Created in COMSOL Multiphysics 5.6

Total Internal Reflection

Introduction

Total internal reflection (TIR) is the phenomenon that occurs when a light wave propagates in a material with a high refractive index toward a material with a low index at an angle larger than the so-called critical angle. At such an angle, the light wave is totally reflected at the interface between the two different materials. This phenomenon can be seen in a variety of optical devices. In LED and OLED devices, TIR is the major cause of their poor extraction efficiency. There are countless numbers of research studies that try to avoid TIR by making some grating structure at the material interface to couple out the trapped light waves for such devices. On the other hand, virtual reality (VR) glasses, augmented reality (AR) glasses, head-up displays (HUD), head-mounted displays (HMD) used in aerospace applications, civil engineering applications, leisure applications, and entertainment applications, utilize TIR to their advantage, in the form of a light waveguide with a narrow and thin structure. In those applications, the light wave that enters at the entrance bounces back and forth in the waveguide and couples out to an exit optic, such as a reflector or a grating, before reaching the eye.

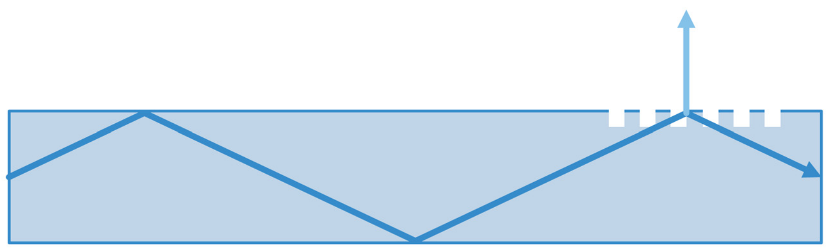


Figure 1: Application example of TIR in a waveguide with a outcoupling grating.

Model Definition

In this model, an incoming wave of 1 μm wavelength, having a Gaussian profile with a 75 μm beam waist radius, enters the left boundary of a 2D rectangular waveguide of 350 μm width and 20 mm length. The wave enters the waveguide at such a small angle of incidence that the propagation angle at the interface between the waveguide and the surrounding air is much larger than the critical angle. Thus, the wave is captured inside the waveguide by TIR. As the waveguide is long compared to the wavelength, the Electromagnetic Waves, Beam Envelopes interface is used to reduce the number of mesh elements required along the propagation direction. The Bidirectional formulation is specifically suited for this model, since there are two wave directions, $\mathbf{k}_1 = (k_x, k_y)$ and $\mathbf{k}_2 = (k_x, -k_y)$. The wave vector components here are $k_x = k_0 n\cos\theta_{\text{in}}$ and $k_y = k_0 n\sin\theta_{\text{in}}$, where n is the refractive index of the waveguide, k_0 is the wave number in vacuum, and

θ_{in} = 10 deg is the angle of incidence in the waveguide material with respect to the boundary normal.

Results and Discussion

Figure 2 shows the electric field norm. As the *y* dimension is scaled ten times, the propagation direction looks much steeper than it actually is. The mesh was deliberately made very fine, to resolve the interference pattern close to the top and bottom boundaries.

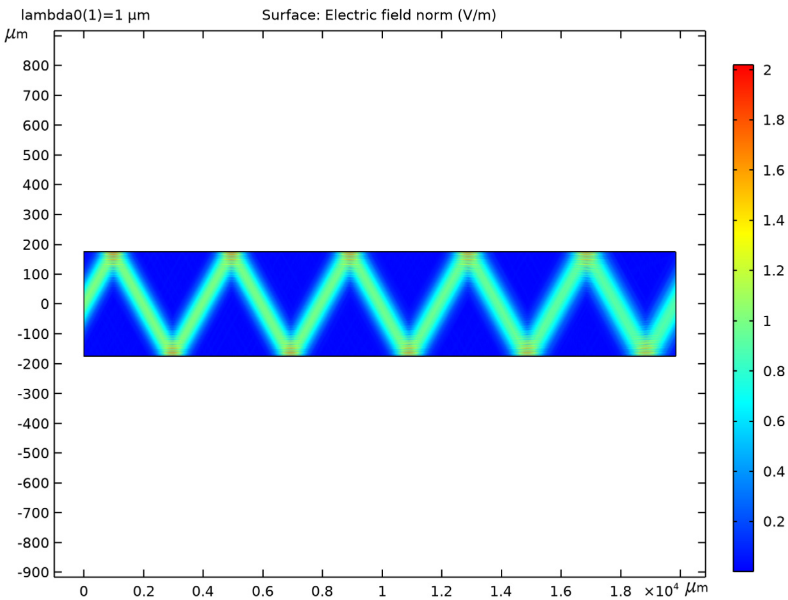


Figure 2: Simulation result showing the electric field norm. The plot is scaled ten times in the y direction.

