



Surface Cracked Cylinder

Introduction

This model reproduces a NAFEMS benchmark (Ref. 1) in which a pressurized cylinder has a horizontal semi-elliptic crack on its inner surface. The energy release rate is calculated along the crack front using the J-integral method. The stress intensity factors calculated from the J-integral are then compared to the reference data.

Model Definition

Since the model consists on a cylinder with a horizontal crack on its mid plane, only a quarter of the whole geometry is built. A tube around the elliptical crack front is built to generate a swept mesh along it. The swept mesh makes the J-integral curve along the crack front smoother.

A **Symmetry** condition is applied on the thickness boundaries of the cylinder. On the crack boundaries, it is overridden by a **Crack** feature. The crack is symmetric to define explicitly the crack front. Due to the symmetry, the stress intensity factors for modes II and III will be zero.

The pressure load can be applied with a **Boundary Load** feature on the inner face of the cylinder, and with the **Face Load** subfeature on the crack face .

The J-integral at a point of the crack front is composed of a closed contour integral and a surface integral. The closed contour integral is composed of a circular integral around the crack front and an integral on the crack face to take into account the force applied on it.

$$J = \int_{\Gamma} W_s \mathbf{m} \cdot \mathbf{e}_1 - (\boldsymbol{\sigma} \cdot \mathbf{m}) \cdot (\nabla \mathbf{u} \cdot \mathbf{e}_1) dl + \int_{\Gamma_{\text{face}}} \mathbf{F}_A \cdot (\nabla \mathbf{u} \cdot \mathbf{e}_1) dl + \int_A \nabla [(\boldsymbol{\sigma} \cdot \mathbf{t}_1) \cdot (\nabla \mathbf{u} \cdot \mathbf{e}_1)] dA$$

The stress intensity factor in mode I is then computed from the J-integral

$$K_I = \sqrt{\frac{E^* J}{1 + \beta_K^2}}$$

where E^* is the equivalent Young's modulus. A 2D plane strain condition is assumed, so

$$E^* = \frac{E}{1 - \nu^2}.$$

Results and Discussion

Figure 1 shows the default stress plot, where a stress concentration around the crack front is visible. Figure 2 shows a closer view of this stress concentration

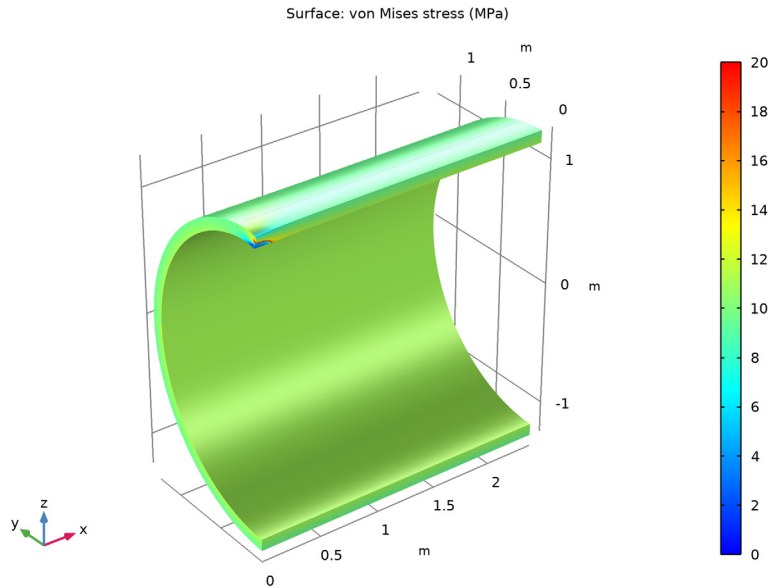


Figure 1: Von Mises stress in the cylinder and at the crack front.

