

# Analyzing Porous Structures on the Microscopic Scale

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## Introduction

Modeling flow through realistic porous structures is difficult due to the complexity of the structure itself. Resolving the flow field in detail is not feasible in real-life applications. Therefore, macroscopic approaches utilizing averaged quantities of the porous structure, such as porosity and permeability, are used. This example analyzes the flow field at the pore scale in detail. The results are used to validate and adapt the macroscopic description, which in turn are used to model large-scale porous geometries.

# Model Definition

The modeled geometry shown in Figure 1 is a representative elementary volume (REV) whose properties are representative for the entire system.



Figure 1: Geometry of the REV's pore volume.

Because only the area of the pore space is required for modeling, the porous matrix is not resolved explicitly. The REV has a quadratic cross section of 2 cm side length and a width of 6 cm. Water flows with a velocity of u = 0.1 mm/s and can flow out freely in the normal direction at the other end. The other boundaries are assumed to be symmetry boundaries.

This does not correspond to the actual situation, but for modeling an REV, symmetry boundary conditions are very well suited.

To characterize the flow inside the porous structure one can estimate the Reynolds number according to

Re = 
$$\frac{\rho u L}{\mu}$$

with the water density  $\rho = 1000 \text{ kg/m}^3$  and viscosity  $\mu = 10^{-3} \text{ Pa} \cdot \text{s}$ . The cross-section side length serves as the characteristic length scale, *L*. This results in Re = 2 and the Stokes equation can be used to describe the flow where inertia terms are neglected.

Finally, the goal of the model is to obtain the averaged values for porosity and permeability to describe a macroscopic model with, for example, Darcy's law or the Brinkman equation. The porosity is defined as the fraction of pore space volume  $V_{fluid}$  to total volume  $V_{tot}$ :

$$\varepsilon = \frac{V_{\text{fluid}}}{V_{\text{tot}}}$$

To calculate the permeability  $\kappa$  (m<sup>2</sup>) the following relationship is used:

$$\mathbf{u} = -\frac{\kappa}{\mu} \nabla p$$

Approximating the pressure gradient  $\nabla p$  by the pressure difference between the inlet and the outlet,  $\Delta p$ , divided by the side length *L*, and replacing the velocity vector **u** by the outlet velocity  $u_{\text{out}}$  in the flow direction gives the expression

$$\kappa = u_{\text{out}} \mu \frac{L}{\Delta p}$$

# Results and Discussion

First of all it is interesting to have a look at the mesh, or rather the mesh quality. Although the mesh quality in itself says nothing about the quality of the results, a good mesh quality is beneficial for convergence. The overall mesh quality as shown in Figure 2 is very good.



Figure 2: Mesh quality plot.

The velocity field in the REV is shown in Figure 3. Characteristic for the flow in a porous material are areas with high and areas with low velocity. In some areas the flow also stagnates. This behavior is characterized by the permeability.



Surface: 1 (1) Line: Velocity magnitude (m/s) Streamline: Velocity field

Figure 3: Velocity in the REV.

From the simulation the values for the porosity and permeability are obtained, with  $\varepsilon$  = 0.373 and  $\kappa$   $\approx$  3  $\cdot$   $10^{-8} m^2$ .

# Notes About the COMSOL Implementation

The geometry is characterized by narrow regions and a complex free surface. This has a great influence on the mesh. The automatic meshing algorithm can be tuned by certain manual settings, which is beneficial for both mesh quality and performance.

Application Library path: Subsurface\_Flow\_Module/Fluid\_Flow/
pore\_scale\_flow\_3d

# Modeling Instructions

From the File menu, choose New.

## NEW

In the New window, click 🙆 Model Wizard.

## MODEL WIZARD

- I In the Model Wizard window, click 间 3D.
- 2 In the Select Physics tree, select Fluid Flow>Single-Phase Flow>Creeping Flow (spf).
- 3 Click Add.
- 4 Click  $\bigcirc$  Study.

5 In the Select Study tree, select General Studies>Stationary.

6 Click **M** Done.

## GEOMETRY I

- I In the Model Builder window, under Component I (compl) click Geometry I.
- 2 In the Settings window for Geometry, locate the Units section.
- 3 From the Length unit list, choose cm.

