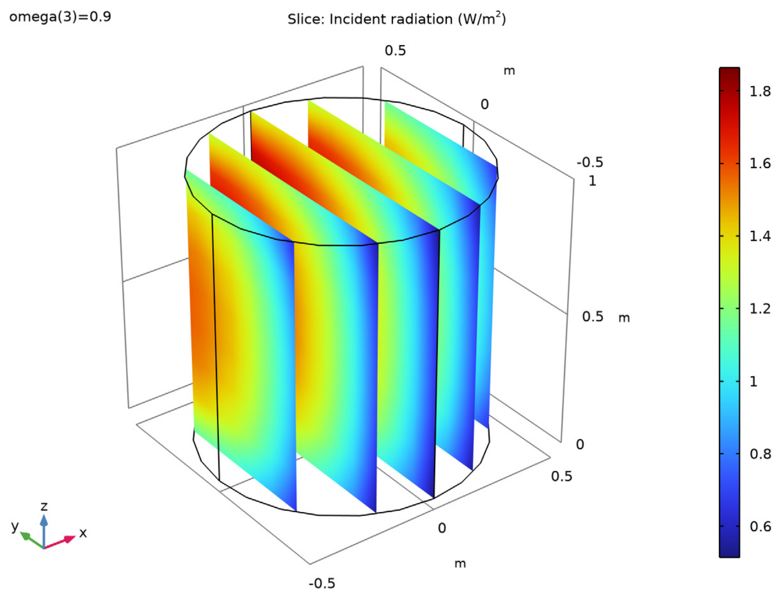
1 In the **Model Builder** window, expand the **Study 2>Solver Configurations>Solution 2 (sol2)** node, then click **Dependent Variables 1**.
2 In the **Settings** window for **Dependent Variables**, locate the **General** section.
3 From the **Defined by study step** list, choose **User defined**.
4 Locate the **Initial Values of Variables Solved For** section. From the **Method** list, choose **Solution**.
5 From the **Solution** list, choose **Solution 1 (sol1)**.
6 In the **Model Builder** window, expand the **Solution 2 (sol2)** node.
7 Right-click **Stationary Solver 1** and choose **Fully Coupled**.
8 Right-click **AMG, radiation variables** and choose **Enable**.

9 In the **Study** toolbar, click **Compute**.

RESULTS

Incident Radiation (rpm) 1

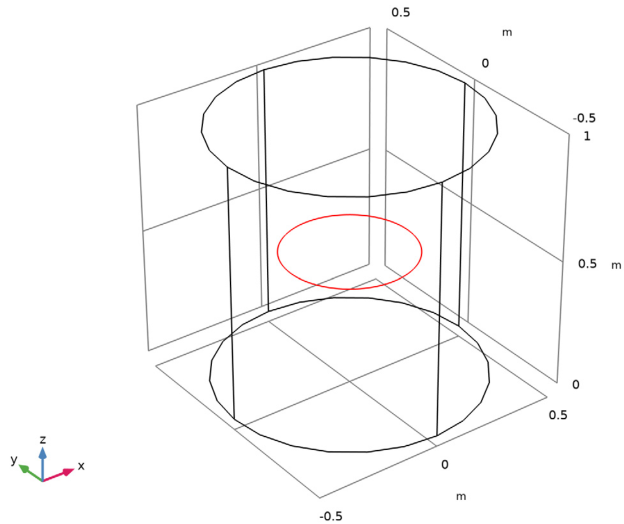
The first default plot shows the incident radiation distribution in 3D and the second default plot represents the net radiative heat flux distribution in 3D, see figure below.



Parameterized Curve 3D 1

1. In the **Results** toolbar, click **More Datasets** and choose **Parameterized Curve 3D**.
2. In the **Settings** window for **Parameterized Curve 3D**, locate the **Data** section.
3. From the **Dataset** list, choose **Study 2/Solution 2 (sol2)**.
4. Locate the **Parameter** section. In the **Name** text field, type `phi`.
5. In the **Maximum** text field, type `2*pi`.
6. Locate the **Expressions** section. In the **x** text field, type `R*cos(phi)/2`.
7. In the **y** text field, type `R*sin(phi)/2`.
8. In the **z** text field, type `L/2`.

9 Click **Plot**.



Add a new 1D Plot to represent the incident radiation in function of the azimuthal angle and compare with Figure 5.

Incident Radiation vs. Azimuthal Angle, 1D

1 In the **Results** toolbar, click **1D Plot Group**.

2 In the **Settings** window for **1D Plot Group**, type `Incident Radiation vs. Azimuthal Angle, 1D` in the **Label** text field.