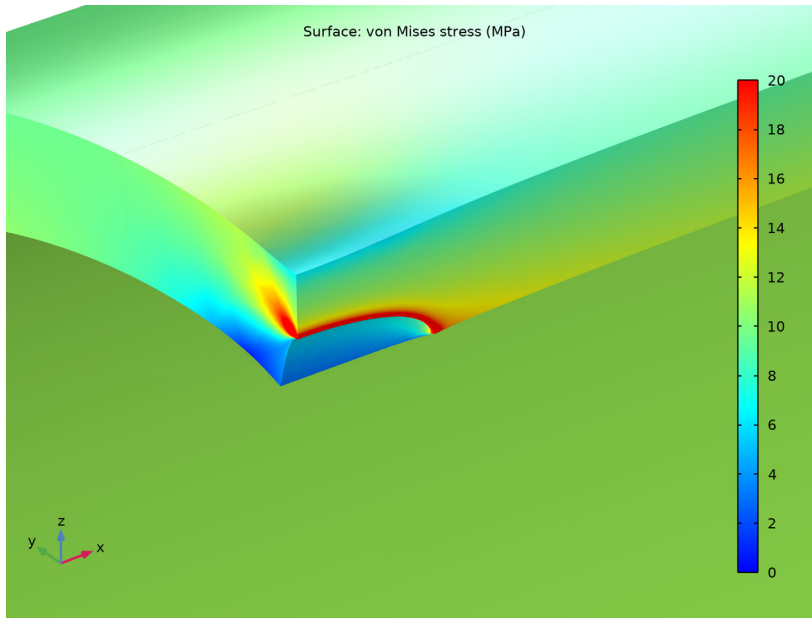


Figure 2: Close view of the stress concentration at the crack front.

The J-integral calculated along the crack front represents the energy release rate per surface area when the crack is locally extended in the crack plane direction. The crack extension

direction, the J-integral intensity and the surface contour used for integral calculation are plotted in [Figure 3](#)

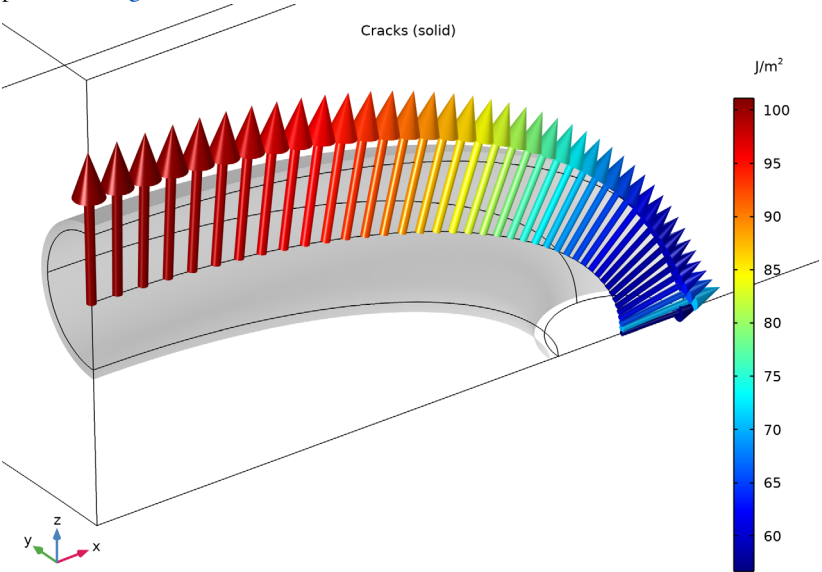


Figure 3: 3D plot of J-integral: crack growth direction, intensity and contour used for the calculation.

[Figure 4](#) plots the J-integral as function of the parametric angle along the elliptic crack. A jump in the curve at zero angle can be noticed. This glitch is often seen near free boundaries due to the singularities that contribute to the surface integral.

