Created in COMSOL Multiphysics 5.6

Evaluation of Dynamic Coefficients of a Plain Journal Bearing

Introduction

When analyzing rotors, it is common that bearings are modeled through their effective dynamic coefficients about a static equilibrium position. This example demonstrates a method to compute such coefficients for a plain journal bearing. Computed coefficients are compared to analytical values obtained from solving Reynolds equation, using a short bearing approximation. To make the comparison meaningful, the length of the bearing is taken to be much smaller than its diameter.

Model Definition

The plain journal bearing has a radius of 0.1 m, and a length of 0.04 m. The angular velocity of the journal is 1000 rad/s, and the clearance between the journal and the bearing is 0.0001 m. The viscosity and density of the lubricant are taken as 0.02 Pa·s and 866 kg/m^3, respectively. To find the equilibrium position corresponding to different static loads, the journal weight is varied from 500 N to 50,000 N.

Bearing stiffness and damping coefficients are computed for the equilibrium positions by solving a perturbed form of Reynolds equation.

The dimensionless stiffness and damping coefficients obtained from an analytical solution of Reynolds equation (Ref. 1) are:

$$k_{22} = \frac{4[16\varepsilon^2 + \pi^2(2-\varepsilon^2)]}{Q}$$

$$k_{23} = \frac{\pi[\pi^2(1-\varepsilon^2) - 16\varepsilon^4]}{Q\varepsilon\sqrt{1-\varepsilon^2}}$$

$$k_{32} = -\frac{\pi[\pi^2(1+2\varepsilon^2)(1-\varepsilon^2) + 32\varepsilon^2(1+\varepsilon^2)]}{Q(1-\varepsilon^2)}$$

$$k_{33} = \frac{4[\pi^2(1+2\varepsilon^2)(1-\varepsilon^2) + 32\varepsilon^2(1+\varepsilon^2)]}{Q(1-\varepsilon^2)}$$

and

$$c_{22} = \frac{2\pi[\pi^2(1+2\varepsilon^2) - 16\varepsilon^2]\sqrt{1-\varepsilon^2}}{Q\varepsilon}$$

$$c_{23} = \frac{8[16\varepsilon^2 - \pi^2(1+2\varepsilon^2)]}{Q}$$

$$c_{32} = c_{23}$$

$$c_{33} = \frac{2\pi[48\varepsilon^2 + \pi^2(1-\varepsilon^2)^2]}{Q\varepsilon\sqrt{1-\varepsilon^2}}$$

The parameter ε is the relative eccentricity of the journal. Q is given by

$$Q = [\pi^2 + (16-\pi^2)\varepsilon^2]^{3/2}$$

Scaling factors for the above dimensionless parameters are $k_0 = W/C$ for stiffness and $c_0 = W/(C\Omega)$ for damping. The bearing load is W, C is the clearance, and Ω is the angular speed of the journal.

Results and Discussion

Figure 1 shows how the journal eccentricity changes with the static load on the bearing. The figure shows that with increasing load, its effect on eccentricity decreases. This clearly depicts the nonlinear behavior of the bearing.

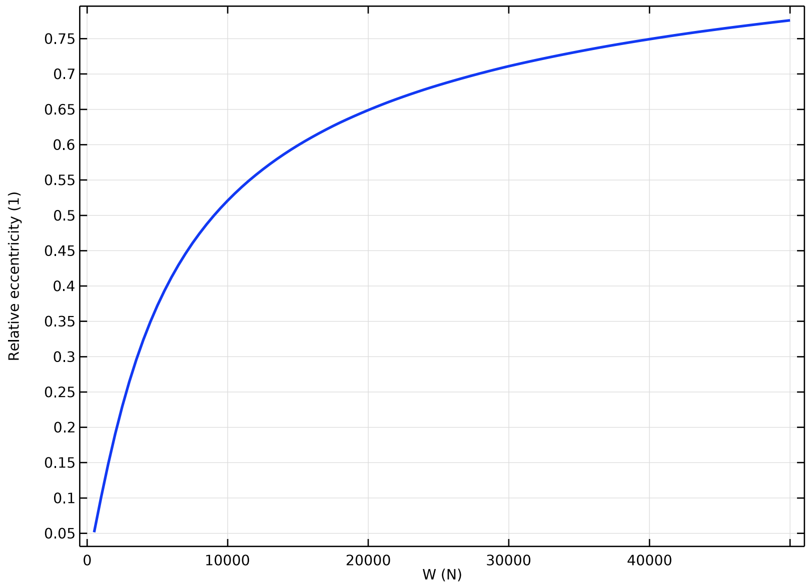

Figure 1: Eccentricity vs load.