Figure 3 shows a zoom-in of the first reflection of the beam at the top waveguide-air interface. The interference pattern can be resolved here, thanks to the excessively fine mesh and the use of extra fine resolution when rendering the plot.

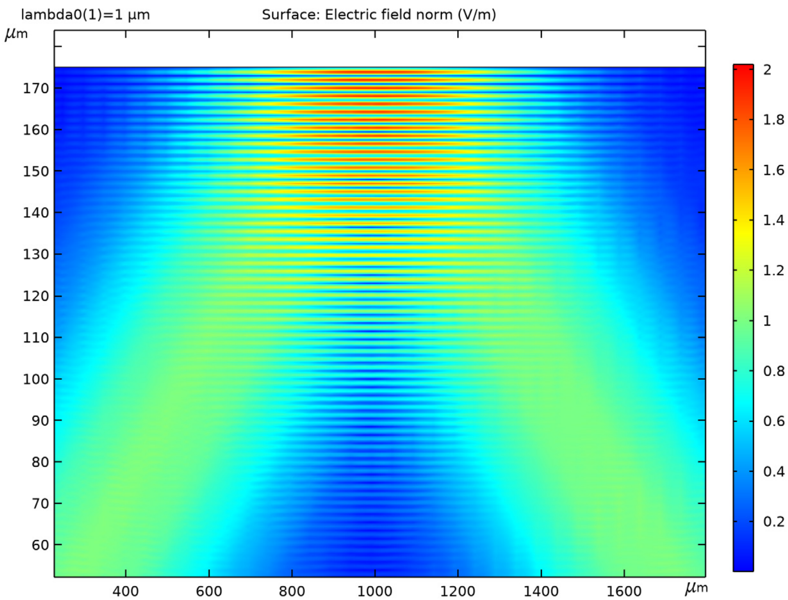


Figure 3: A zoom-in on the first reflection at the top waveguide-air interface.

[Figure 4](#) shows the electric field norm of the first wave. Notice the slight divergence of the beam as it propagates back and forth inside the waveguide.

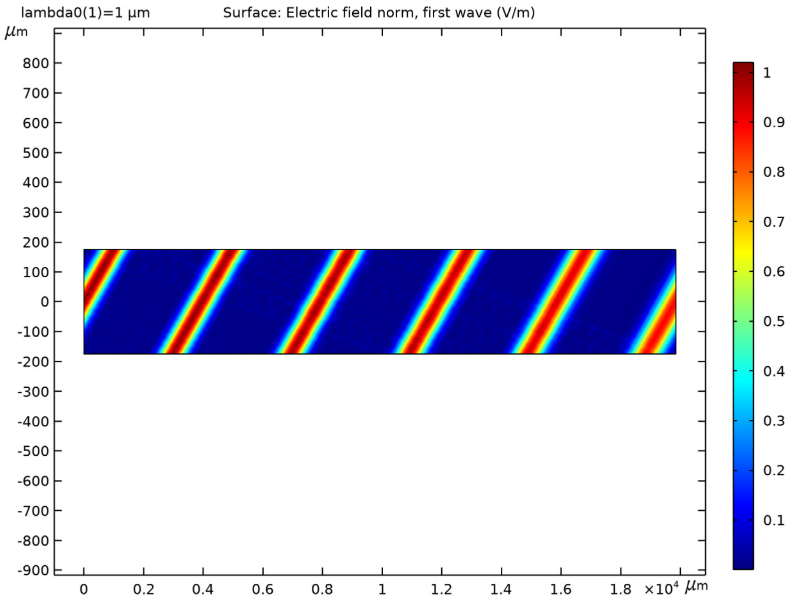


Figure 4: The norm of the electric field for the first wave.

Figure 5 is similar to Figure 4, but shows the second wave.

