



# Estimating Permeability from Microscale Porous Structures

## Introduction

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This tutorial model demonstrates how to compute porosity and permeability of a sphere packing from a fully resolved microscopic model. These values are then used to model the sphere packing on the macroscopic scale using Darcy's Law.

## Model Definition

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The geometry used to calculate the permeability in the microscopic scale is shown in the figure below.

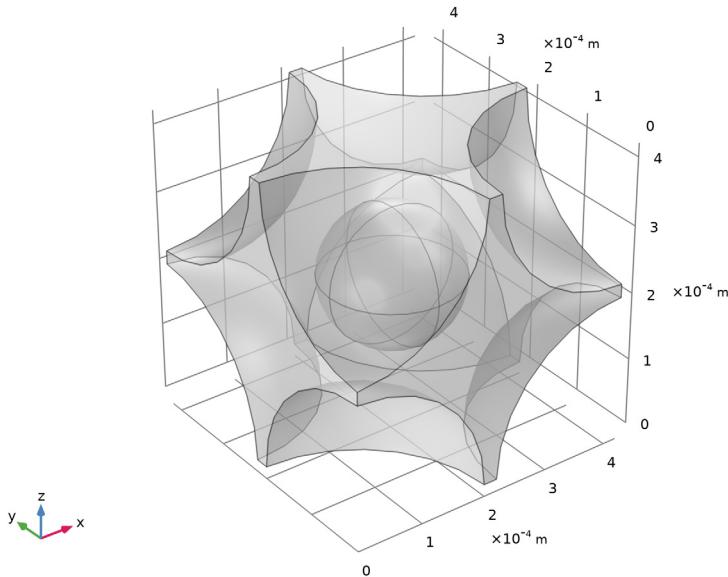


Figure 1: Geometry of the unit cell of a sphere packing.

A body-centered cubic (BCC) lattice is built with spheres of two different diameters. The flow around the spheres is described using the Creeping Flow interface. A pressure difference of 1 Pa is applied between inlet (top boundary) and outlet (bottom boundary). The boundaries that correspond to the solid-fluid interface are defined as walls and the remaining boundaries are symmetry boundaries.

From this set up the porosity and permeability can be calculated. The porosity is defined as the fraction of pore space volume  $V_{\text{fluid}}$  to total volume  $V_{\text{tot}}$

$$\varepsilon = \frac{V_{\text{fluid}}}{V_{\text{tot}}}.$$

To calculate the permeability  $\kappa$  ( $\text{m}^2$ ) the following relationship is used

$$\mathbf{u} = -\frac{\kappa}{\mu} \nabla p$$

whereas the pressure gradient  $\nabla p$  is replaced by the pressure difference between inlet and outlet  $\Delta p$  divided by the size of the unit cell  $L$  and the velocity vector  $\mathbf{u}$  is replaced by the outlet velocity  $u_{\text{out}}$  in flow direction ( $z$  direction), hence

$$\kappa = u_{\text{out}} \mu \frac{L}{\Delta p}.$$

The resulting values are then used to model the sphere packing on the macroscopic scale. The block height is 10 times larger than the height of the unit cell and the base area is 3 times larger.

To verify that a macroscopic approach using homogenization by deploying Darcy's Law gives accurate values, the model compares the mass flow per unit area ( $\text{L}^2$ ).

### *Results and Discussion*

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The velocity field in the unit cell is shown in [Figure 2](#).

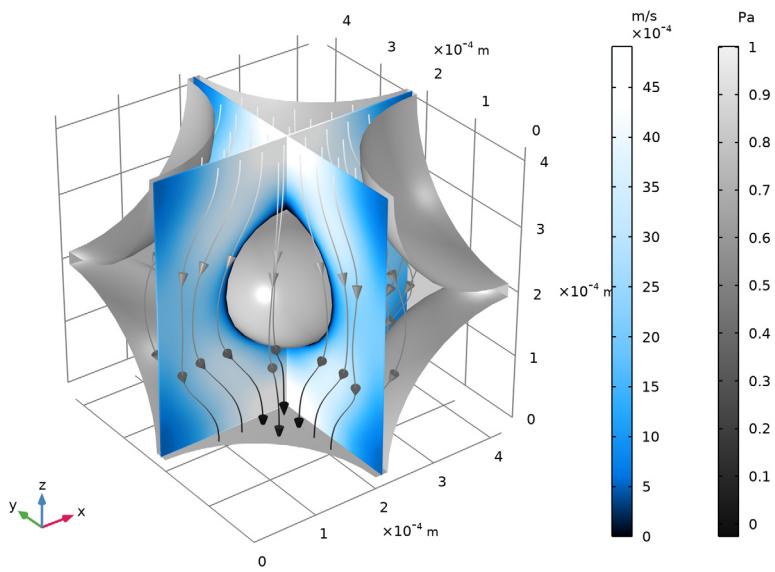


Figure 2: Velocity in the unit cell.

