



Ion Funnel

Introduction

An electrodynamic ion funnel provides an efficient means of transferring ions from regions of high pressure to high vacuum. The ion funnel can couple devices that generally operate at pressures of different orders of magnitude, such as ion mobility spectrometers and mass spectrometers, allowing mixtures of ionized gases to be separated and analyzed while minimizing losses. For this reason, ion funnels can be used to improve instrument sensitivity in a wide variety of applications, such as the analysis of complex biological molecules or the detection of explosives.

This example uses the Charged Particle Tracing interface to model the movement of ions through an electrodynamic ion funnel. The Electric Force feature is used to apply DC and RF potentials to guide ions through the funnel, while the Elastic Collision Force feature is used to model collisions with background gas molecules.

Note: This application requires the Particle Tracing Module.

Model Definition

The ion funnel is a converging series of insulated ring-shaped electrodes, each subjected to an RF potential, with adjacent electrodes out of phase. The RF potential confines ions radially while a DC bias guides the ions toward successively narrower electrodes. The superposition of the DC and RF fields focuses the ions and offsets the effects of thermal dispersion and coulombic repulsion.

The ion funnel contains a buffer gas of neutral argon at 1 Torr, which is assumed to follow a Maxwellian velocity distribution:

$$f(v_i) = \sqrt{\frac{m_p}{2\pi k_B T_0}} \exp\left(-\frac{m_p v_i^2}{2k_B T_0}\right)$$

where

- v_i (SI unit: m/s) is the i th velocity component,
- m_p (SI unit: kg) is the molecular mass,
- T_0 (SI unit: K) is the temperature, and
- $k = 1.3806488 \times 10^{-23}$ J/K is the Boltzmann constant.

In this example, the background gas molecules have molar mass 0.04 kg/mol, for a molecular mass of 6.6422×10^{-26} kg. The temperature is assumed to be 293.15 K.

The number density of the gas can be computed using the ideal gas law,

$$p = \frac{nRT}{N_A}$$

where $R = 8.3144621$ J/(mol·K) is the universal gas constant and $N_A = 6.02214129 \times 10^{23}$ 1/mol is the Avogadro constant. For a pressure of 1 Torr this yields $n = 3.294 \times 10^{22}$ atoms/m³.

The interaction of ions with the background gas is modeled using the **Collisions** node with an **Elastic** subnode. At each time step taken by the solver, for each model particle a background gas particle is sampled at random from the Maxwellian distribution. The frequency of elastic collisions is then computed from the collision cross section, background gas number density, and the relative velocity of the model particle with respect to the randomly sampled background gas molecule:

$$\nu = N_d \sigma |\mathbf{v}_p - \mathbf{v}_g|$$

where the collision cross section σ (SI unit: m²) is usually a function of the particle kinetic energy. The collision probability is then computed as a function of the collision frequency and the time step size:

$$P = 1 - \exp(-\nu \Delta t)$$

The model uses three physics interfaces: Electrostatics, Electric Currents, and Charged Particle Tracing. The Electrostatics and Electric Currents interfaces are used to compute the DC and AC fields, respectively. These fields are then coupled to the Charged Particle Tracing interface, which models the motion of the ions due to the electric fields and interaction with neutral particles in the background gas. Interactions between the ions are neglected. In order to accurately model collisions of ions with the background gas, the average time between elastic collisions should be significantly greater than the maximum time step taken by the solver. Strict or manual time stepping is recommended.

Results and Discussion

The electric potential in the ion funnel is plotted in [Figure 1](#). The gradual DC bias guides positive ions from the larger end of the funnel to the smaller end. The AC voltage, which is out of phase between adjacent electrodes, causes the gradient of the electric potential to become very large close to the electrodes, keeping the ions confined within the funnel.