#### Import I (imp1)

- I In the **Home** toolbar, click **Import**.
- 2 In the Settings window for Import, locate the Import section.
- 3 Click Browse.
- **4** Browse to the model's Application Libraries folder and double-click the file pore\_scale\_flow\_3d.mphbin.
- 5 Click Import.

## GLOBAL DEFINITIONS

Add some parameters that are used to set up the model.

## Parameters 1

- I In the Model Builder window, under Global Definitions click Parameters I.
- 2 In the Settings window for Parameters, locate the Parameters section.

Name	Expression	Value	Description
rho_f	1000[kg/m^3]	1000 kg/m <sup>3</sup>	Fluid density
mu_f	1e-3[Pa*s]	0.001 Pa·s	Fluid viscosity
u_in	1e-4[m/s]	IE-4 m/s	Inlet velocity
width	2[cm]	0.02 m	REV width
length	6[cm]	0.06 m	REV length
V_tot	width^2*length	2.4E-5 m <sup>3</sup>	Total REV volume

**3** In the table, enter the following settings:

## MATERIALS

Water

- I In the Model Builder window, under Component I (compl) right-click Materials and choose Blank Material.
- 2 In the Settings window for Material, type Water in the Label text field.
- 3 Locate the Material Contents section. In the table, enter the following settings:

Property	Variable	Value	Unit	Property group
Density	rho	rho_f	kg/m³	Basic
Dynamic viscosity	mu	mu_f	Pa∙s	Basic

## CREEPING FLOW (SPF)

- I In the Model Builder window, under Component I (compl) click Creeping Flow (spf).
- 2 In the Settings window for Creeping Flow, click to expand the Discretization section.
- 3 From the Discretization of fluids list, choose PI+PI.

Linear elements reduce the number of degrees of freedom to be solved. Because the geometry already requires a fine mesh, using linear elements decreases computational time and memory requirements while maintaining sufficient accuracy.

Inlet 1

- I In the Physics toolbar, click 📄 Boundaries and choose Inlet.
- **2** Select Boundary 2 only.
- 3 In the Settings window for Inlet, locate the Velocity section.
- **4** In the  $U_0$  text field, type u\_in.
- 5 Locate the Boundary Selection section. Click 🗞 Create Selection.

6 In the Create Selection dialog box, type Inlet in the Selection name text field.

7 Click OK.

#### Outlet I

- I In the Physics toolbar, click 📄 Boundaries and choose Outlet.
- **2** Select Boundaries 16 and 22 only.
- 3 In the Settings window for Outlet, locate the Pressure Conditions section.
- 4 Select the Normal flow check box.
- 5 Locate the Boundary Selection section. Click he Create Selection.
- 6 In the Create Selection dialog box, type Outlet in the Selection name text field.
- 7 Click OK.

Create a few more selections to use them throughout the model set up.

## DEFINITIONS

In the Model Builder window, expand the Definitions node.

Wall

- I In the Model Builder window, expand the Component I (compl)>Definitions>Selections node.
- 2 Right-click Component I (compl)>Definitions>Selections and choose Explicit.
- 3 In the Settings window for Explicit, type Wall in the Label text field.
- 4 Locate the Input Entities section. From the Geometric entity level list, choose Boundary.
- **5** Select Boundaries 4, 10, and 30 only.

It might be easier to select the correct boundary by using the **Selection List** window. To open this window, in the **Home** toolbar click **Windows** and choose **Selection List**. (If you are running the cross-platform desktop, you find **Windows** in the main menu.)

## All boundaries

- I In the Definitions toolbar, click 🛯 🐂 Explicit.
- 2 In the Settings window for Explicit, locate the Input Entities section.
- 3 From the Geometric entity level list, choose Boundary.
- 4 Select the All boundaries check box.
- 5 In the Label text field, type All boundaries.

#### Symmetry

I In the **Definitions** toolbar, click 🛅 **Difference**.