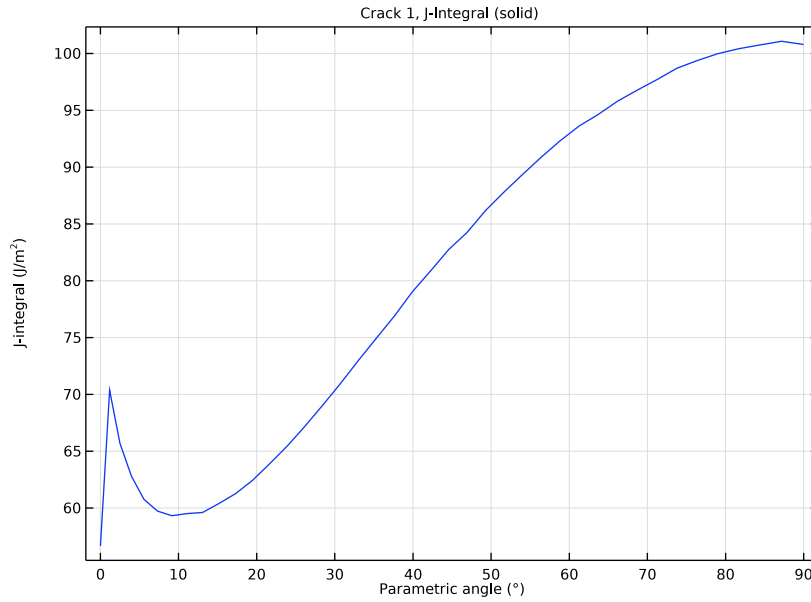


Figure 4: J-integral along the crack front.

From the J-integral the stress intensity factor in mode I can be calculated and compared to the benchmark in [Ref. 1](#), see [Figure 5](#).

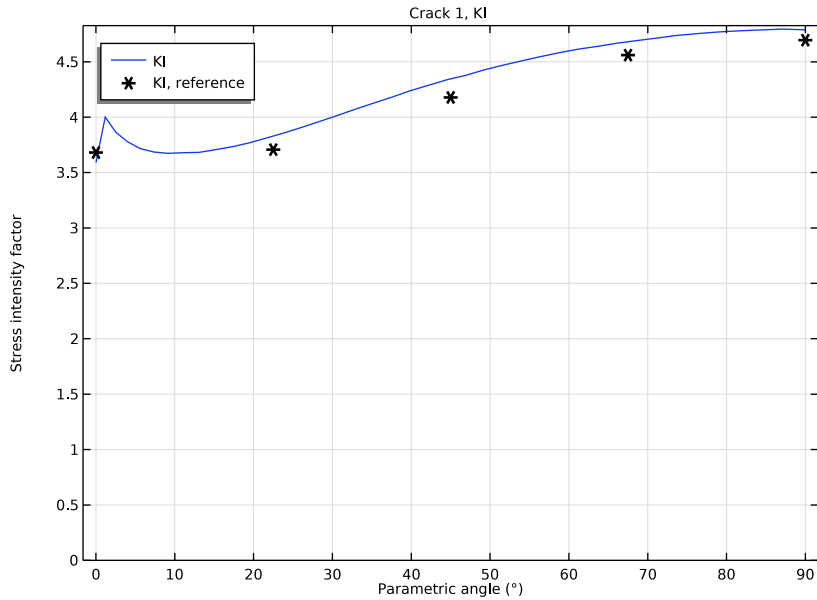


Figure 5: Stress intensity factor along the crack front compared to the benchmark.

Reference


1. R. Judge and B. Madsen, *Three Dimensional Test Cases in Linear Elastic Fracture Mechanics, part 4: Surface Cracked Cylinder*, NAFEMS, 1993.

Application Library path: Structural_Mechanics_Module/Fracture_Mechanics/surface_cracked_cylinder




Modeling Instructions

From the **File** menu, choose **New**.