From the simulation the values for the porosity and permeability are obtained, with  $\varepsilon = 0.49571$  and  $\kappa \approx 3 \cdot 10^{-10} \text{ m}^2$ .

Figure 3 shows the pressure distribution in the macroscopic model.

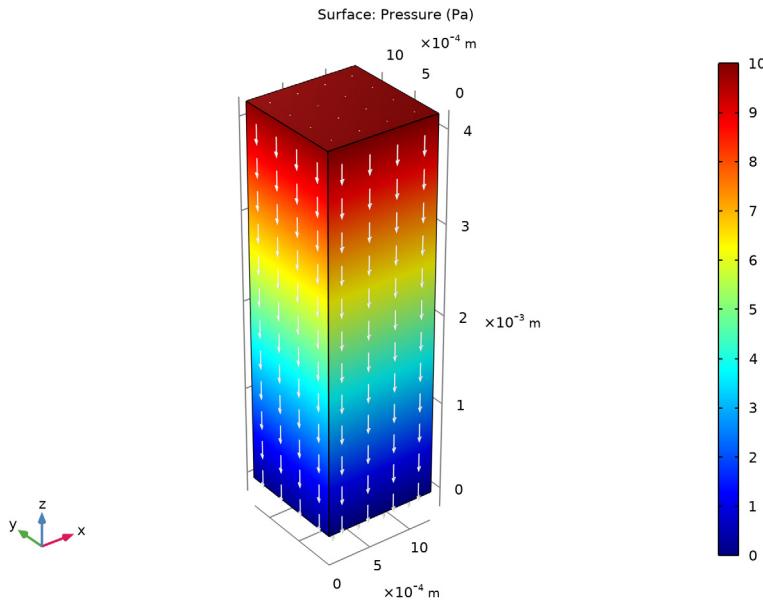


Figure 3: Pressure field for the macroscopic model.

The mass flow per unit area is about  $0.72 \text{ kg}/(\text{m}^2 \cdot \text{s})$  in both models.

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**Application Library path:** Subsurface\_Flow\_Module/Fluid\_Flow/permeability\_estimation

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### Modeling Instructions

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From the **File** menu, choose **New**.

#### NEW

In the **New** window, click  **Model Wizard**.

#### MODEL WIZARD

- 1 In the **Model Wizard** window, click  **3D**.
- 2 In the **Select Physics** tree, select **Fluid Flow>Single-Phase Flow>Creeping Flow (spf)**.

- 3 Click **Add**.
- 4 Click  **Study**.
- 5 In the **Select Study** tree, select **General Studies>Stationary**.
- 6 Click  **Done**.

## GEOMETRY I

Insert the geometry sequence from a file.

- 1 In the **Geometry** toolbar, click  **Insert Sequence**.
- 2 Browse to the model's Application Libraries folder and double-click the file `permeability_estimation_geom_sequence.mph`.
- 3 In the **Geometry** toolbar, click  **Build All**.
- 4 Click the  **Go to Default View** button in the **Graphics** toolbar.

Add some parameters to the **Parameters** list, that are used to set up the physics. The list already contains the parameters used to create the geometry and were imported automatically with the geometry sequence.

## GLOBAL DEFINITIONS

*Parameters I*

- 1 In the **Model Builder** window, under **Global Definitions** click **Parameters I**.
- 2 In the **Settings** window for **Parameters**, locate the **Parameters** section.
- 3 In the table, enter the following settings:

Name	Expression	Value	Description
p_in	1[Pa]	1 Pa	Inlet pressure
rho_f	1000[kg/m^3]	1000 kg/m <sup>3</sup>	Fluid density
mu_f	1e-3[Pa*s]	0.001 Pa·s	Fluid viscosity

## MATERIALS

*Material I (mat1)*

- 1 In the **Model Builder** window, under **Component 1 (comp1)** right-click **Materials** and choose **Blank Material**.
- 2 In the **Settings** window for **Material**, locate the **Material Contents** section.