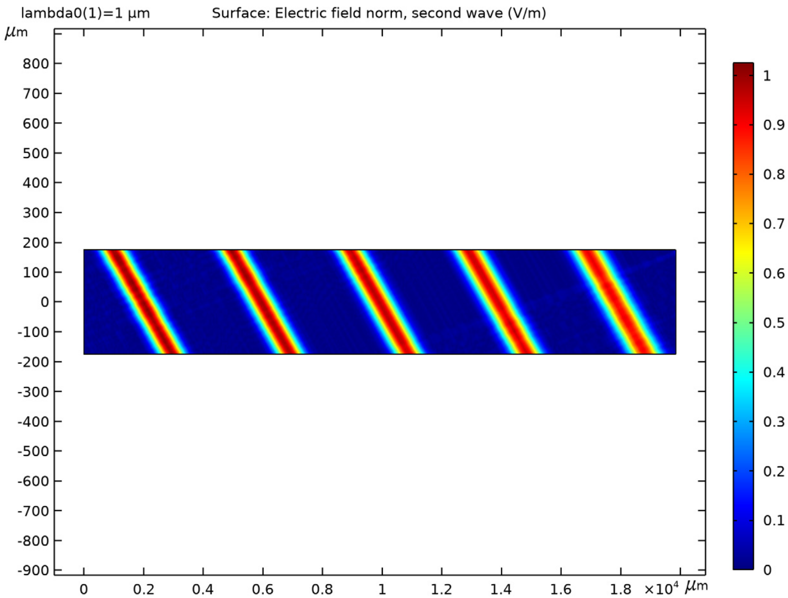


Figure 5: The norm of the electric field for the second wave.

Notes About the COMSOL Implementation

The Gaussian input beam is specified using the **Gaussian beam incident field** option for the **Matched boundary condition**. There you specify the beam radius at the focal plane, w_0, and the distance to the focal plane, p_0, along the propagation direction, which is specified by the wave vector for the first wave, $\mathbf{k}_1$, from the reference point on the input boundary. This reference point is defined here as the average position on the boundary selected for the **Matched boundary condition**.

The **Impedance** boundary condition is used to truncate the modeling domain at the interface between the waveguide and the surrounding air. When the **Propagation direction** option is set to **From wave vector**, the Impedance boundary condition uses the fact that the field is composed of the two waves propagating with wave vectors $\mathbf{k}_1$ and $\mathbf{k}_2$, respectively, to calculate the coupling between the first and the second wave and vice versa. In this case, there is total internal reflection at the waveguide-air boundaries, so 100% of the incident wave is reflected back into the other wave. For other cases, when total internal reflection does not apply, reflection and transmission coefficients are calculated from the Fresnel coefficients.

Application Library path: `Wave_Optics_Module/Waveguides_and_Couplers/total_internal_reflection`

Modeling Instructions

From the **File** menu, choose **New**.

NEW

In the **New** window, click **Model Wizard**.

MODEL WIZARD

1 In the **Model Wizard** window, click **2D**.

2 In the **Select Physics** tree, select **Optics>Wave Optics>Electromagnetic Waves, Beam Envelopes (ewbe)**.

3 Click **Add**.

4 Click **Study**.

5 In the **Select Study** tree, select **Preset Studies for Selected Physics Interfaces>Wavelength Domain**.

6 Click **Done**.

GLOBAL DEFINITIONS

Parameters 1

1 In the **Model Builder** window, under **Global Definitions** click **Parameters 1**.

2 In the **Settings** window for **Parameters**, locate the **Parameters** section.

3 In the table, enter the following settings:

Name	Expression	Value	Description
lda0	1[um]	1E-6 m	Wavelength
k0	2*pi/lda0	6.2832E6 1/m	Vacuum wave number
n	1.5	1.5	Refractive index in waveguide
k	k0*n	9.4248E6 1/m	Wave number in waveguide
theta	10[deg]	0.17453 rad	Incidence angle
k1x	k*cos(theta)	9.2816E6 1/m	Wave vector, first wave, x component
k1y	k*sin(theta)	1.6366E6 1/m	Wave vector, first wave, y component
d	350[um]	3.5E-4 m	Waveguide width
L	5*4*d/2/tan(theta)	0.019849 m	Waveguide length
w0	75[um]	7.5E-5 m	Beam radius

The waveguide length is defined above for the beam to make five bouncing cycles as it propagates in the waveguide.