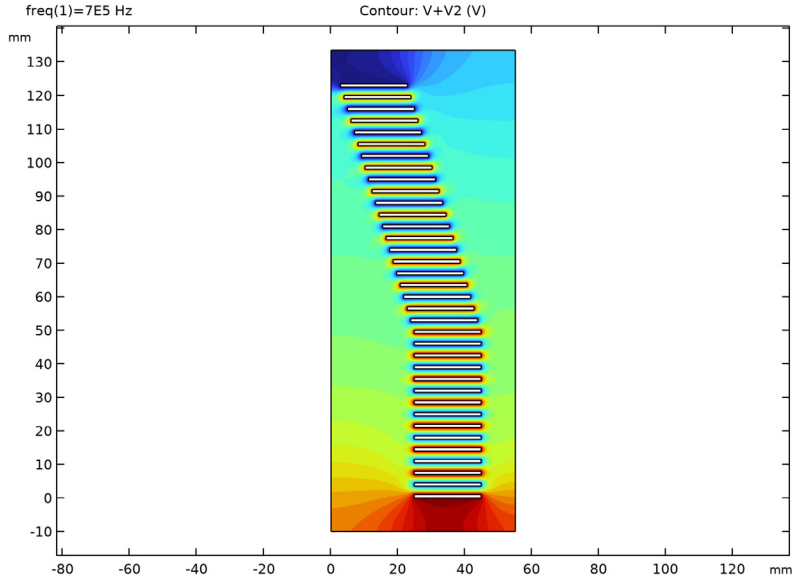


Figure 1: The combined DC and AC potential is plotted in the ion funnel at time $t=0$.

The ion trajectories are plotted in [Figure 2](#). Because the ions are confined to an area of reduced size, they can be transported to another device, such as a mass spectrometer, more efficiently.

The x - and y -coordinates of the particles at the narrow end of the funnel are plotted in [Figure 3](#). Although the ions are released along the positive x -axis, they are uniformly distributed around the z -axis by the time they reach the end of the funnel. Because the **Collisions** node uses random numbers to determine whether a collision takes place at each time step, the results may be slightly different from those shown in [Figure 2](#) and [Figure 3](#).

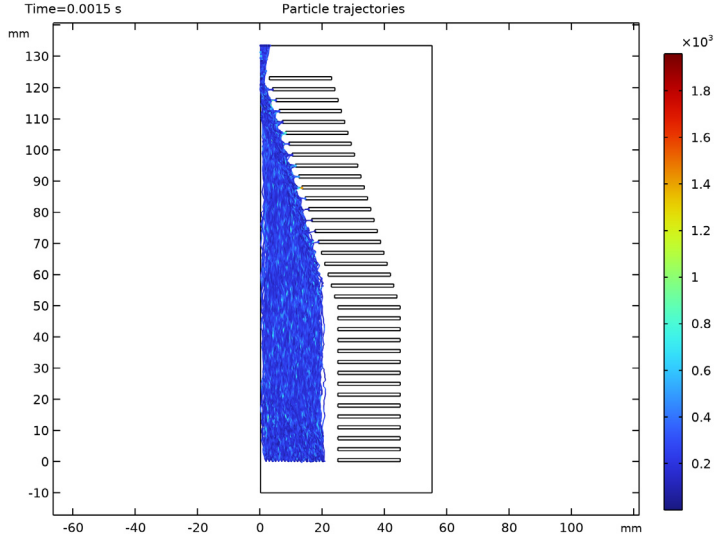


Figure 2: The trajectories of positive ions in the funnel.

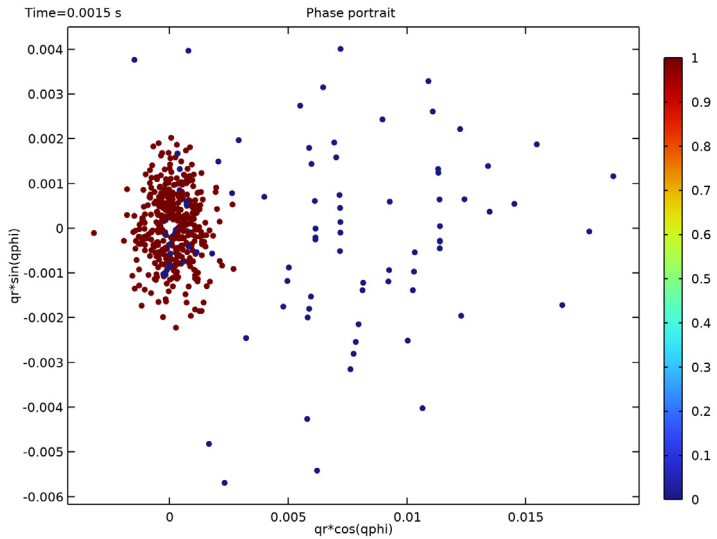


Figure 3: Phase portrait showing the x - and y -coordinates of the ions at the narrow end of the funnel. The color expression is red for particles that have exited the funnel and blue otherwise.