3 In the table, enter the following settings:

Property	Variable	Value	Unit	Property group
Density	rho	rho_f	kg/m <sup>3</sup>	Basic
Dynamic viscosity	mu	mu_f	Pa·s	Basic

### CREEPING FLOW (SPF)

#### Inlet 1

- 1 In the **Model Builder** window, under **Component 1 (comp1)** right-click **Creeping Flow (spf)** and choose **Inlet**.
- 2 In the **Settings** window for **Inlet**, locate the **Boundary Condition** section.
- 3 From the list, choose **Pressure**.
- 4 Locate the **Pressure Conditions** section. In the  $p_0$  text field, type  $p_{in}$ .
- 5 Select Boundary 6 only.

#### Outlet 1

- 1 In the **Physics** toolbar, click  **Boundaries** and choose **Outlet**.
- 2 Select Boundary 5 only.

#### Symmetry 1

- 1 In the **Physics** toolbar, click  **Boundaries** and choose **Symmetry**.
- 2 Select Boundaries 1, 2, 9, and 22 only.

Use the **Mass Properties** to create a variable for the fluid volume and add variables that calculate porosity and permeability.

### DEFINITIONS

- 1 Click the  **Show More Options** button in the **Model Builder** toolbar.
- 2 In the **Show More Options** dialog box, in the tree, select the check box for the node **General>Variable Utilities**.
- 3 Click **OK**.

#### Mass Properties 1 (mass1)

- 1 In the **Model Builder** window, under **Component 1 (comp1)** right-click **Definitions** and choose **Variable Utilities>Mass Properties**.
- 2 Clear all check boxes, except the one for **Create volume variable**.

### Variables 1

- 1 In the **Model Builder** window, right-click **Definitions** and choose **Variables**.
- 2 In the **Settings** window for **Variables**, locate the **Variables** section.
- 3 In the table, enter the following settings:

Name	Expression	Unit	Description
por	mass1.volume/L^3		Porosity
u_out	spf.out1.Mflow/rho_f/ L^2	m/s	Outlet velocity
k0	u_out*mu_f*L/p_in	m <sup>2</sup>	Permeability

### MESH 1

To ensure a good mesh quality while keeping the model size small restrict the boundary layer thickness.

- 1 In the **Model Builder** window, under **Component 1 (comp1)** click **Mesh 1**.
- 2 In the **Settings** window for **Mesh**, locate the **Mesh Settings** section.
- 3 From the **Sequence type** list, choose **User-controlled mesh**.

### Boundary Layer Properties 1

- 1 In the **Model Builder** window, expand the **Component 1 (comp1)>Mesh 1>Boundary Layers 1** node, then click **Boundary Layer Properties 1**.
- 2 In the **Settings** window for **Boundary Layer Properties**, locate the **Boundary Layer Properties** section.
- 3 From the **Thickness of first layer** list, choose **Manual**.
- 4 Click  **Build All**.

### STUDY 1

#### Solution 1 (sol1)

- 1 In the **Study** toolbar, click  **Show Default Solver**.  
The model size is relatively small and a direct solver can be used.
- 2 In the **Model Builder** window, expand the **Solution 1 (sol1)** node.
- 3 Right-click **Study 1>Solver Configurations>Solution 1 (sol1)>Stationary Solver 1** and choose **Direct**.
- 4 In the **Model Builder** window, expand the **Study 1>Solver Configurations>Solution 1 (sol1)>Stationary Solver 1** node.

5 Right-click **Study 1>Solver Configurations>Solution 1 (sol1)>Stationary Solver 1>Direct, fluid flow variables (spf)** and choose **Compute**.

## RESULTS

Two default plots are created automatically. Modify the velocity plot to obtain [Figure 2](#).

### *Slice*

- 1 In the **Model Builder** window, expand the **Velocity (spf)** node, then click **Slice**.
- 2 In the **Settings** window for **Slice**, locate the **Plane Data** section.
- 3 In the **Planes** text field, type 1.
- 4 Locate the **Coloring and Style** section. From the **Color table** list, choose **JupiterAuroraBorealis**.

### *Slice 2*

- 1 Right-click **Slice** and choose **Duplicate**.
- 2 In the **Settings** window for **Slice**, locate the **Plane Data** section.
- 3 From the **Plane** list, choose **zx-planes**.
- 4 In the **Planes** text field, type 1.
- 5 Click to expand the **Inherit Style** section. From the **Plot** list, choose **Slice**.
- 6 In the **Velocity (spf)** toolbar, click  **Plot**.