Figure 2 shows the computed attitude angle with respect to loading direction, compared to the analytical curve. For small loads the curves coincide. With increasing loads, the journal becomes increasingly eccentric in the bearing. This produces a difference in shear forces at the minimum and maximum film thickness locations. The difference results in a net force on the journal. In high eccentricity cases, the journal equilibrium location is determined by the balance of external loads on the bearing, and the pressure and shear forces.

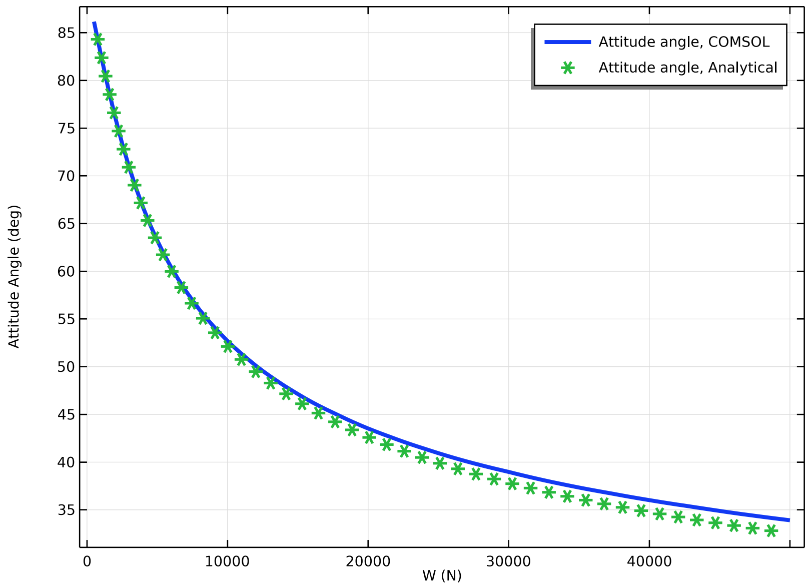

Figure 2: Attitude angle vs load.

The maximum film pressure and minimum film thickness are two important performance parameters for a bearing. These are plotted in Figure 3.

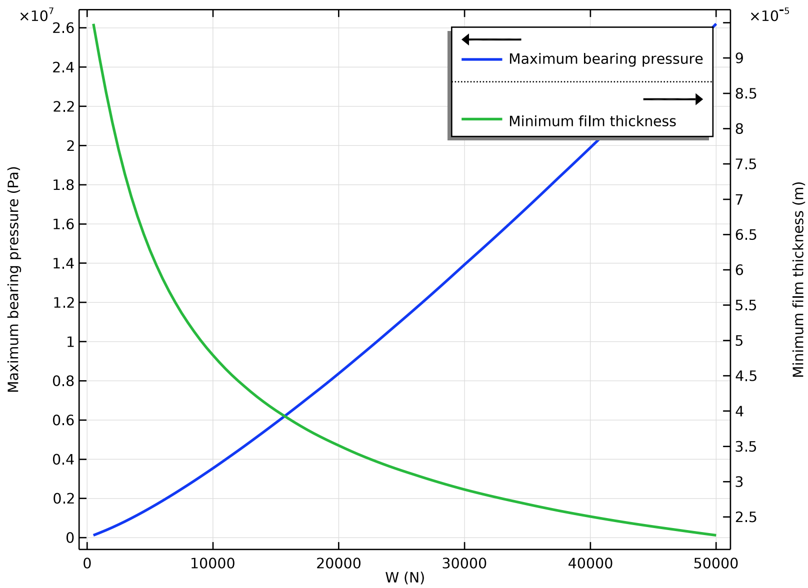

Figure 3: Maximum pressure and minimum film thickness vs load.