Reference


I. A. V. Phelps, “The application of scattering cross sections to ion flux models in discharge sheaths,” *J. Appl. Phys.* vol. 76, pp. 747-753, 1994.

Application Library path: ACDC_Module/Particle_Tracing/ion_funnel




Modeling Instructions

From the **File** menu, choose **New**.

NEW

In the **New** window, click  **Model Wizard**.


MODEL WIZARD

- 1 In the **Model Wizard** window, click  **2D Axisymmetric**.
- 2 In the **Select Physics** tree, select **AC/DC>Electric Fields and Currents>Electrostatics (es)**.
- 3 Click **Add**.
- 4 In the **Select Physics** tree, select **AC/DC>Electric Fields and Currents>Electric Currents (ec)**.
- 5 Click **Add**.
- 6 In the **Select Physics** tree, select **AC/DC>Particle Tracing>Charged Particle Tracing (cpt)**.
- 7 Click **Add**.
- 8 Click  **Study**.
- 9 In the **Select Study** tree, select **Preset Studies for Some Physics Interfaces>Stationary**.
- 10 Click  **Done**.

GLOBAL DEFINITIONS

To save time, load the parameters from a file.


Parameters 1

- 1 In the **Model Builder** window, under **Global Definitions** click **Parameters 1**.
- 2 In the **Settings** window for **Parameters**, locate the **Parameters** section.
- 3 Click  **Load from File**.
- 4 Browse to the model's Application Libraries folder and double-click the file `ion_funnel_parameters.txt`.


GEOMETRY 1

- 1 In the **Model Builder** window, under **Component 1 (comp1)** click **Geometry 1**.
- 2 In the **Settings** window for **Geometry**, locate the **Units** section.
- 3 From the **Length unit** list, choose **mm**.

Rectangle 1 (r1)



- 1 In the **Geometry** toolbar, click  **Rectangle**.
- 2 In the **Settings** window for **Rectangle**, locate the **Size and Shape** section.
- 3 In the **Width** text field, type w_{elec} .
- 4 In the **Height** text field, type t_{elec} .
- 5 Locate the **Position** section. In the **r** text field, type r_{max} .

Rectangle 2 (r2)



- 1 In the **Geometry** toolbar, click  **Rectangle**.
- 2 In the **Settings** window for **Rectangle**, locate the **Size and Shape** section.
- 3 In the **Width** text field, type w_{elec} .
- 4 In the **Height** text field, type t_{elec} .
- 5 Locate the **Position** section. In the **r** text field, type r_{max} .
- 6 In the **z** text field, type $t_{elec}+t_{gap}$.

Create arrays containing the electrodes which are in phase with each other. This setup allows the number of electrodes to be changed at any time by changing the corresponding parameters.


Array 1 (arr1)

- 1 In the **Geometry** toolbar, click  **Transforms** and choose **Array**.
- 2 Select the object **r1** only.
- 3 In the **Settings** window for **Array**, locate the **Size** section.
- 4 From the **Array type** list, choose **Linear**.
- 5 In the **Size** text field, type $(N_{straight}+1)/2$.
- 6 Locate the **Displacement** section. In the **z** text field, type $2*(t_{elec}+t_{gap})$.
- 7 Locate the **Selections of Resulting Entities** section. Select the **Resulting objects selection** check box.
- 8 From the **Show in physics** list, choose **Boundary selection**.
- 9 Click  **Build Selected**.



Array 2 (arr2)

- 1 In the **Geometry** toolbar, click  **Transforms** and choose **Array**.
- 2 Select the object **r2** only.
- 3 In the **Settings** window for **Array**, locate the **Size** section.
- 4 From the **Array type** list, choose **Linear**.
- 5 In the **Size** text field, type $(N_{\text{straight}} - 1) / 2$.
- 6 Locate the **Displacement** section. In the **z** text field, type $2 * (telec + tgap)$.
- 7 Locate the **Selections of Resulting Entities** section. Select the **Resulting objects selection** check box.
- 8 From the **Show in physics** list, choose **Boundary selection**.
- 9 Click  **Build Selected**.


Rectangle 3 (r3)

- 1 In the **Geometry** toolbar, click  **Rectangle**.
- 2 In the **Settings** window for **Rectangle**, locate the **Size and Shape** section.
- 3 In the **Width** text field, type w_{elec} .
- 4 In the **Height** text field, type $telec$.
- 5 Locate the **Position** section. In the **r** text field, type r_{min} .
- 6 In the **z** text field, type $h_{funnel} - telec$.