NEW

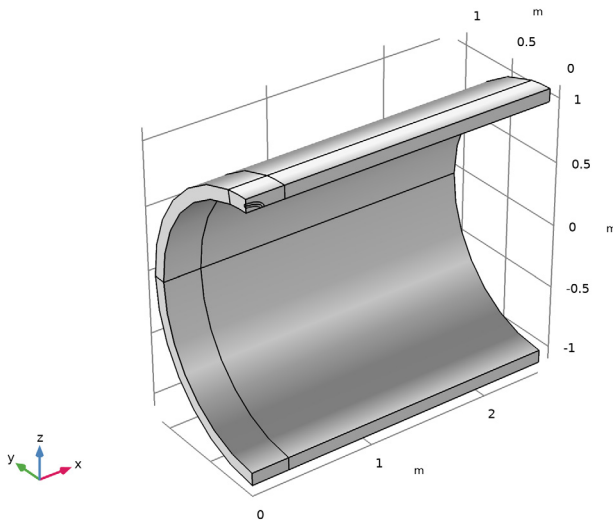
In the **New** window, click  **Model Wizard**.

MODEL WIZARD

- 1 In the **Model Wizard** window, click  **3D**.
- 2 In the **Select Physics** tree, select **Structural Mechanics>Solid Mechanics (solid)**.
- 3 Click **Add**.
- 4 Click  **Study**.
- 5 In the **Select Study** tree, select **General Studies>Stationary**.
- 6 Click  **Done**.

GOMETRY I

- 1 In the **Geometry** toolbar, click  **Insert Sequence**.
- 2 Browse to the model's Application Libraries folder and double-click the file `surface_cracked_cylinder.mph`.
- 3 In the **Geometry** toolbar, click  **Build All**.



GLOBAL DEFINITIONS

Parameters I


- 1 In the **Model Builder** window, under **Global Definitions** click **Parameters I**.
- 2 In the **Settings** window for **Parameters**, locate the **Parameters** section.

3 In the table, enter the following settings:

Name	Expression	Value	Description
p	1 [MPa]	1E6 Pa	Pressure load

MATERIALS

Material 1 (mat1)

1 In the **Materials** toolbar, click  **Blank Material**.

2 In the **Settings** window for **Material**, locate the **Material Contents** section.

3 In the table, enter the following settings:

Property	Variable	Value	Unit	Property group
Young's modulus	E	207 [GPa]	Pa	Basic
Poisson's ratio	nu	0.3	l	Basic
Density	rho	8000	kg/m ³	Basic

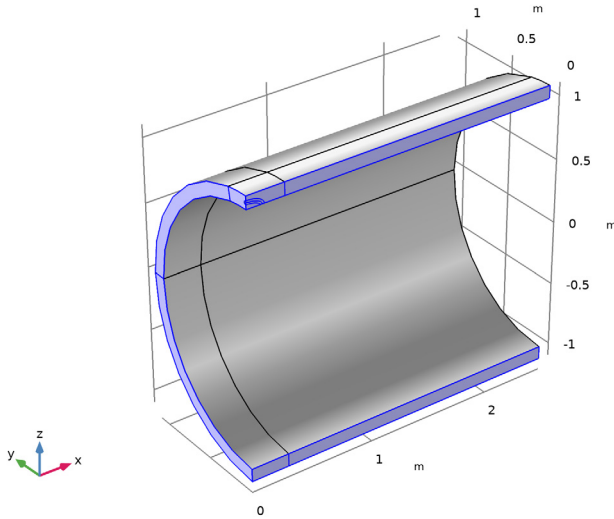
SOLID MECHANICS (SOLID)

Symmetry 1


1 In the **Model Builder** window, under **Component 1 (comp1)** right-click **Solid Mechanics (solid)** and choose **More Constraints>Symmetry**.

2 Select Boundaries 1, 2, 5, 6, 8, 9, 11, 12, 15, 22, and 26 only.

The **Symmetry** condition is applied on the whole thickness of the cylinder. It will be overridden by the **Crack** feature on the crack surface.