Figure 4 and Figure 5 compare the computed values of the dimensionless stiffness and dimensionless damping coefficient with the corresponding analytical values. The computed values match the analytical values.

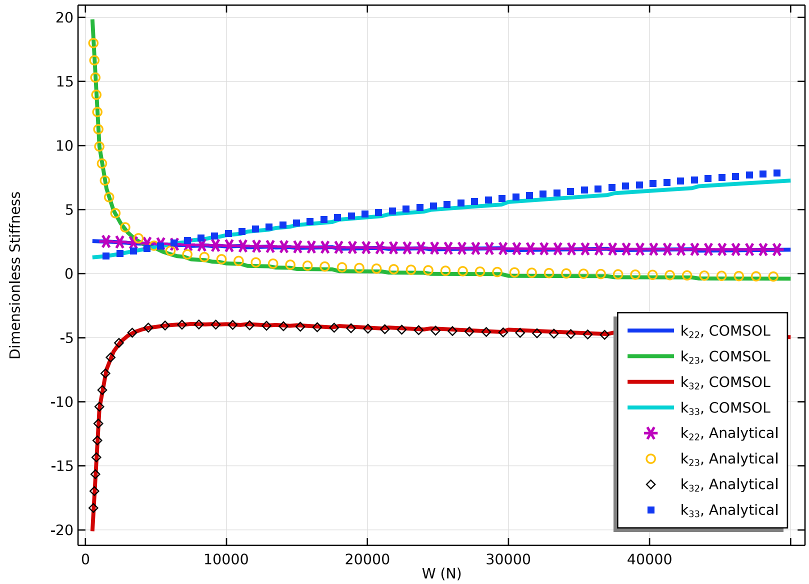

Figure 4: Dimensionless stiffness.

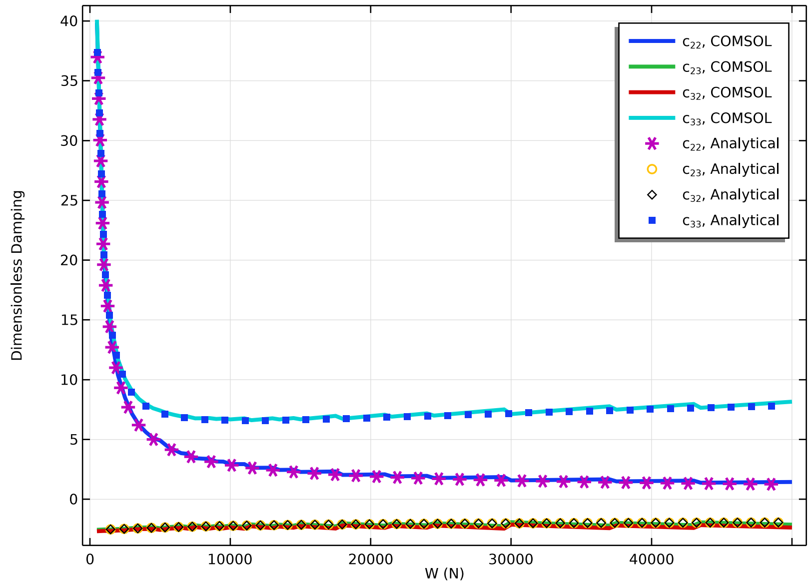

Figure 5: Dimensionless damping.

Reference

1. J.S. Rao, *Rotor Dynamics*, section 7.6, pp. 179–191, New Age International (P) Limited, 2014.

Application Library path: `Rotordynamics_Module/Verification_Examples/journal_bearing_dynamic_coefficients`

Modeling Instructions

From the **File** menu, choose **New**.

NEW

In the **New** window, click **Model Wizard**.