Rectangle 4 (r4)


- 1 In the **Geometry** toolbar, click  **Rectangle**.
- 2 In the **Settings** window for **Rectangle**, locate the **Size and Shape** section.
- 3 In the **Width** text field, type w_{elec} .
- 4 In the **Height** text field, type $telec$.
- 5 Locate the **Position** section. In the **r** text field, type $r_{min} + (r_{max} - r_{min}) / N_{inclined}$.
- 6 In the **z** text field, type $h_{funnel} - (2 * telec + tgap)$.
- 7 Click the  **Zoom Extents** button in the **Graphics** toolbar.

Array 3 (arr3)

- 1 In the **Geometry** toolbar, click  **Transforms** and choose **Array**.
- 2 Select the object **r3** only.
- 3 In the **Settings** window for **Array**, locate the **Size** section.
- 4 From the **Array type** list, choose **Linear**.




- 5 In the **Size** text field, type $(N_{\text{inclined}}+1)/2$.
- 6 Locate the **Displacement** section. In the **r** text field, type $2*(r_{\text{max}}-r_{\text{min}})/N_{\text{inclined}}$.
- 7 In the **z** text field, type $-2*(\text{telec}+\text{tgap})$.
- 8 Locate the **Selections of Resulting Entities** section. Select the **Resulting objects selection** check box.
- 9 From the **Show in physics** list, choose **Boundary selection**.

Array 4 (arr4)

- 1 In the **Geometry** toolbar, click  **Transforms** and choose **Array**.
- 2 Select the object **r4** only.
- 3 In the **Settings** window for **Array**, locate the **Size** section.
- 4 From the **Array type** list, choose **Linear**.
- 5 In the **Size** text field, type $(N_{\text{inclined}}-1)/2$.
- 6 Locate the **Displacement** section. In the **r** text field, type $2*(r_{\text{max}}-r_{\text{min}})/N_{\text{inclined}}$.
- 7 In the **z** text field, type $-2*(\text{telec}+\text{tgap})$.
- 8 Locate the **Selections of Resulting Entities** section. Select the **Resulting objects selection** check box.
- 9 From the **Show in physics** list, choose **Boundary selection**.


Add a rectangle to enclose the modeling domain. Note that the rectangle is positioned a small distance away from the axis of symmetry. This makes the particle tracing model more robust, since the centrifugal force acting on the particles approaches infinity as the radial coordinate approaches zero.

Rectangle 5 (r5)


- 1 In the **Geometry** toolbar, click  **Rectangle**.
- 2 In the **Settings** window for **Rectangle**, locate the **Size and Shape** section.
- 3 In the **Width** text field, type $r_{\text{max}}+\text{welec}+10[\text{mm}]$.
- 4 In the **Height** text field, type $h_{\text{funnel}}+20[\text{mm}]$.
- 5 Locate the **Position** section. In the **r** text field, type $0.2[\text{mm}]$.
- 6 In the **z** text field, type $-10[\text{mm}]$.
- 7 Click  **Build All Objects**.
- 8 Click the  **Zoom Extents** button in the **Graphics** toolbar.

DEFINITIONS


Odd electrodes

- 1 In the **Definitions** toolbar, click  **Union**.
- 2 In the **Settings** window for **Union**, type Odd electrodes in the **Label** text field.
- 3 Locate the **Geometric Entity Level** section. From the **Level** list, choose **Boundary**.
- 4 Locate the **Input Entities** section. Under **Selections to add**, click **+ Add**.
- 5 In the **Add** dialog box, in the **Selections to add** list, choose **Array 1** and **Array 4**.
- 6 Click **OK**.


Even electrodes

- 1 In the **Definitions** toolbar, click  **Union**.
- 2 In the **Settings** window for **Union**, type Even electrodes in the **Label** text field.
- 3 Locate the **Geometric Entity Level** section. From the **Level** list, choose **Boundary**.
- 4 Locate the **Input Entities** section. Under **Selections to add**, click **+ Add**.
- 5 In the **Add** dialog box, in the **Selections to add** list, choose **Array 2** and **Array 3**.
- 6 Click **OK**.

All electrodes

- 1 In the **Definitions** toolbar, click  **Union**.
- 2 In the **Settings** window for **Union**, type All electrodes in the **Label** text field.
- 3 Locate the **Geometric Entity Level** section. From the **Level** list, choose **Boundary**.
- 4 Locate the **Input Entities** section. Under **Selections to add**, click **+ Add**.
- 5 In the **Add** dialog box, in the **Selections to add** list, choose **Odd electrodes** and **Even electrodes**.
- 6 Click **OK**.