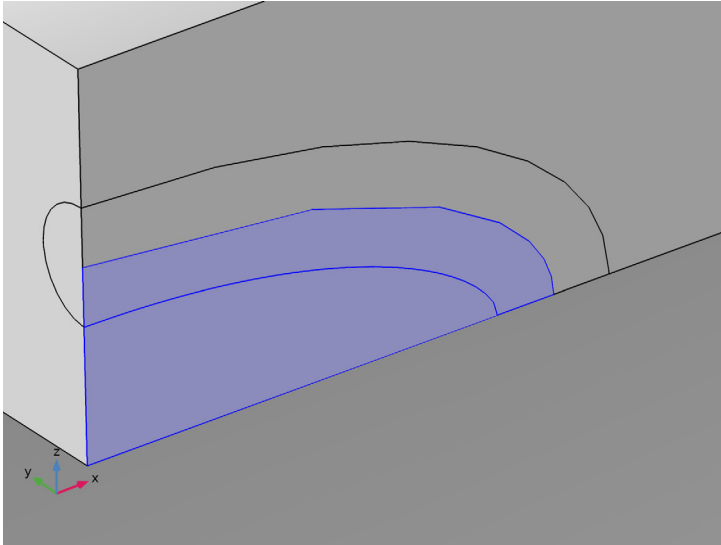
Prescribed Displacement I

- 1 In the **Physics** toolbar, click  **Points** and choose **Prescribed Displacement**.
- 2 Select Point 11 only.
- 3 In the **Settings** window for **Prescribed Displacement**, locate the **Prescribed Displacement** section.
- 4 Select the **Prescribed in z direction** check box.

Crack I


- 1 In the **Physics** toolbar, click  **Boundaries** and choose **Crack**.

2 Select Boundaries 6 and 9 only.



3 In the **Settings** window for **Crack**, locate the **Crack Definition** section.

4 From the **Crack surface** list, choose **Symmetric**.

5 Click to expand the **Crack Front** section. Click  **Clear Selection**.

6 Select Edge 13 only.

J-Integral 1

In the **Physics** toolbar, click  **Attributes** and choose **J-Integral**.

Boundary Load 1

1 In the **Physics** toolbar, click  **Boundaries** and choose **Boundary Load**.

Apply pressure load on the inner face of the cylinder. The pressure on the crack will be applied under the **Crack** feature.

2 Select Boundaries 4, 7, 17, 24, 27, and 31 only.

3 In the **Settings** window for **Boundary Load**, locate the **Force** section.

4 From the **Load type** list, choose **Pressure**.

5 In the p text field, type p .

Crack 1

In the **Model Builder** window, click **Crack 1**.

Face Load 1

- 1 In the **Physics** toolbar, click  **Attributes** and choose **Face Load**.
- 2 In the **Settings** window for **Face Load**, locate the **Load** section.
- 3 In the p text field, type p .

MESH 1

Free Triangular 1

- In the **Mesh** toolbar, click  **Boundary** and choose **Free Triangular**.

Size

- 1 In the **Model Builder** window, click **Size**.
- 2 In the **Settings** window for **Size**, locate the **Element Size** section.
- 3 Click the **Custom** button.
- 4 Locate the **Element Size Parameters** section. In the **Maximum element size** text field, type $th/2$.
- 5 In the **Minimum element size** text field, type $a/200$.

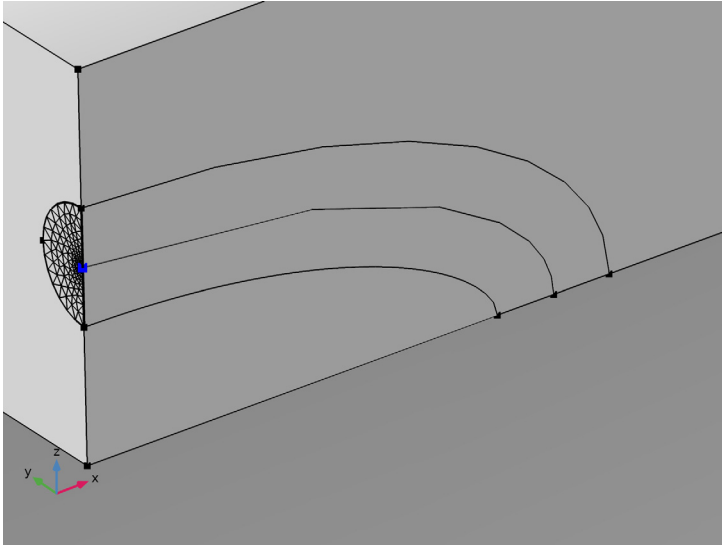
Free Triangular 1

- 1 In the **Model Builder** window, click **Free Triangular 1**.
- 2 Select Boundary 8 only.