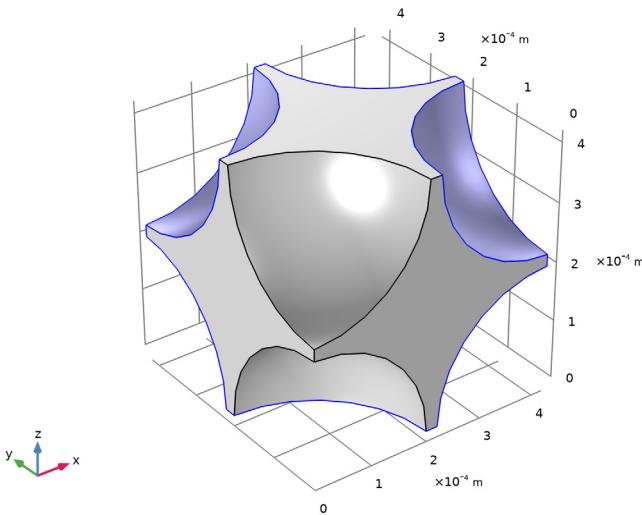
### *Surface 1*

- 1 In the **Model Builder** window, right-click **Velocity (spf)** and choose **Surface**.
- 2 In the **Settings** window for **Surface**, locate the **Expression** section.
- 3 In the **Expression** text field, type 1.
- 4 Locate the **Coloring and Style** section. From the **Coloring** list, choose **Uniform**.
- 5 From the **Color** list, choose **Gray**.

### *Selection 1*

- 1 Right-click **Surface 1** and choose **Selection**.
- 2 In the **Settings** window for **Selection**, locate the **Selection** section.

3 From the **Selection** list, choose **All boundaries** and remove the top, bottom and front boundaries as shown below.



#### Streamline 1

- 1 In the **Model Builder** window, right-click **Velocity (spf)** and choose **Streamline**.
- 2 Select Boundary 6 only.
- 3 In the **Settings** window for **Streamline**, locate the **Streamline Positioning** section.
- 4 In the **Number** text field, type 40.
- 5 Locate the **Coloring and Style** section. Find the **Point style** subsection. From the **Type** list, choose **Arrow**.
- 6 Select the **Number of arrows** check box.
- 7 In the associated text field, type 80.
- 8 In the **Velocity (spf)** toolbar, click **Plot**.

#### Color Expression 1

- 1 Right-click **Streamline 1** and choose **Color Expression**.
- 2 In the **Settings** window for **Color Expression**, locate the **Expression** section.
- 3 In the **Expression** text field, type  $p$ .
- 4 Locate the **Coloring and Style** section. From the **Color table** list, choose **GrayPrint**.

### *Velocity (spf)*

- 1 In the **Model Builder** window, click **Velocity (spf)**.
- 2 In the **Settings** window for **3D Plot Group**, click to expand the **Title** section.
- 3 From the **Title type** list, choose **None**.
- 4 Locate the **Plot Settings** section. Clear the **Plot dataset edges** check box.
- 5 Locate the **Color Legend** section. Select the **Show units** check box.
- 6 In the **Velocity (spf)** toolbar, click  **Plot**.

### *Global Evaluation* |

- 1 In the **Results** toolbar, click  **Global Evaluation**.
- 2 In the **Settings** window for **Global Evaluation**, click **Add Expression** in the upper-right corner of the **Expressions** section. From the menu, choose **Component 1 (compl)>Definitions>Variables>k0 - Permeability - m<sup>2</sup>**.
- 3 Click **Add Expression** in the upper-right corner of the **Expressions** section. From the menu, choose **Component 1 (compl)>Definitions>Variables>por - Porosity**.
- 4 Click  **Evaluate**.

### **TABLE**

- 1 Go to the **Table** window.

The permeability is about  $3^{-10} \text{ m}^2$  and the porosity is 0.49571.

### **ROOT**

Now, add a new component to set up a Darcy's Law interface and use these values.

### **ADD COMPONENT**

In the **Model Builder** window, right-click the root node and choose **Add Component>3D**.

### **GEOMETRY 2**

#### *Block 1 (blk1)*

- 1 In the **Geometry** toolbar, click  **Block**.
- 2 In the **Settings** window for **Block**, locate the **Size and Shape** section.
- 3 In the **Width** text field, type  $3*L$ .
- 4 In the **Depth** text field, type  $3*L$ .
- 5 In the **Height** text field, type  $10*L$ .
- 6 Click  **Build All Objects**.