3 Locate the **Data** section. From the **Dataset** list, choose **Parameterized Curve 3D 1**.

Line Graph 1

1 In the **Incident Radiation vs. Azimuthal Angle, 1D** toolbar, click **Line Graph**.

2 In the **Settings** window for **Line Graph**, locate the **Legends** section.

3 Select the **Show legends** check box.

Incident Radiation vs. Azimuthal Angle, 1D

1 In the **Model Builder** window, click **Incident Radiation vs. Azimuthal Angle, 1D**.

2 In the **Settings** window for **1D Plot Group**, locate the **Title** section.

3 From the **Title type** list, choose **None**.

4 Locate the **Plot Settings** section. Select the **x-axis label** check box.

5 In the associated text field, type `Azimuthal angle (rad)`.

6 Select the **y-axis label** check box.

Line Graph 1

1 In the **Model Builder** window, click **Line Graph 1**.

2 In the **Settings** window for **Line Graph**, locate the **x-Axis Data** section.

3 From the **Parameter** list, choose **Expression**.

4 In the **Expression** text field, type `phi`.

5 In the **Incident Radiation vs. Azimuthal Angle, 1D** toolbar, click **Plot**.

Modeling Instructions — Case 2

GLOBAL DEFINITIONS

Parameters 1

For case 2, you need to modify the value of the single-scattering albedo that you defined previously and add a parameter for the Legendre coefficient `a1` in the scattering phase function.

1 In the **Model Builder** window, under **Global Definitions** click **Parameters 1**.

2 In the **Settings** window for **Parameters**, locate the **Parameters** section.

3 In the table, enter the following settings:

Name	**Expression**	**Value**	**Description**
omega	0.5	0.5	Single-scattering albedo
a1	0.99	0.99	Legendre coefficient

RADIATION IN PARTICIPATING MEDIA (RPM)

Participating Medium 1

1 In the **Model Builder** window, under **Component 1 (comp1)>**
Radiation in Participating Media (rpm) click **Participating Medium 1**.

2 In the **Settings** window for **Participating Medium**, locate the **Scattering** section.

3 From the **Scattering type** list, choose **Linear anisotropic**.

4 In the a_1 text field, type `a1`.

ADD STUDY

1 In the **Home** toolbar, click **Add Study** to open the **Add Study** window.

2 Go to the **Add Study** window.

3 Find the **Studies** subsection. In the **Select Study** tree, select **General Studies>Stationary**.

4 Click **Add Study** in the window toolbar.

STUDY 3

Step 1: Stationary

In the **Home** toolbar, click **Add Study** to close the **Add Study** window.

Parametric Sweep

1 In the **Study** toolbar, click **Parametric Sweep**.

2 In the **Settings** window for **Parametric Sweep**, locate the **Study Settings** section.

3 Click **Add**.

4 In the table, enter the following settings:

Parameter name	Parameter value list	Parameter unit
a1 (Legendre coefficient)	-0.99 0 0.99	

Note that the value $a_1 = 0$ gives the same solution as for $\omega = 0.5$ in case 1.

Solution 3 (sol3)

In the **Study** toolbar, click **Show Default Solver**.

Solver Configurations

In the **Model Builder** window, expand the **Study 3>Solver Configurations** node.

Solution 3 (sol3)

1 In the **Model Builder** window, expand the **Study 3>Solver Configurations>Solution 3 (sol3)** node, then click **Dependent Variables 1**.

2 In the **Settings** window for **Dependent Variables**, locate the **General** section.

3 From the **Defined by study step** list, choose **User defined**.

4 Locate the **Initial Values of Variables Solved For** section. From the **Method** list, choose **Solution**.

5 From the **Solution** list, choose **Solution 2 (sol2)**.

6 In the **Model Builder** window, expand the **Study 3>Solver Configurations>Solution 3 (sol3)>Stationary Solver 1** node.