Exterior boundaries

- 1 In the **Definitions** toolbar, click  **Explicit**.
- 2 In the **Settings** window for **Explicit**, type Exterior boundaries in the **Label** text field.
- 3 Locate the **Input Entities** section. From the **Geometric entity level** list, choose **Boundary**.
- 4 Select Boundaries 2, 3, and 144 only.

All boundaries

- 1 In the **Definitions** toolbar, click  **Union**.
- 2 In the **Settings** window for **Union**, type All boundaries in the **Label** text field.

- 3 Locate the **Geometric Entity Level** section. From the **Level** list, choose **Boundary**.
- 4 Locate the **Input Entities** section. Under **Selections to add**, click **+ Add**.
- 5 In the **Add** dialog box, in the **Selections to add** list, choose **All electrodes** and **Exterior boundaries**.
- 6 Click **OK**.


MATERIALS

Material 1 (mat1)


- 1 In the **Model Builder** window, under **Component 1 (comp1)** right-click **Materials** and choose **Blank Material**.
- 2 In the **Settings** window for **Material**, locate the **Material Contents** section.
- 3 In the table, enter the following settings:

Property	Variable	Value	Unit	Property group
Relative permittivity	epsilon_r_iso ; epsilon_r_ii = epsilon_r_iso, epsilon_r_ij = 0	1		Basic
Electrical conductivity	sigma_iso ; sigma_ii = sigma_iso, sigma_ij = 0	0	S/m	Basic


ELECTROSTATICS (ES)

- 1 In the **Model Builder** window, under **Component 1 (comp1)** click **Electrostatics (es)**.
- 2 In the **Settings** window for **Electrostatics**, locate the **Domain Selection** section.
- 3 Click  **Clear Selection**.
- 4 Select Domain 1 only.


Electric Potential 1

- 1 In the **Physics** toolbar, click  **Boundaries** and choose **Electric Potential**.
- 2 In the **Settings** window for **Electric Potential**, locate the **Boundary Selection** section.
- 3 From the **Selection** list, choose **All boundaries**.
- 4 Locate the **Electric Potential** section. In the V_0 text field, type EDC*z.


ELECTRIC CURRENTS (EC)

- 1 In the **Model Builder** window, under **Component 1 (comp1)** click **Electric Currents (ec)**.
- 2 In the **Settings** window for **Electric Currents**, locate the **Domain Selection** section.
- 3 Click  **Clear Selection**.
- 4 Select Domain 1 only.


Terminal 1

- 1 In the **Physics** toolbar, click  **Boundaries** and choose **Terminal**.
- 2 In the **Settings** window for **Terminal**, locate the **Boundary Selection** section.
- 3 From the **Selection** list, choose **Odd electrodes**.
- 4 Locate the **Terminal** section. From the **Terminal type** list, choose **Voltage**.
- 5 In the V_0 text field, type V_{pp} .

Terminal 2

- 1 In the **Physics** toolbar, click  **Boundaries** and choose **Terminal**.
- 2 In the **Settings** window for **Terminal**, locate the **Boundary Selection** section.
- 3 From the **Selection** list, choose **Even electrodes**.
- 4 Locate the **Terminal** section. From the **Terminal type** list, choose **Voltage**.
- 5 In the V_0 text field, type $-V_{pp}$.


CHARGED PARTICLE TRACING (CPT)

- 1 In the **Model Builder** window, under **Component 1 (comp1)** click **Charged Particle Tracing (cpt)**.
- 2 In the **Settings** window for **Charged Particle Tracing**, locate the **Domain Selection** section.
- 3 Click  **Clear Selection**.
- 4 Select Domain 1 only.
- 5 Locate the **Additional Variables** section. Select the **Include out-of-plane degrees of freedom** check box.



Particle Properties 1

- 1 In the **Model Builder** window, under **Component 1 (comp1)**>**Charged Particle Tracing (cpt)** click **Particle Properties 1**.
- 2 In the **Settings** window for **Particle Properties**, locate the **Particle Mass** section.
- 3 In the m_p text field, type $0.146[\text{kg/mol}]/N_A\text{const}$.
- 4 Locate the **Charge Number** section. In the Z text field, type 1.