MODEL WIZARD

1 In the **Model Wizard** window, click **3D**.

2 In the **Select Physics** tree, select **Structural Mechanics>Rotordynamics>Hydrodynamic Bearing (hdb)**.

3 Click **Add**.

4 Click **Study**.

5 In the **Select Study** tree, select **General Studies>Stationary**.

6 Click **Done**.

GLOBAL DEFINITIONS

Parameters 1

1 In the **Model Builder** window, under **Global Definitions** click **Parameters 1**.

2 In the **Settings** window for **Parameters**, locate the **Parameters** section.

3 In the table, enter the following settings:

Name	Expression	Value	Description
Rj	0.1[m]	0.1 m	Journal radius
H	0.04[m]	0.04 m	Journal width
C	1e-4[m]	1E-4 m	Clearance
Omega	1000[rad/s]	1000 rad/s	Angular velocity
mu0	0.02[Pa*s]	0.02 Pa·s	Lubricant viscosity
rho0	866[kg/m^3]	866 kg/m³	Lubricant density
W	500[N]	500 N	Static load on bearing

GEOMETRY 1

Cylinder 1 (cyl1)

1 In the **Geometry** toolbar, click **Cylinder**.

2 In the **Settings** window for **Cylinder**, locate the **Object Type** section.

3 From the **Type** list, choose **Surface**.

4 Locate the **Size and Shape** section. In the **Radius** text field, type `Rj`.

5 In the **Height** text field, type `H`.

6 Locate the **Axis** section. From the **Axis type** list, choose **x-axis**.

7 Click **Build All Objects**.

Define the variables for the analytical stiffness and damping.

DEFINITIONS

Variables 1

1 In the **Model Builder** window, expand the **Component 1 (comp1)>Definitions** node.

2 Right-click **Definitions** and choose **Variables**.

3 In the **Settings** window for **Variables**, locate the **Variables** section.

4 In the table, enter the following settings:

Name	Expression	Unit	Description
k0	W/C	N/m	Stiffness scaling
c0	W/(C*Omega)	N·s/m	Damping scaling
e	hdb.hjb1.ec_rel		Eccentricity
phi0	atan2(pi*sqrt(1-e^2), 4*e)	rad	Attitude angle
Q	(pi^2+(16-pi^2)*e^2)^1.5		Auxiliary variable
k22	4*(16*e^2+pi^2*(2-e^2))/Q		Dimensionless stiffness, 22 component
k23	pi*(pi^2*(1-e^2)-16*e^4)/(e*sqrt(1-e^2)*Q)		Dimensionless stiffness, 23 component
k32	-pi*(pi^2*(1+2*e^2)*(1-e^2)+32*e^2*(1+e^2))/(e*sqrt(1-e^2)*Q)		Dimensionless stiffness, 32 component
k33	4*(pi^2*(1+2*e^2)*(1-e^2)+32*e^2*(1+e^2))/((1-e^2)*Q)		Dimensionless stiffness, 33 component
c22	2*pi*sqrt(1-e^2)*(pi^2*(1+2*e^2)-16*e^2)/(e*Q)		Dimensionless damping, 22 component
c23	8*(16*e^2-pi^2*(1+2*e^2))/Q		Dimensionless damping, 23 component
c32	c23		Dimensionless damping, 32 component
c33	2*pi*(48*e^2+pi^2*(1-e^2)^2)/(e*sqrt(1-e^2)*Q)		Dimensionless damping, 33 component

HYDRODYNAMIC BEARING (HDB)

1 In the **Model Builder** window, under **Component 1 (comp1)** click **Hydrodynamic Bearing (hdb)**.
2 In the **Settings** window for **Hydrodynamic Bearing**, locate the **Dynamic Coefficients** section.
3 Select the **Calculate dynamic coefficients** check box.

Hydrodynamic Journal Bearing 1

1 In the **Model Builder** window, under **Component 1 (comp1)>Hydrodynamic Bearing (hdb)** click **Hydrodynamic Journal Bearing 1**.
2 In the **Settings** window for **Hydrodynamic Journal Bearing**, locate the **Bearing Properties** section.
3 In the C text field, type `C`.
4 Locate the **Journal Properties** section. From the **Specify** list, choose **Load**.
5 Specify the $\mathbf{W}_\mathrm{j}$ vector as

0	x
0	y
-W	z

6 Specify the $\mathbf{u}_{J0}$ vector as

0	x
0	y
0	z

7 In the Ω text field, type `Omega`.

Choose the **Gümbel** boundary condition to consider the film with positive pressure only to participate in the load equilibrium.