7 In the **Model Builder** window, expand the **Solution 3 (sol3)** node.

8 Right-click **Stationary Solver 1** and choose **Fully Coupled**.

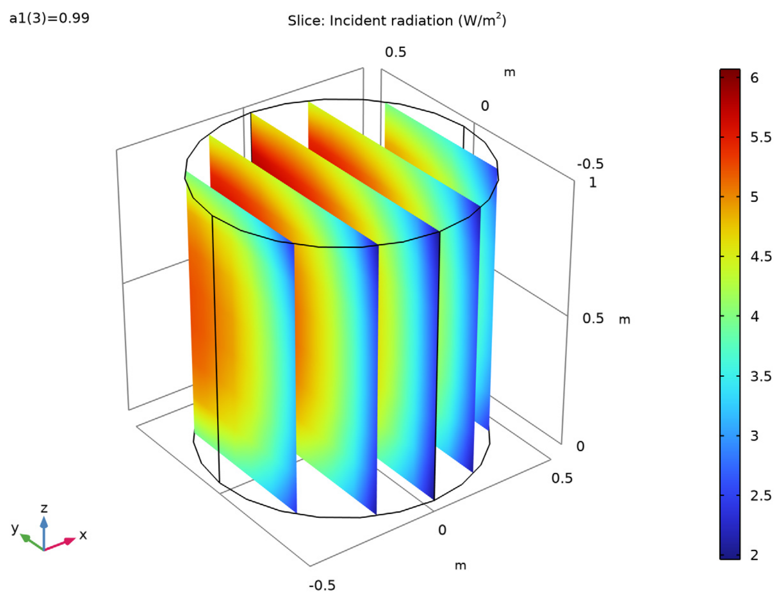
9 Right-click **AMG, radiation variables** and choose **Enable**.

10 In the **Study** toolbar, click **Compute**.

RESULTS

Incident Radiation (rpm) 2

The first default plot shows the incident radiation distribution in 3D and the second default plot represents the net radiative heat flux distribution in 3D, see figure below.



Parameterized Curve 3D 2

1 In the **Results** toolbar, click **More Datasets** and choose **Parameterized Curve 3D**.

2 In the **Settings** window for **Parameterized Curve 3D**, locate the **Data** section.

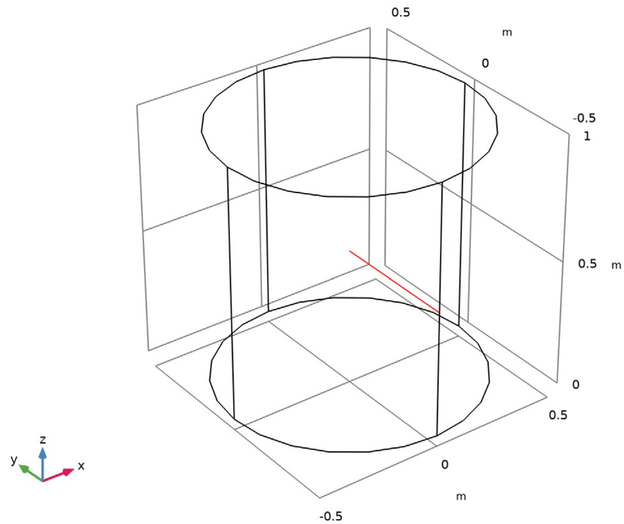
3 From the **Dataset** list, choose **Study 3/Solution 3 (sol3)**.

4 Locate the **Expressions** section. In the **y** text field, type `-s*R`.

5 In the **z** text field, type `L/2`.

With the above definition, $s = -y/R$ equals the optical thickness along the negative y-axis for the given constant values of x and z.

6 Click **Plot**.



Add a new 1D Plot to represent the incident radiation in function of the normalized optical thickness and compare with Figure 6.

Incident Radiation vs. Normalized Optical Thickness, 1D

1 In the **Results** toolbar, click **1D Plot Group**.

2 In the **Settings** window for **1D Plot Group**, type `Incident Radiation vs. Normalized Optical Thickness, 1D` in the **Label** text field.

3 Locate the **Data** section. From the **Dataset** list, choose **Parameterized Curve 3D 2**.

Line Graph 1

1 In the **Incident Radiation vs. Normalized Optical Thickness, 1D** toolbar, click **Line Graph**.

2 In the **Settings** window for **Line Graph**, locate the **x-Axis Data** section.

3 From the **Parameter** list, choose **Expression**.

4 In the **Expression** text field, type `s`.

5 Locate the **Legends** section. Select the **Show legends** check box.

Incident Radiation vs. Normalized Optical Thickness, 1D

1 In the **Model Builder** window, click **Incident Radiation vs. Normalized Optical Thickness, 1D**.

2 In the **Settings** window for **1D Plot Group**, locate the **Title** section.

3 From the **Title type** list, choose **None**.

4 Locate the **Plot Settings** section. Select the **x-axis label** check box.

5 In the associated text field, type `Normalized optical thickness (y/R)`.

6 Select the **y-axis label** check box.

7 In the **Incident Radiation vs. Normalized Optical Thickness, 1D** toolbar, click **Plot**.