Wall 2

- 1 In the **Physics** toolbar, click  **Boundaries** and choose **Wall**.
- 2 Select Boundary 1 only.
- 3 In the **Settings** window for **Wall**, locate the **Wall Condition** section.
- 4 From the **Wall condition** list, choose **Bounce**.


Release from Grid 1

- 1 In the **Physics** toolbar, click  **Global** and choose **Release from Grid**.
- 2 In the **Settings** window for **Release from Grid**, locate the **Initial Coordinates** section.
- 3 Click  **R Range**.
- 4 In the **Range** dialog box, type 2 in the **Start** text field.
- 5 In the **Step** text field, type 1.
- 6 In the **Stop** text field, type 20.
- 7 Click **Replace**.
- 8 In the **Settings** window for **Release from Grid**, locate the **Initial Velocity** section.
- 9 From the **Initial velocity** list, choose **Maxwellian**.
- 10 In the N_v text field, type 5.

Electric Force 1

- 1 In the **Physics** toolbar, click  **Domains** and choose **Electric Force**.
- 2 Select Domain 1 only.
- 3 In the **Settings** window for **Electric Force**, locate the **Electric Force** section.
- 4 From the **E** list, choose **Electric field (es/ccn1)**.
- 5 Locate the **Advanced Settings** section. Select the **Use piecewise polynomial recovery on field** check box.

Electric Force 2


- 1 In the **Physics** toolbar, click  **Domains** and choose **Electric Force**.
- 2 Select Domain 1 only.
- 3 In the **Settings** window for **Electric Force**, locate the **Electric Force** section.
- 4 From the **E** list, choose **Electric field (ec/cucn1)**.
- 5 Locate the **Advanced Settings** section. From the **Time dependence of field** list, choose **Time harmonic**.
- 6 Select the **Use piecewise polynomial recovery on field** check box.

Now define the cross sections used for the collision model.

DEFINITIONS


Enter the analytic approximation for momentum cross section for elastic scattering between SF₆⁺ ions and neutral Ar atoms from [Ref. 1](#), which depends on the kinetic energy of the particles.

Analytic 1 (an1)

- 1 In the **Home** toolbar, click  **Functions** and choose **Global>Analytic**.
- 2 In the **Settings** window for **Analytic**, type Qm in the **Function name** text field.
- 3 Locate the **Definition** section. In the **Expression** text field, type $1.15e-18*x^{(-0.1)*(1+0.015/x)^{0.6}}$.
- 4 Locate the **Units** section. In the **Arguments** text field, type eV.
- 5 In the **Function** text field, type m².


Enter the analytic approximation for isotropic elastic collision between SF₆⁺ ions and neutral Ar atoms from [Ref. 1](#), which depends on the kinetic energy of the particles.

Analytic 2 (an2)

- 1 In the **Home** toolbar, click  **Functions** and choose **Global>Analytic**.
- 2 In the **Settings** window for **Analytic**, type Qi in the **Function name** text field.
- 3 Locate the **Definition** section. In the **Expression** text field, type $2e-19/(x^{(0.5)*(1+x)})+3e-19*x/(1+x/3)^{(2.3)}$.
- 4 Locate the **Units** section. In the **Arguments** text field, type eV.
- 5 In the **Function** text field, type m².

CHARGED PARTICLE TRACING (CPT)

Collisions 1

- 1 In the **Physics** toolbar, click  **Domains** and choose **Collisions**.
- 2 Select Domain 1 only.
- 3 In the **Settings** window for **Collisions**, locate the **Fluid Properties** section.
- 4 In the N_d text field, type ND.
- 5 Locate the **Collision Statistics** section. Select the **Count all collisions** check box.

Elastic 1


- 1 In the **Physics** toolbar, click  **Attributes** and choose **Elastic**.
- 2 In the **Settings** window for **Elastic**, locate the **Collision Frequency** section.

- 3 In the σ text field, type $Q_i(\text{cpt} \cdot E_p)$.
- 4 Locate the **Collision Statistics** section. Select the **Count collisions** check box.

Collisions I

In the **Model Builder** window, click **Collisions I**.

Resonant Charge Exchange I


- 1 In the **Physics** toolbar, click  **Attributes** and choose **Resonant Charge Exchange**.
- 2 In the **Settings** window for **Resonant Charge Exchange**, locate the **Collision Frequency** section.
- 3 In the σ text field, type $(Q_m(\text{cpt} \cdot E_p) - Q_i(\text{cpt} \cdot E_p)) / 2$.
- 4 Locate the **Collision Statistics** section. Select the **Count collisions** check box.