8 Locate the **Film Boundary Condition** section. From the **Film type** list, choose **Gümbel**.
9 Locate the **Fluid Properties** section. From the μ list, choose **User defined**. In the associated text field, type `mu0`.
10 From the ρ list, choose **User defined**. In the associated text field, type `rho0`.

Use a mapped mesh to resolve the pressure.

MESH 1

Mapped 1

1 In the **Mesh** toolbar, click **Boundary** and choose **Mapped**.

2 In the **Settings** window for **Mapped**, locate the **Boundary Selection** section.

3 From the **Selection** list, choose **All boundaries**.

Distribution 1

1 Right-click **Mapped 1** and choose **Distribution**.

2 Select Edges 1, 2, 4, and 6 only.

3 In the **Settings** window for **Distribution**, locate the **Distribution** section.

4 In the **Number of elements** text field, type `15`.

Distribution 2

1 In the **Model Builder** window, right-click **Mapped 1** and choose **Distribution**.

2 Select Edge 7 only.

3 In the **Settings** window for **Distribution**, locate the **Distribution** section.

4 In the **Number of elements** text field, type `4`.

5 Click **Build All**.

STUDY 1

Step 1: Stationary

1 In the **Model Builder** window, under **Study 1** click **Step 1: Stationary**.

2 In the **Settings** window for **Stationary**, click to expand the **Study Extensions** section.

3 Select the **Auxiliary sweep** check box.

4 Click **Add**.

5 In the table, enter the following settings:

Parameter name	Parameter value list	Parameter unit
W (Static load on bearing)	`range(500,500,50000)`	N

6 In the **Home** toolbar, click **Compute**.

RESULTS

Fluid Pressure (hdb)

In the **Fluid Pressure (hdb)** toolbar, click **Plot**.

Use the following instructions to plot the eccentricity versus load curve shown in Figure 1.

Eccentricity

1 In the **Home** toolbar, click **Add Plot Group** and choose **1D Plot Group**.

2 In the **Settings** window for **1D Plot Group**, type `Eccentricity` in the **Label** text field.

Global 1

1 Right-click **Eccentricity** and choose **Global**.

2 In the **Settings** window for **Global**, click **Replace Expression** in the upper-right corner of the **y-Axis Data** section. From the menu, choose **Component 1 (comp1)> Hydrodynamic Bearing>Hydrodynamic Journal Bearing 1>hdb.hjb1.ec_rel - Relative eccentricity**.

3 Click to expand the **Coloring and Style** section. In the **Width** text field, type 2.

4 In the **Eccentricity** toolbar, click **Plot**.

5 Right-click **Global 1** and choose **Show Legends**.

Eccentricity

1 In the **Model Builder** window, click **Eccentricity**.

2 In the **Settings** window for **1D Plot Group**, click to expand the **Title** section.

3 From the **Title type** list, choose **None**.

4 In the **Eccentricity** toolbar, click **Plot**.

To compare the computed and analytical attitude angles shown in Figure 2, follow the below instructions.

Attitude Angle

1 In the **Home** toolbar, click **Add Plot Group** and choose **1D Plot Group**.

2 In the **Settings** window for **1D Plot Group**, type `Attitude Angle` in the **Label** text field.

Global 1

1 Right-click **Attitude Angle** and choose **Global**.

2 In the **Settings** window for **Global**, click **Replace Expression** in the upper-right corner of the **y-Axis Data** section. From the menu, choose **Component 1 (comp1)> Hydrodynamic Bearing>Hydrodynamic Journal Bearing 1>hdb.hjb1.phia - Attitude angle - rad**.

