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Sagnac Interferometer

Introduction

The Sagnac effect is a phenomenon that arises when light propagates around a closed loop in a rotating frame of reference. Although the effect is fundamentally relativistic in nature, it can still be observed in a pure geometrical optics simulation when the frame of reference rotates slowly. An understanding of the Sagnac effect is essential to modern guidance and navigation systems, an area in which optical gyroscopes (or gyros) often prove to be a cost-effective alternative to mechanical gyros; they benefit from comparatively low maintenance costs because they have no moving parts.

This is a model of a simple Sagnac interferometer consisting of two mirrors and a beam splitter arranged in a triangle. The entire modeling domain rotates; as a result, the rays propagating in opposite directions in the triangle have different optical path lengths due to the Sagnac effect. This can be used to deduce the angular velocity of the system.

Model Definition

The model geometry is shown in Figure 1. The two mirrors and the beam splitter form an equilateral triangle in which light propagates in both directions.

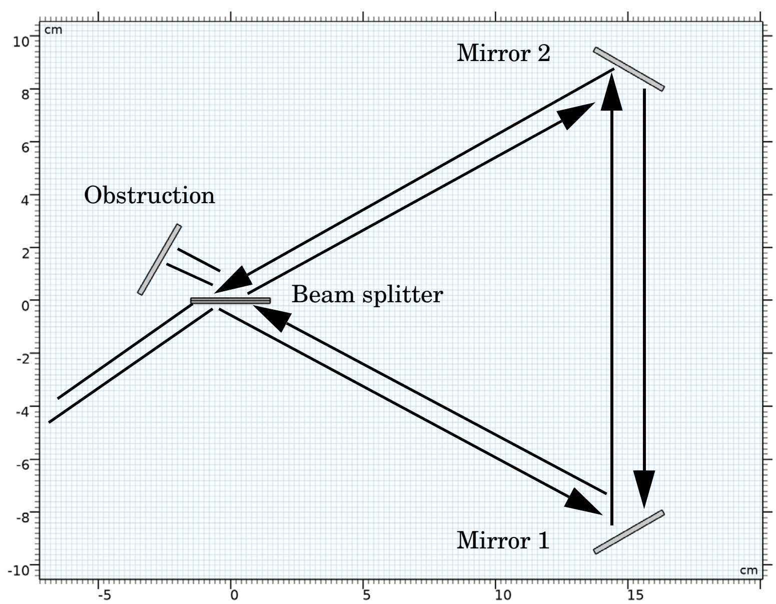

Figure 1: Diagram of the Sagnac interferometer. The entire assembly rotates, and rays propagate through the triangle in both directions.

The entire apparatus rotates at a constant angular velocity Ω (SI unit: rad/s).

In an active ring laser gyro, at least one side of the triangle would typically include a lasing medium, and the light exiting the triangle would typically pass through a prism to combine the outgoing beams. However, in this greatly simplified Sagnac interferometer model, these components are not considered.

For certain numerical considerations in the following sections it is convenient to know some geometry dimensions. Also assume the rays propagate in a vacuum, $n = 1$.

TABLE 1: INTERFEROMETER SPECIFICATION.

Name	Expression	Value	Description
λ_0	N/A	632.8 nm	Vacuum wavelength
R	N/A	10 cm	Ring radius
b	$b = R\sqrt{3}$	17.3 cm	Triangle side length
P	$P = 3b$	52.0 cm	Triangle perimeter
A	$A = b^2\sqrt{3}/4$	130. cm^2	Triangle area

THE SAGNAC EFFECT

The Sagnac effect is most easily illustrated by two counterpropagating beams of light, each constrained within a ring that is rotating at constant angular velocity Ω. This is shown in Figure 1; the beam propagating in the direction of rotation is shown as a solid line, whereas the beam propagating opposite the direction of rotation is shown as a dashed line. Assume that these diagrams are being observed from an inertial frame of reference.