## ADD PHYSICS

- 1 In the **Home** toolbar, click  **Add Physics** to open the **Add Physics** window.
- 2 Go to the **Add Physics** window.
- 3 In the tree, select **Fluid Flow>Porous Media and Subsurface Flow>Darcy's Law (dl)**.
- 4 Find the **Physics interfaces in study** subsection. In the table, clear the **Solve** check box for **Study 1**.
- 5 Click **Add to Component 2** in the window toolbar.
- 6 In the **Home** toolbar, click  **Add Physics** to close the **Add Physics** window.

## DARCY'S LAW (DL)

### Fluid and Matrix Properties 1

- 1 In the **Model Builder** window, under **Component 2 (comp2)>Darcy's Law (dl)** click **Fluid and Matrix Properties 1**.
- 2 In the **Settings** window for **Fluid and Matrix Properties**, locate the **Fluid Properties** section.
- 3 From the  $\rho$  list, choose **User defined**. In the associated text field, type `rho_f`.
- 4 From the  $\mu$  list, choose **User defined**. In the associated text field, type `mu_f`.
- 5 Locate the **Matrix Properties** section. From the  $\epsilon_p$  list, choose **User defined**. In the associated text field, type `0.49571`.
- 6 From the  $\kappa$  list, choose **User defined**. In the associated text field, type `3.0292E-10`.

### Pressure 1

- 1 In the **Physics** toolbar, click  **Boundaries** and choose **Pressure**.
- 2 Select Boundary 4 only.

The macroscopic model is ten times larger in the direction of the pressure gradient than the unit cell. To compare the results of the unit cell with the results of the macroscopic model set the pressure to  $10*p_{in}$ .

- 3 In the **Settings** window for **Pressure**, locate the **Pressure** section.
- 4 In the  $p_0$  text field, type `10*p_in`.

### Pressure 2

- 1 In the **Physics** toolbar, click  **Boundaries** and choose **Pressure**.
- 2 Select Boundary 3 only.

## ADD STUDY

- 1 In the **Home** toolbar, click  **Add Study** to open the **Add Study** window.

- 2 Go to the **Add Study** window.
- 3 Find the **Studies** subsection. In the **Select Study** tree, select **General Studies>Stationary**.
- 4 Disable the Creeping Flow interface for this study.
- 5 Click **Add Study** in the window toolbar.
- 6 In the **Home** toolbar, click  **Add Study** to close the **Add Study** window.

## STUDY 2

- 1 In the **Model Builder** window, click **Study 2**.
- 2 In the **Home** toolbar, click  **Compute**.

## RESULTS

Again default plots are created. Modify the pressure plot to obtain [Figure 3](#).

### Arrow Surface 1

- 1 In the **Model Builder** window, right-click **Pressure (dl)** and choose **Arrow Surface**.
- 2 In the **Settings** window for **Arrow Surface**, click to expand the **Title** section.
- 3 From the **Title type** list, choose **None**.
- 4 Locate the **Coloring and Style** section. From the **Color** list, choose **White**.
- 5 In the **Pressure (dl)** toolbar, click  **Plot**.

Finally, compare the models using predefined mass flow variables. For the unit cell divide the variable by the area of the unit cell which is  $L^2$  and not the area of the outlet boundary. For the macroscopic model use  $(3*L)^2$ . The resulting values should be almost identical.

### Global Evaluation 2

- 1 In the **Results** toolbar, click  **Global Evaluation**.
- 2 In the **Settings** window for **Global Evaluation**, click **Replace Expression** in the upper-right corner of the **Expressions** section. From the menu, choose **Component 1 (compl)> Creeping Flow>Auxiliary variables>spf.out1.massFlowRate - Outward mass flow rate across feature selection - kg/s.**
- 3 Locate the **Expressions** section. In the table, enter the following settings:

Expression	Unit	Description
<code>spf.out1.massFlowRate/L^2</code>	<code>kg/(m^2*s)</code>	Unit mass flow

- 4 Click  **Evaluate**.

### Global Evaluation 3

- 1 In the **Results** toolbar, click  **Global Evaluation**.
- 2 In the **Settings** window for **Global Evaluation**, locate the **Data** section.
- 3 From the **Dataset** list, choose **Study 2/Solution 2 (3) (sol2)**.
- 4 Click **Replace Expression** in the upper-right corner of the **Expressions** section. From the menu, choose **Component 2 (comp2)>Darcy's Law>Mass flow>dl.pr2.Mflow - Mass flow - kg/s**.
- 5 Locate the **Expressions** section. In the table, enter the following settings:

Expression	Unit	Description
$dl.pr2.Mflow/(3*L)^2$	kg / (m^2*s)	Unit mass flow

- 6 Click the **Evaluate** button. The results appear in the **Table** window.