Size 1

- 1 Right-click **Free Triangular 1** and choose **Size**.
- 2 In the **Settings** window for **Size**, locate the **Geometric Entity Selection** section.
- 3 From the **Geometric entity level** list, choose **Point**.
- 4 Select Point 5 only.
- 5 Locate the **Element Size** section. Click the **Custom** button.
- 6 Locate the **Element Size Parameters** section. Select the **Maximum element size** check box.
- 7 In the associated text field, type $a/100$.
- 8 Select the **Minimum element size** check box.
- 9 In the associated text field, type $a/200$.
- 10 Select the **Maximum element growth rate** check box.
- 11 In the associated text field, type 1.2 .

12 Click  **Build Selected**.



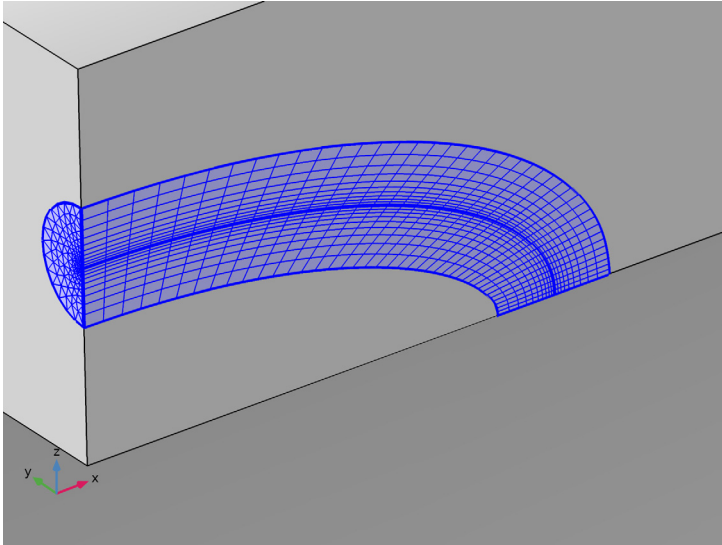
Swept 1

- 1 In the **Mesh** toolbar, click  **Swept**.
- 2 In the **Settings** window for **Swept**, locate the **Domain Selection** section.
- 3 From the **Geometric entity level** list, choose **Domain**.
- 4 Select Domain 3 only.


Distribution 1

- 1 Right-click **Swept 1** and choose **Distribution**.
- 2 In the **Settings** window for **Distribution**, locate the **Distribution** section.
- 3 From the **Distribution type** list, choose **Predefined**.
- 4 In the **Number of elements** text field, type 40.
- 5 In the **Element ratio** text field, type 6.
- 6 Select the **Reverse direction** check box.

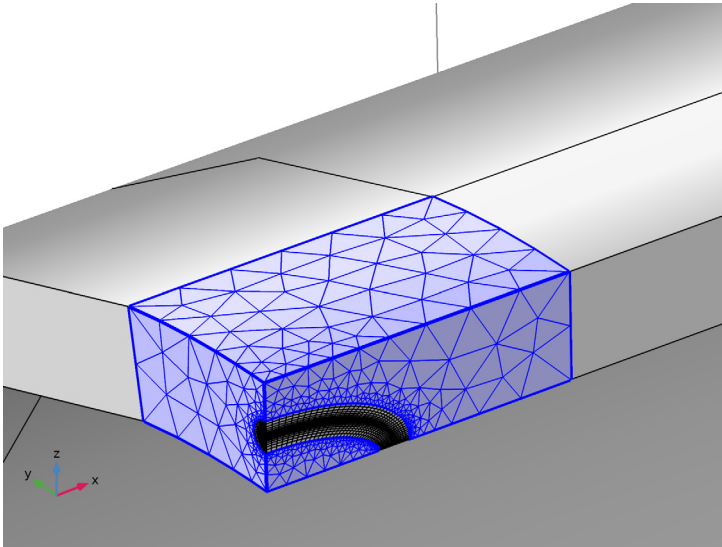
7 Click  **Build Selected**.