3 Locate the **y-Axis Data** section. In the table, enter the following settings:

Expression	Unit	Description
hdb.hjb1.phia	deg	Attitude angle, COMSOL

4 Locate the **Coloring and Style** section. In the **Width** text field, type `3`.

Global 2

1 In the **Model Builder** window, right-click **Attitude Angle** and choose **Global**.

2 In the **Settings** window for **Global**, locate the **y-Axis Data** section.

3 In the table, enter the following settings:

Expression	**Unit**	**Description**
`phi0`	`deg`	`Attitude angle, Analytical`

4 Locate the **Coloring and Style** section. Find the **Line style** subsection. From the **Line** list, choose **None**.

5 Find the **Line markers** subsection. From the **Marker** list, choose **Cycle**.

6 In the **Number** text field, type `50`.

7 In the **Attitude Angle** toolbar, click **Plot**.

Attitude Angle

1 In the **Model Builder** window, click **Attitude Angle**.

2 In the **Settings** window for **1D Plot Group**, locate the **Plot Settings** section.

3 Select the **y-axis label** check box.

4 In the associated text field, type `Attitude Angle (deg)`.

5 Locate the **Title** section. From the **Title type** list, choose **None**.

Duplicate the eccentricity plot and follow the instructions below to plot the maximum pressure, and minimum film thickness curves, as shown in Figure 3.

Pressure and Film Thickness

1 In the **Model Builder** window, right-click **Eccentricity** and choose **Duplicate**.

2 In the **Settings** window for **1D Plot Group**, type `Pressure and Film Thickness` in the **Label** text field.

Global 1

1 In the **Model Builder** window, expand the **Pressure and Film Thickness** node, then click **Global 1**.

2 In the **Settings** window for **Global**, click **Replace Expression** in the upper-right corner of the **y-Axis Data** section. From the menu, choose **Component 1 (comp1)>Hydrodynamic Bearing>Journal and bearing properties>hdb.hjb1.p_max - Maximum bearing pressure - Pa**.

Global 2

1 Right-click **Results>Pressure and Film Thickness>Global 1** and choose **Duplicate**.

2 In the **Settings** window for **Global**, click **Replace Expression** in the upper-right corner of the **y-Axis Data** section. From the menu, choose **Component 1 (comp1)> Hydrodynamic Bearing>Journal and bearing properties>hdb.hjb1.h_min - Minimum film thickness - m**.

Pressure and Film Thickness

1 In the **Model Builder** window, click **Pressure and Film Thickness**.

2 In the **Settings** window for **1D Plot Group**, locate the **Plot Settings** section.

3 Select the **Two y-axes** check box.

4 In the table, select the **Plot on secondary y-axis** check box for **Global 2**.

5 Right-click **Pressure and Film Thickness** and choose **Show Legends**.

6 In the **Pressure and Film Thickness** toolbar, click **Plot**.

Figure 4 compares the computed dimensionless stiffness to its analytical counterpart. Follow the instructions below to generate this plot.

Bearing Stiffness

1 In the **Home** toolbar, click **Add Plot Group** and choose **1D Plot Group**.

2 In the **Settings** window for **1D Plot Group**, type `Bearing Stiffness` in the **Label** text field.

Global 1

1 Right-click **Bearing Stiffness** and choose **Global**.

2 In the **Settings** window for **Global**, click **Replace Expression** in the upper-right corner of the **y-Axis Data** section. From the menu, choose **Component 1 (comp1)> Hydrodynamic Bearing>Dynamic coefficients>hdb.hjb1.k22 - Bearing stiffness, local yy component - N/m**.

3 Click **Add Expression** in the upper-right corner of the **y-Axis Data** section. From the menu, choose **Component 1 (comp1)>Hydrodynamic Bearing>Dynamic coefficients> hdb.hjb1.k23 - Bearing stiffness, local yz component - N/m**.

4 Click **Add Expression** in the upper-right corner of the **y-Axis Data** section. From the menu, choose **Component 1 (comp1)>Hydrodynamic Bearing>Dynamic coefficients> hdb.hjb1.k32 - Bearing stiffness, local zy component - N/m**.

5 Click **Add Expression** in the upper-right corner of the **y-Axis Data** section. From the menu, choose **Component 1 (comp1)>Hydrodynamic Bearing>Dynamic coefficients> hdb.hjb1.k33 - Bearing stiffness, local zz component - N/m**.