DEFINITIONS

In the **Model Builder** window, expand the **Component 1 (comp1)>Definitions** node.

Axis

1 In the **Model Builder** window, expand the **Component 1 (comp1)>Definitions>View 1** node, then click **Axis**.

2 In the **Settings** window for **Axis**, locate the **Axis** section.

3 From the **View scale** list, choose **Manual**.

4 In the **y scale** text field, type 10, to make the view of the waveguide less wide. Notice though that the propagation angle now looks much steeper.

GEOMETRY 1

Now, define the geometry.

Rectangle 1 (r1)

1 In the **Geometry** toolbar, click **Rectangle**.

2 In the **Model Builder** window, click **Geometry 1**.

3 In the **Settings** window for **Geometry**, locate the **Units** section.

4 From the **Length unit** list, choose **µm**.

5 In the **Model Builder** window, click **Rectangle 1 (r1)**.

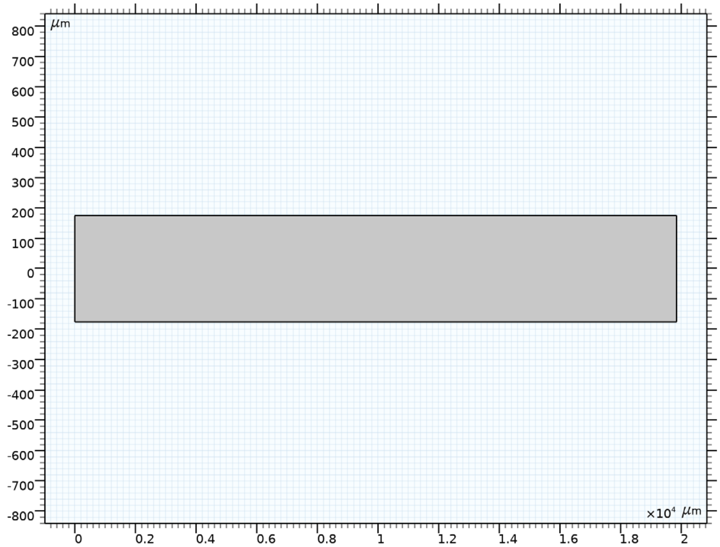
6 In the **Settings** window for **Rectangle**, locate the **Size and Shape** section.

7 In the **Width** text field, type `L`.

8 In the **Height** text field, type `d`.

9 Locate the **Position** section. In the **y** text field, type `-d/2`.

10 Click **Build All Objects**.



MATERIALS

Material 1 (mat1)

1 In the **Model Builder** window, under **Component 1 (comp1)** right-click **Materials** and choose **Blank Material**.

2 In the **Settings** window for **Material**, locate the **Material Contents** section.

3 In the table, enter the following settings:

Property	**Variable**	**Value**	**Unit**	**Property group**
Refractive index, real part	n_iso ; nii = n_iso, nij = 0	n	1	Refractive index
Refractive index, imaginary part	ki_iso ; kiii = ki_iso, kiij = 0	0	1	Refractive index

ELECTROMAGNETIC WAVES, BEAM ENVELOPES (EWBE)

Make the simulation for out-of-plane polarization. The two waves, defining the propagation, have the same x components for the wave vector, but opposite y components.