- 2 In the Settings window for Difference, type Symmetry in the Label text field.
- **3** Locate the **Geometric Entity Level** section. From the **Level** list, choose **Boundary**.
- 4 Locate the Input Entities section. Under Selections to add, click + Add.
- 5 In the Add dialog box, select All boundaries in the Selections to add list.
- 6 Click OK.
- 7 In the Settings window for Difference, locate the Input Entities section.
- 8 Under Selections to subtract, click + Add.
- 9 In the Add dialog box, in the Selections to subtract list, choose Inlet, Outlet, and Wall.10 Click OK.

## CREEPING FLOW (SPF)

#### Symmetry I

- I In the Physics toolbar, click 🔚 Boundaries and choose Symmetry.
- 2 In the Settings window for Symmetry, locate the Boundary Selection section.
- 3 From the Selection list, choose Symmetry.

## MESH I

The physics-controlled mesh is a good starting point. Modify the sequence to help the meshing algorithm and improve overall quality.

- I In the Model Builder window, under Component I (compl) click Mesh I.
- 2 In the Settings window for Mesh, locate the Physics-Controlled Mesh section.
- **3** From the **Element size** list, choose **Extra fine**.
- 4 Locate the Mesh Settings section. From the Sequence type list, choose Usercontrolled mesh.

#### Size

- I In the Model Builder window, under Component I (compl)>Mesh I click Size.
- 2 In the Settings window for Size, click to expand the Element Size Parameters section.
- 3 Locate the Element Size section. Click the Custom button.
- **4** Locate the **Element Size Parameters** section. In the **Maximum element size** text field, type 0.125.

Size 1

- I In the Model Builder window, click Size I.
- 2 In the Settings window for Size, locate the Element Size section.

- **3** Click the **Custom** button.
- 4 Locate the Element Size Parameters section. Select the Maximum element size check box.
- **5** In the associated text field, type **0.06**.

## Corner Refinement I

- I In the Model Builder window, right-click Corner Refinement I and choose Disable.
- 2 In the Settings window for Corner Refinement, click 📗 Build Selected.

Corner refinement will not improve the mesh significantly, hence it is disabled.

Free Triangular 1

- I In the Mesh toolbar, click  $\bigwedge$  Boundary and choose Free Triangular.
- 2 In the Settings window for Free Triangular, locate the Boundary Selection section.
- 3 From the Selection list, choose Wall.
- 4 Click 🖷 Build Selected.

Creating a surface mesh for the freeform surface first, helps the meshing algorithm.



To ensure a good quality of the mesh, we use the following settings for the tetrahedral mesh.

Free Tetrahedral I

I In the Model Builder window, click Free Tetrahedral I.

- 2 In the Settings window for Free Tetrahedral, click to expand the Element Quality Optimization section.
- **3** Select the **Avoid too large elements** check box.
- 4 Select the Avoid too small elements check box.
- 5 From the Optimization level list, choose Medium.
- 6 Click 📗 Build All.
- 7 In the Mesh toolbar, click A Plot.

## RESULTS

Mesh I

Compare with Figure 2. The overall mesh quality is very good. The amount of low quality mesh elements (skewness) is low. Mesh quality is not an indication for the accuracy of the solution, but it affects the convergence behavior.

## STUDY I

- I In the Model Builder window, click Study I.
- 2 In the Settings window for Study, locate the Study Settings section.
- **3** Clear the **Generate default plots** check box.
- **4** In the **Home** toolbar, click **= Compute**.

Create the plot as shown in Figure 3.

5 In the Results toolbar, click More Datasets and choose Surface.

## RESULTS

Surface 1

- I In the Settings window for Surface, locate the Selection section.
- 2 From the Selection list, choose Wall.

#### Velocity

- I In the **Results** toolbar, click **a 3D Plot Group**.
- 2 In the Settings window for 3D Plot Group, type Velocity in the Label text field.
- 3 Locate the Plot Settings section. Clear the Plot dataset edges check box.

## Surface 1

- I Right-click Velocity and choose Surface.
- 2 In the Settings window for Surface, locate the Data section.

- 3 From the Dataset list, choose Surface I.
- **4** Locate the **Expression** section. In the **Expression** text field, type **1**.
- 5 Locate the Coloring and Style section. From the Coloring list, choose Uniform.
- 6 From the Color list, choose Gray.