Free Tetrahedral 1

- 1 In the **Mesh** toolbar, click  **Free Tetrahedral**.
- 2 In the **Settings** window for **Free Tetrahedral**, locate the **Domain Selection** section.
- 3 From the **Geometric entity level** list, choose **Domain**.
- 4 Select Domain 2 only.


5 Click  **Build Selected**.



Swept 2

- 1 In the **Mesh** toolbar, click  **Swept**.
- 2 In the **Settings** window for **Swept**, locate the **Domain Selection** section.
- 3 From the **Geometric entity level** list, choose **Domain**.
- 4 Select Domains 1 and 4 only.

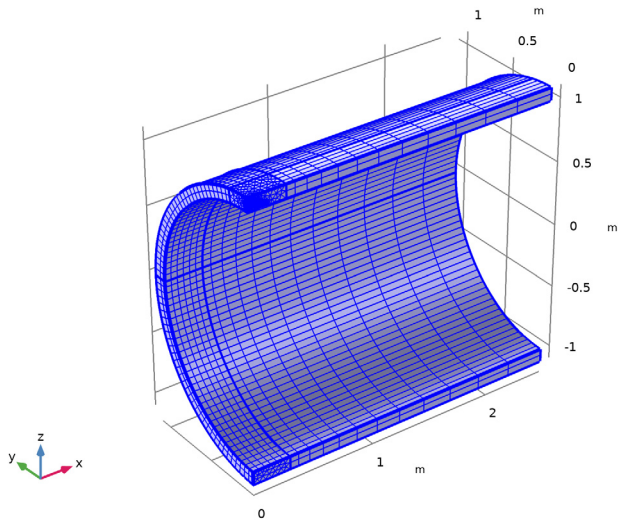
Swept 3

In the **Mesh** toolbar, click  **Swept**.


Distribution 1

- 1 Right-click **Swept 3** and choose **Distribution**.
- 2 In the **Settings** window for **Distribution**, locate the **Distribution** section.
- 3 From the **Distribution type** list, choose **Predefined**.
- 4 In the **Number of elements** text field, type 12.
- 5 In the **Element ratio** text field, type 4.

6 Click  **Build All**.



STUDY I

In the **Home** toolbar, click  **Compute**.


RESULTS

Surface I

- 1 In the **Model Builder** window, expand the **Stress (solid)** node, then click **Surface I**.
- 2 In the **Settings** window for **Surface**, locate the **Expression** section.
- 3 From the **Unit** list, choose **MPa**.
- 4 Click to expand the **Range** section. Select the **Manual color range** check box.
- 5 In the **Minimum** text field, type 0.
- 6 In the **Maximum** text field, type 20.

Stress (solid)

- 1 In the **Model Builder** window, click **Stress (solid)**.
- 2 In the **Settings** window for **3D Plot Group**, locate the **Plot Settings** section.
- 3 Clear the **Plot dataset edges** check box.

4 In the **Stress (solid)** toolbar, click  **Plot**.

The first default plot shows von Mises stress. On a closer view the maximum stress is visible along the crack front, see [Figure 2](#).


There is also a default arrow plot that shows the crack direction and the intensity of J-integral, [Figure 3](#). The contour used to calculate the J-integral is displayed as well.

Cracks (solid)

To plot the J-integral ([Figure 4](#)) along crack front, you may introduce the parametric angle along the ellipse.

DEFINITIONS

Variables I

- 1 In the **Home** toolbar, click  **Variables** and choose **Local Variables**.
- 2 In the **Settings** window for **Variables**, locate the **Geometric Entity Selection** section.
- 3 From the **Geometric entity level** list, choose **Domain**.
- 4 Select Domains 2 and 3 only.
- 5 Locate the **Variables** section. In the table, enter the following settings:


Name	Expression	Unit	Description
angle	$\text{atan2}((Z-R1)/a, X/c)$	rad	Parametric angle

STUDY I

In the **Study** toolbar, click  **Update Solution**.