6 Locate the **y-Axis Data** section. In the table, enter the following settings:

Expression	Unit	Description
`hdb.hjb1.k22/k0`	1	`k<sub>22</sub>, COMSOL`
`hdb.hjb1.k23/k0`	1	`k<sub>23</sub>, COMSOL`
`hdb.hjb1.k32/k0`	1	`k<sub>32</sub>, COMSOL`
`hdb.hjb1.k33/k0`	1	`k<sub>33</sub>, COMSOL`

7 Locate the **Coloring and Style** section. In the **Width** text field, type `3`.

Global 2

1 In the **Model Builder** window, right-click **Bearing Stiffness** and choose **Global**.

2 In the **Settings** window for **Global**, locate the **y-Axis Data** section.

3 In the table, enter the following settings:

Expression	Unit	Description
`k22`	1	`k<sub>22</sub>, Analytical`
`k23`	1	`k<sub>23</sub>, Analytical`
`k32`	1	`k<sub>32</sub>, Analytical`
`k33`	1	`k<sub>33</sub>, Analytical`

4 Locate the **Coloring and Style** section. Find the **Line style** subsection. From the **Line** list, choose **None**.

5 Find the **Line markers** subsection. From the **Marker** list, choose **Cycle**.

6 In the **Number** text field, type `50`.

Bearing Stiffness

1 In the **Model Builder** window, click **Bearing Stiffness**.

2 In the **Settings** window for **1D Plot Group**, locate the **Plot Settings** section.

3 Select the **y-axis label** check box.

4 In the associated text field, type `Dimensionless Stiffness`.

5 Locate the **Title** section. From the **Title type** list, choose **None**.

6 Locate the **Legend** section. From the **Position** list, choose **Lower right**.

Figure 5 compares the computed dimensionless damping to its analytical counterpart. Follow the instructions below to generate this plot.

Bearing Damping Coefficient

1 In the **Home** toolbar, click **Add Plot Group** and choose **1D Plot Group**.

2 In the **Settings** window for **1D Plot Group**, type `Bearing Damping Coefficient` in the **Label** text field.

Global 1

1 Right-click **Bearing Damping Coefficient** and choose **Global**.

2 In the **Settings** window for **Global**, click **Replace Expression** in the upper-right corner of the **y-Axis Data** section. From the menu, choose **Component 1 (comp1)> Hydrodynamic Bearing>Dynamic coefficients>hdb.hjb1.c22 - Bearing damping coefficient, local yy component - N·s/m**.

3 Locate the **y-Axis Data** section. In the table, enter the following settings:

Expression	Unit	Description
hdb.hjb1.c22/c0	1	c₂₂, COMSOL
hdb.hjb1.c23/c0	1	c₂₃, COMSOL
hdb.hjb1.c32/c0	1	c₃₂, COMSOL
hdb.hjb1.c33/c0	1	c₃₃, COMSOL

4 Locate the **Coloring and Style** section. In the **Width** text field, type `3`.

Global 2

1 In the **Model Builder** window, right-click **Bearing Damping Coefficient** and choose **Global**.

2 In the **Settings** window for **Global**, locate the **y-Axis Data** section.

3 In the table, enter the following settings:

Expression	Unit	Description
c22	1	c₂₂, Analytical
c23	1	c₂₃, Analytical
c32	1	c₃₂, Analytical
c33	1	c₃₃, Analytical

4 Locate the **Coloring and Style** section. Find the **Line style** subsection. From the **Line** list, choose **None**.

5 Find the **Line markers** subsection. From the **Marker** list, choose **Cycle**.

6 In the **Number** text field, type `50`.

7 In the **Bearing Damping Coefficient** toolbar, click **Plot**.

Bearing Damping Coefficient

1 In the **Model Builder** window, click **Bearing Damping Coefficient**.

2 In the **Settings** window for **1D Plot Group**, locate the **Title** section.

3 From the **Title type** list, choose **None**.

4 Locate the **Plot Settings** section. Select the **y-axis label** check box.

5 In the associated text field, type `Dimensionless Damping`.