1 In the **Model Builder** window, under **Component 1 (comp1)** click **Electromagnetic Waves, Beam Envelopes (ewbe)**.

2 In the **Settings** window for **Electromagnetic Waves, Beam Envelopes**, locate the **Components** section.

3 From the **Electric field components solved for** list, choose **Out-of-plane vector**.

4 Locate the **Wave Vectors** section. Specify the $\mathbf{k}_1$ vector as

k1x	x
k1y	y

5 Specify the $\mathbf{k}_2$ vector as

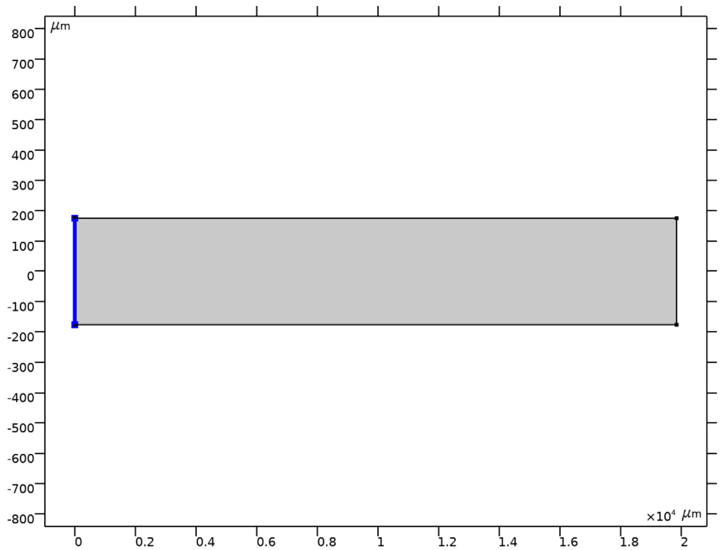
ewbe.k1x	x
-ewbe.k1y	y

Matched Boundary Condition 1

The incident Gaussian beam is defined using a Matched boundary condition. The beam will be launched in the direction of the first wave vector, as previously defined in the **Wave vector** section in the settings for the physics interface.

1 In the **Physics** toolbar, click **Boundaries** and choose **Matched Boundary Condition**.

2 Select Boundary 1 only.



3 In the **Settings** window for **Matched Boundary Condition**, locate the **Matched Boundary Condition** section.

4 From the **Incident field** list, choose **Gaussian beam**.

5 In the w_0 text field, type `w0`.

6 Specify the $\mathbf{E}_{g0}$ vector as

0	x
0	y
1[V/m]	z

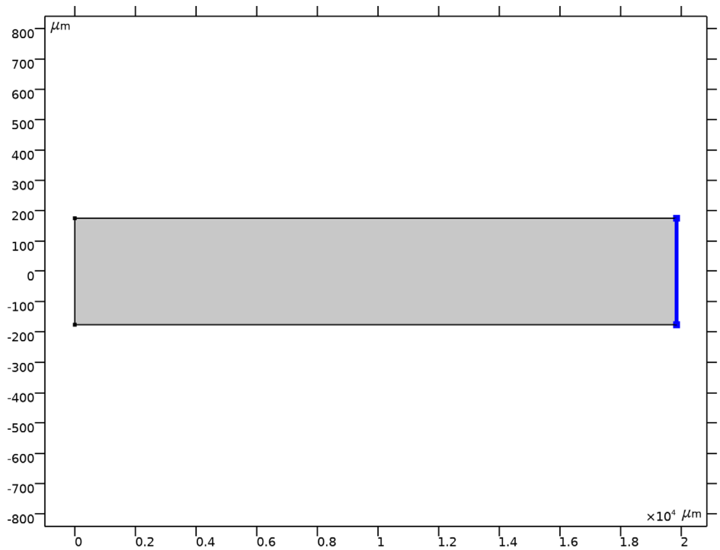
7 Find the **Scattered field** subsection. Select the **No scattered field** check box, as we know that there will not be any scattered wave exiting this boundary.

Matched Boundary Condition 2

A Matched boundary condition is used on the exit boundary to perfectly absorb the outgoing wave that here will propagate in the direction of the first wave, as defined in the **Wave vector** section in the settings for the physics interface.

1 In the **Physics** toolbar, click **Boundaries** and choose **Matched Boundary Condition**.

2 Select Boundary 4 only.



3 In the **Settings** window for **Matched Boundary Condition**, locate the **Matched Boundary Condition** section.

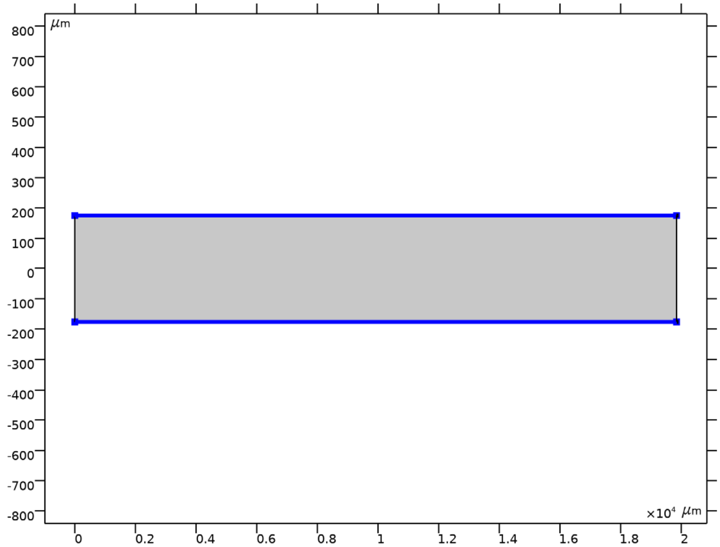
4 From the **Input wave** list, choose **Second wave**, the first wave will be absorbed at this boundary and there will not be any incident wave here.