RESULTS

J-Integral I

- 1 In the **Model Builder** window, expand the **Crack I, J-Integral (solid)** node, then click **J-Integral I**.
- 2 In the **Settings** window for **Line Graph**, locate the **x-Axis Data** section.
- 3 From the **Parameter** list, choose **Expression**.
- 4 In the **Expression** text field, type angle.
- 5 From the **Unit** list, choose °.
- 6 In the **Crack I, J-Integral (solid)** toolbar, click  **Plot**.

Duplicate the last plot group to plot and compare the stress intensity factor to the reference value, [Figure 5](#)


Crack I, KI

- 1 In the **Model Builder** window, right-click **Crack I, J-Integral (solid)** and choose **Duplicate**.
- 2 In the **Settings** window for **ID Plot Group**, type Crack 1, KI in the **Label** text field.

KI

- 1 In the **Model Builder** window, expand the **Crack I, KI** node, then click **J-Integral I**.
- 2 In the **Settings** window for **Line Graph**, type KI in the **Label** text field.
- 3 Click **Replace Expression** in the upper-right corner of the **y-Axis Data** section. From the menu, choose **Component I (comp1)>Solid Mechanics>Cracks>solid.crack1.jint1.KI - Stress intensity factor, mode I**.
- 4 Locate the **y-Axis Data** section. In the **Expression** text field, type `solid.crack1.jint1.KI/1e6`.

Crack I, KI

- 1 In the **Model Builder** window, click **Crack I, KI**.
- 2 In the **Settings** window for **ID Plot Group**, locate the **Plot Settings** section.
- 3 Select the **x-axis label** check box.
- 4 Select the **y-axis label** check box.
- 5 In the associated text field, type `Stress intensity factor`.
- 6 Locate the **Axis** section. Select the **Manual axis limits** check box.
- 7 In the **y minimum** text field, type 0.
- 8 In the **Crack I, KI** toolbar, click  **Plot**.

Import the stress intensity factors from benchmark to compare with the computed results.

Table I



- 1 In the **Results** toolbar, click  **Table**.
- 2 In the **Settings** window for **Table**, locate the **Data** section.
- 3 Click **Import**.
- 4 Browse to the model's Application Libraries folder and double-click the file `surface_cracked_cylinder_results.txt`.

Table Graph I

- 1 Right-click **Crack I, KI** and choose **Table Graph**.
- 2 In the **Settings** window for **Table Graph**, locate the **Coloring and Style** section.
- 3 From the **Color** list, choose **From theme**.
- 4 Find the **Line style** subsection. From the **Line** list, choose **None**.

- 5 Find the **Line markers** subsection. From the **Marker** list, choose **Asterisk**.
- 6 From the **Positioning** list, choose **In data points**.
- 7 In the **Crack I, KI** toolbar, click  **Plot**.

Crack I, KI

- 1 In the **Model Builder** window, click **Crack I, KI**.
- 2 In the **Settings** window for **ID Plot Group**, locate the **Legend** section.
- 3 Select the **Show legends** check box.
- 4 From the **Position** list, choose **Upper left**.

KI

- 1 In the **Model Builder** window, click **KI**.
- 2 In the **Settings** window for **Line Graph**, click to expand the **Legends** section.
- 3 Select the **Show legends** check box.
- 4 From the **Legends** list, choose **Manual**.
- 5 In the table, enter the following settings:

Legends
KI

Table Graph I

- 1 In the **Model Builder** window, click **Table Graph I**.
- 2 In the **Settings** window for **Table Graph**, click to expand the **Legends** section.
- 3 Select the **Show legends** check box.
- 4 From the **Legends** list, choose **Manual**.
- 5 In the table, enter the following settings:

Legends
KI, reference