#### Line 1

- I In the Model Builder window, right-click Velocity and choose Line.
- 2 In the Settings window for Line, locate the Data section.
- 3 From the Dataset list, choose Surface I.
- 4 Locate the Coloring and Style section. From the Coloring list, choose Uniform.
- **5** From the **Color** list, choose **Black**.
- 6 In the Velocity toolbar, click **O** Plot.
- 7 From the **Color** list, choose **Custom**. Choose a darker shade of gray.

## Streamline 1

- I Right-click Velocity and choose Streamline.
- 2 In the Settings window for Streamline, locate the Selection section.
- **3** From the **Selection** list, choose **Inlet**.
- 4 Locate the Streamline Positioning section. In the Number text field, type 40.
- **5** Locate the **Coloring and Style** section. Find the **Line style** subsection. From the **Type** list, choose **Tube**.
- 6 Select the Radius scale factor check box.
- 7 In the associated text field, type 0.0075.

#### Color Expression 1

- I Right-click Streamline I and choose Color Expression.
- 2 In the Settings window for Color Expression, locate the Coloring and Style section.
- 3 From the Color table list, choose AuroraBorealis.

Next, determine the porosity and permeability for our REV in order to perform simulations at the macroscopic level.

#### DEFINITIONS

Integration 1 (intop1)

- I In the Definitions toolbar, click *P* Nonlocal Couplings and choose Integration.
- 2 In the Settings window for Integration, locate the Source Selection section.

3 From the Selection list, choose All domains.

## Average Inlet

- I In the Definitions toolbar, click *N* Nonlocal Couplings and choose Average.
- 2 In the Settings window for Average, locate the Source Selection section.
- 3 From the Geometric entity level list, choose Boundary.
- 4 From the Selection list, choose Inlet.
- 5 In the Label text field, type Average Inlet.

#### Average Outlet

- I In the Definitions toolbar, click Nonlocal Couplings and choose Average.
- 2 In the Settings window for Average, locate the Source Selection section.
- **3** From the **Geometric entity level** list, choose **Boundary**.
- 4 From the Selection list, choose Outlet.
- 5 In the Label text field, type Average Outlet.

Variables I

- I In the Model Builder window, right-click Definitions and choose Variables.
- 2 In the Settings window for Variables, locate the Variables section.
- **3** In the table, enter the following settings:

Name	Expression	Unit	Description
por	<pre>intop1(1)/V_tot</pre>		Porosity
dPdL	-(aveop2(p)-aveop1(p))/ length	N/m³	Pressure drop
u_out	<pre>spf.out1.massFlowRate/ rho_f/width^2</pre>	m/s	Superficial outlet velocity
kappa	u_out*mu_f/dPdL	m²	Permeability

Since you have introduced new variables, the solution needs to be updated. It is not necessary to compute the study again.

## STUDY I

In the **Study** toolbar, click *C* **Update Solution**.

## RESULTS

Global Evaluation 1

I In the Results toolbar, click (8.5) Global Evaluation.

- 2 In the Settings window for Global Evaluation, click Add Expression in the upper-right corner of the Expressions section. From the menu, choose Component I (compl)> Definitions>Variables>por Porosity.
- 3 Click Add Expression in the upper-right corner of the Expressions section. From the menu, choose Component 1 (comp1)>Definitions>Variables>kappa - Permeability - m<sup>2</sup>.
- 4 Click **= Evaluate**.

The results are shown in the **Table** window. The porosity is 0.373 and the permeability is about  $3e-8m^2$ . These results can now be used for calculations of large scale models.

Streamline 1

To make the results even more descriptive, create an animation.

- I In the Model Builder window, click Streamline I.
- 2 In the Settings window for Streamline, locate the Coloring and Style section.
- **3** Find the **Point style** subsection. From the **Type** list, choose **Interactive arrow**.
- 4 In the Extra release times text field, type 2000.

Animation I

I In the Velocity toolbar, click **IIII** Animation and choose Player.

You can adjust the number of frames to obtain a smooth animation of the interactive arrows. This animation visualizes very nicely that there are regions in the porous medium with both very slow and very high velocity.