Impedance Boundary Condition 1

The Impedance boundary condition, with the **Propagation direction** set to **From wave vector**, models how the waves reflect and refract at the boundaries between the waveguide and the surrounding air. In this case, the waves will be fully reflected at these boundaries. However, this setting is also useful under partially reflecting conditions.

1 In the **Physics** toolbar, click **Boundaries** and choose **Impedance Boundary Condition**.

2 Select Boundaries 2 and 3 only.



3 In the **Settings** window for **Impedance Boundary Condition**, locate the **Propagation Direction** section.

4 From the list, choose **From wave vector**.

5 Locate the **Impedance Boundary Condition** section. From the n list, choose **User defined**. From the k list, choose **User defined**. This specifies the refractive index of the air surrounding the waveguide.

MESH 1

The mesh should resolve the beam width, both as the beam enters and exits at the left and right boundaries, respectively, but also as it is reflected at the top and bottom boundaries. However, here a much finer mesh is used, just to resolve the interference patterns next to the top and bottom boundaries, where the two waves overlap.

1 In the **Model Builder** window, under **Component 1 (comp1)** click **Mesh 1**.

2 In the **Settings** window for **Mesh**, locate the **Electromagnetic Waves, Beam Envelopes (ewbe)** section.

3 In the N_T text field, type 100.

4 In the N_L text field, type 400.

STUDY 1

Step 1: Wavelength Domain

1 In the **Model Builder** window, under **Study 1** click **Step 1: Wavelength Domain**.

2 In the **Settings** window for **Wavelength Domain**, locate the **Study Settings** section.

3 In the **Wavelengths** text field, type `lda0`.

4 In the **Home** toolbar, click **Compute**.

RESULTS

Electric Field (ewbe)

The plot shows the total internal reflection (TIR) in the waveguide.