STUDY I

Step 1: Stationary

- 1 In the **Model Builder** window, under **Study I** click **Step 1: Stationary**.
- 2 In the **Settings** window for **Stationary**, locate the **Physics and Variables Selection** section.
- 3 In the table, clear the **Solve for** check boxes for **Electric Currents (ec)** and **Charged Particle Tracing (cpt)**.

Frequency Domain

- 1 In the **Study** toolbar, click  **Study Steps** and choose **Frequency Domain> Frequency Domain**.
- 2 In the **Settings** window for **Frequency Domain**, locate the **Study Settings** section.
- 3 In the **Frequencies** text field, type f_0 .
- 4 Locate the **Physics and Variables Selection** section. In the table, clear the **Solve for** check boxes for **Electrostatics (es)** and **Charged Particle Tracing (cpt)**.
- 5 Click to expand the **Values of Dependent Variables** section. Find the **Values of variables not solved for** subsection. From the **Settings** list, choose **User controlled**.
- 6 From the **Method** list, choose **Solution**.
- 7 From the **Study** list, choose **Study I, Stationary**.

Step 1: Stationary

- 1 In the **Model Builder** window, click **Step 1: Stationary**.
- 2 In the **Settings** window for **Stationary**, click to expand the **Values of Dependent Variables** section.


3 In the **Study** toolbar, click  **Compute**.

RESULTS



Electric Potential (es)

Create a contour plot of the electric potential when $t = 0$.



Sum of AC and DC Potentials

- 1 In the **Home** toolbar, click  **Add Plot Group** and choose **2D Plot Group**.
- 2 In the **Settings** window for **2D Plot Group**, type Sum of AC and DC Potentials in the **Label** text field.

Contour 1


- 1 Right-click **Sum of AC and DC Potentials** and choose **Contour**.
- 2 In the **Settings** window for **Contour**, locate the **Expression** section.
- 3 In the **Expression** text field, type $V+V2$.
- 4 Locate the **Levels** section. In the **Total levels** text field, type 30.
- 5 Locate the **Coloring and Style** section. From the **Contour type** list, choose **Filled**.
- 6 Clear the **Color legend** check box.
- 7 In the **Sum of AC and DC Potentials** toolbar, click  **Plot**.
- 8 Click the  **Zoom Extents** button in the **Graphics** toolbar. Compare this plot to [Figure 1](#). The plot indicates that the potential gradient is steep in the area surrounding the electrodes. The large electric field magnitude in this area confines ions within the funnel.

ADD STUDY


- 1 In the **Home** toolbar, click  **Add Study** to open the **Add Study** window.
- 2 Go to the **Add Study** window.
- 3 Find the **Studies** subsection. In the **Select Study** tree, select **General Studies> Time Dependent**.
- 4 Find the **Physics interfaces in study** subsection. In the table, clear the **Solve** check boxes for **Electrostatics (es)** and **Electric Currents (ec)**.
- 5 Click **Add Study** in the window toolbar.
- 6 In the **Home** toolbar, click  **Add Study** to close the **Add Study** window.


STUDY 2

Step 1: Time Dependent

- 1 In the **Settings** window for **Time Dependent**, locate the **Study Settings** section.
- 2 Click  **Range**.
- 3 In the **Range** dialog box, type $1e-5$ in the **Step** text field.
- 4 In the **Stop** text field, type $15e-4$.
- 5 Click **Replace**.
- 6 In the **Settings** window for **Time Dependent**, locate the **Study Settings** section.
- 7 From the **Tolerance** list, choose **User controlled**.
- 8 In the **Relative tolerance** text field, type $1e-3$.
- 9 Click to expand the **Values of Dependent Variables** section. Find the **Values of variables not solved for** subsection. From the **Settings** list, choose **User controlled**.
- 10 From the **Method** list, choose **Solution**.
- 11 From the **Study** list, choose **Study 1, Frequency Domain**.


Solution 3 (sol3)

- 1 In the **Study** toolbar, click  **Show Default Solver**.
- 2 In the **Model Builder** window, expand the **Solution 3 (sol3)** node, then click **Time-Dependent Solver 1**.
- 3 In the **Settings** window for **Time-Dependent Solver**, click to expand the **Absolute Tolerance** section.
- 4 From the **Tolerance method** list, choose **Manual**.
- 5 In the **Absolute tolerance** text field, type $1e-4$.
- 6 Click to expand the **Time Stepping** section. From the **Maximum step constraint** list, choose **Constant**.
- 7 In the **Maximum step** text field, type $1e-8$.
- 8 Click to expand the **Output** section. Clear the **Store reaction forces** check box.
- 9 Clear the **Store time derivatives** check box.
- 10 In the **Model Builder** window, expand the **Study 2>Solver Configurations>Solution 3 (sol3)>Time-Dependent Solver 1** node, then click **Advanced**.
- 11 In the **Settings** window for **Advanced**, locate the **General** section.
- 12 From the **Solver log** list, choose **Minimal**.


13 In the **Study** toolbar, click  **Compute**.

RESULTS


Particle Trajectories I

- 1 In the **Model Builder** window, expand the **Particle Trajectories (cpt)** node, then click **Particle Trajectories I**.
- 2 In the **Settings** window for **Particle Trajectories**, locate the **Coloring and Style** section.
- 3 Find the **Line style** subsection. From the **Type** list, choose **Line**.
- 4 In the **Particle Trajectories (cpt)** toolbar, click  **Plot**. Compare the result to [Figure 2](#).


Transverse Particle Positions


- 1 In the **Home** toolbar, click  **Add Plot Group** and choose **2D Plot Group**.
- 2 In the **Settings** window for **2D Plot Group**, type **Transverse Particle Positions** in the **Label** text field.
- 3 Locate the **Data** section. From the **Dataset** list, choose **Particle I**.
- 4 From the **Time (s)** list, choose **0.0015**.
- 5 Locate the **Plot Settings** section. Clear the **Plot dataset edges** check box.

Phase Portrait I

- 1 In the **Transverse Particle Positions** toolbar, click  **More Plots** and choose **Phase Portrait**.
- 2 In the **Settings** window for **Phase Portrait**, locate the **Expression** section.
- 3 From the **x-axis** list, choose **Manual**.
- 4 In the **Expression** text field, type $qr \cdot \cos(qphi)$.
- 5 From the **y-axis** list, choose **Manual**.
- 6 In the **Expression** text field, type $qr \cdot \sin(qphi)$.


Color Expression I

- 1 Right-click **Phase Portrait I** and choose **Color Expression**.
- 2 In the **Settings** window for **Color Expression**, locate the **Expression** section.
- 3 In the **Expression** text field, type $qz > 120$ [mm].
- 4 In the **Transverse Particle Positions** toolbar, click  **Plot**.

- 5 Click the  **Zoom Extents** button in the **Graphics** toolbar. Compare the resulting plot to [Figure 3](#).


Create an animation showing the x - and y -components of the particle positions over time.

Animation 1


- 1 In the **Results** toolbar, click  **Animation** and choose **File**.
- 2 In the **Settings** window for **Animation**, locate the **Target** section.
- 3 From the **Target** list, choose **Player**.
- 4 Locate the **Scene** section. From the **Subject** list, choose **Transverse Particle Positions**.
- 5 Right-click **Animation 1** and choose **Play**.

Finally, create a 1D plot showing the average radial position over time. Set the color expression to be the total radial force, averaged over all particles.



Average Radial Particle Position

- 1 In the **Results** toolbar, click  **ID Plot Group**.
- 2 In the **Settings** window for **ID Plot Group**, type Average Radial Particle Position in the **Label** text field.
- 3 Locate the **Data** section. From the **Dataset** list, choose **Particle 1**.

Particle 1

- 1 In the **Average Radial Particle Position** toolbar, click  **More Plots** and choose **Particle**.
- 2 In the **Settings** window for **Particle**, click **Replace Expression** in the upper-right corner of the **y-Axis Data** section. From the menu, choose **Component 1 (comp1)> Charged Particle Tracing>Particle position>qr - Particle position, r component - m**.
- 3 Locate the **Data Series Operation** section. From the **Operation** list, choose **Average**.

Color Expression 1

- 1 Right-click **Particle 1** and choose **Color Expression**.
- 2 In the **Settings** window for **Color Expression**, click **Replace Expression** in the upper-right corner of the **Expression** section. From the menu, choose **Component 1 (comp1)> Charged Particle Tracing>Forces>Total force - N>cpt.Ftr - Total force, r component**.
- 3 In the **Average Radial Particle Position** toolbar, click  **Plot**.
- 4 Click the  **Zoom Extents** button in the **Graphics** toolbar.