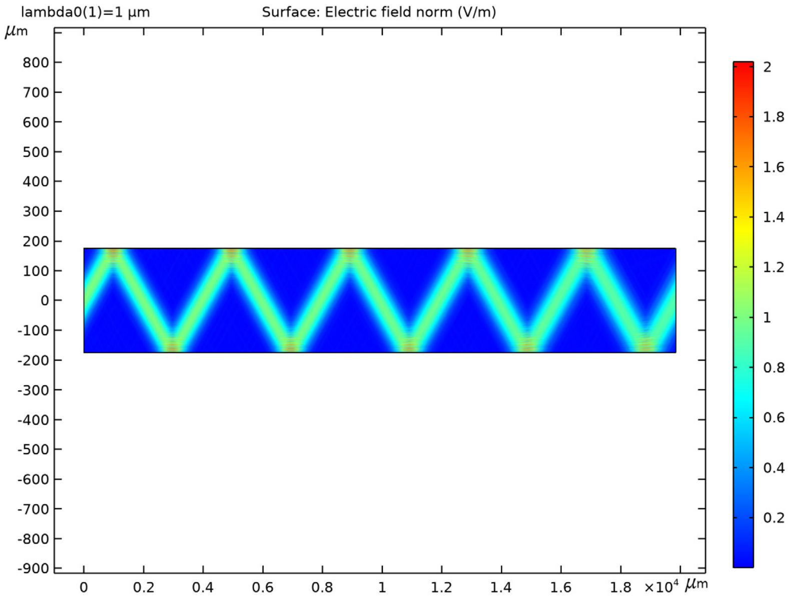
Electric Field

1 In the **Model Builder** window, expand the **Electric Field (ewbe)** node, then click **Electric Field**.

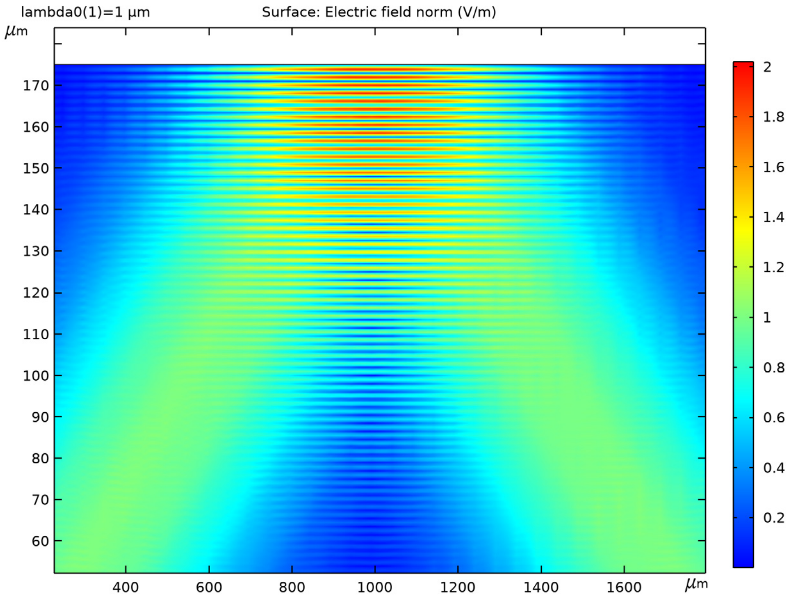
2 In the **Settings** window for **Surface**, click to expand the **Quality** section.

3 From the **Resolution** list, choose **Extra fine**, to resolve the interference patterns close to the top and bottom boundaries.

4 In the **Electric Field (ewbe)** toolbar, click **Plot**.



Zoom-in on the for beam reflection to see the interference pattern between the two waves.



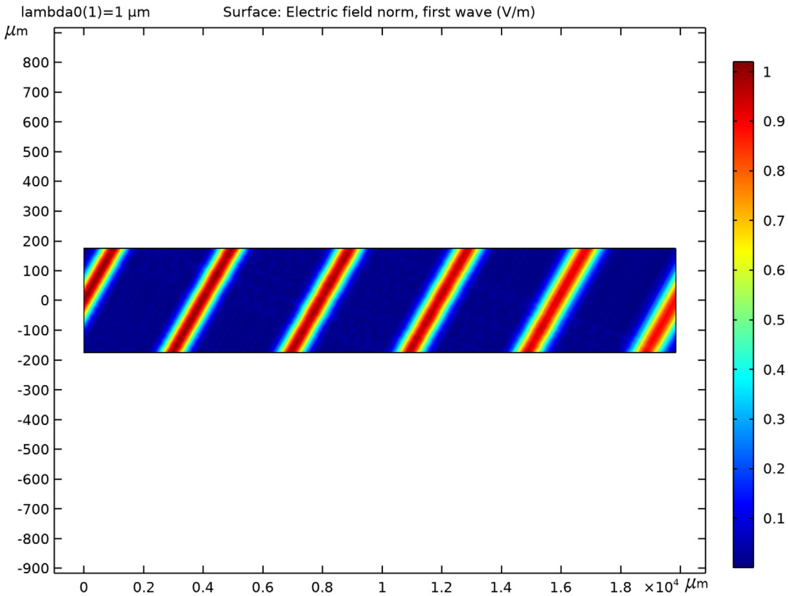
Electric Field, First Wave

1 In the **Home** toolbar, click **Add Plot Group** and choose **2D Plot Group**.
2 In the **Settings** window for **2D Plot Group**, type `Electric Field, First Wave` in the **Label** text field.

Surface 1

1 Right-click **Electric Field, First Wave** and choose **Surface**.
2 In the **Settings** window for **Surface**, locate the **Expression** section.
3 In the **Expression** text field, type `ewbe.normE1`, to plot the norm of the electric field for the first wave.
4 In the **Electric Field, First Wave** toolbar, click **Plot**.

5 Click the **Zoom Extents** button in the **Graphics** toolbar.



Electric Field, Second Wave

1 In the **Home** toolbar, click **Add Plot Group** and choose **2D Plot Group**.

2 In the **Settings** window for **2D Plot Group**, type `Electric Field, Second Wave` in the **Label** text field.

Surface 1

1 Right-click **Electric Field, Second Wave** and choose **Surface**.

2 In the **Settings** window for **Surface**, locate the **Expression** section.

3 In the **Expression** text field, type `ewbe.normE2`, to plot the norm of the electric field for the second wave.

4 In the **Electric Field, Second Wave** toolbar, click **Plot**.