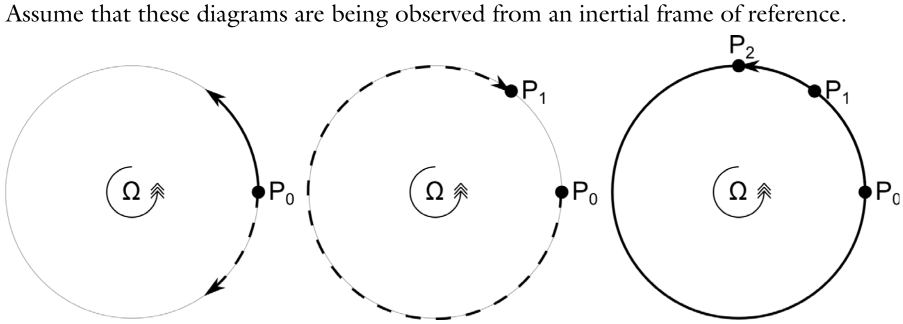

Figure 2: Diagram demonstrating the Sagnac effect in a rotating frame of reference

Initially both beams are released simultaneously from point P_0. Since the ring is rotating, the ray indicated by the dashed line reaches the release point at a new location P_1 before it reaches the original location P_0. Conversely, the ring indicated by the solid line reaches the release point at a third location P_2, having already passed through the original location at P_0. Thus the dashed line travels a shorter distance due to the rotation whereas the solid line travels a longer distance.

The counterpropagating beams thus recombine after having propagated for slightly different distances and times. It follows that there will be a phase difference between the beams when they recombine; this could be observed, for example, as a shift of the interference fringes.

In Ref. 1 it is shown that the magnitude of the optical path difference due to the Sagnac effect is not affected by the shape of the path but only its enclosed area. Assuming the axis of rotation is perpendicular to the plane of the interferometer, so that the angular velocity can be treated as a scalar, the difference in transit time for the counterpropagating beams is

$$\Delta\tau = \frac{4\Omega A}{c_0^2}$$

where the physical constant $c_0 = 299{,}792{,}458\ \mathrm{m/s}$ is the speed of light in a vacuum. The corresponding difference in optical path length is

$$\Delta L = \frac{4\Omega A}{c_0} \quad (1)$$

MEASURING THE OPTICAL PATH DIFFERENCE

For the area given in Table 1 and an angular velocity of $1\ \mathrm{deg/h}$ (which is not unreasonable for spacecraft), Equation 1 gives a path difference of about $8 \times 10^{-16}\ \mathrm{m}$, approximately the radius of a proton. Such a difference is impractically small to measure, so instead of comparing optical paths directly, most devices instead report frequency differences.

$$\frac{\Delta\nu}{\nu} = \frac{\Delta\tau}{\tau} = \frac{\Delta L}{L}$$

For the above parameter values, the frequency difference is about $1\ \mathrm{rad/s}$. This value, called the beat frequency, is much easier to read compared to the path difference or transit time difference.

The ratio of beat frequency to the angular velocity is sometimes called the scale factor S,

$$S = \frac{\Delta\nu}{\Omega}$$

The scale factor is a measure of the sensitivity to small rotations.

NUMERICAL PRECISION

This model uses the Geometrical Optics interface, in which rays are traced through the model geometry while being reflected or refracted at surfaces. Because the ray tracing calculation uses double-precision arithmetic, the smallest relative difference that can be detected between two optical paths is

$$\left(\frac{\Delta L}{L}\right)_{\mathrm{min}} \approx \varepsilon = 2^{-52} \approx 2.2204 \times 10^{-16}$$

where ε is sometimes called machine precision or machine epsilon. At smaller values, the difference in optical path for the counterpropagating beams returns zero due to cancellation error. From Equation 1 it is possible to compute the angular velocity corresponding to the smallest measurable optical path difference in double-precision arithmetic,

$$\left(\frac{\Delta L}{L}\right)_{\mathrm{min}} = \frac{4A}{Lc}\Omega_{\mathrm{min}}$$

Solving for Ω_{min} yields

$$\Omega_{\mathrm{min}} = \frac{Lc\varepsilon}{4A}$$

For the parameter values given in Table 1, Ω_{min} is approximately 6.66×10^{-7} rad/s or 0.137 deg/h. Thus the number of degrees of precision the result can be no greater than $\log 10(\Omega/\Omega_{\mathrm{min}})$.

Results and Discussion

The ray trajectories are shown in Figure 3, where the color expression indicates the ray index, $i \in \{1, 2, 3, 4\}$. The rays that hit the obstruction have indices 1 and 4; this is because the incident ray splits once when entering the ring, and each of the two counterpropagating rays splits again while exiting the ring.

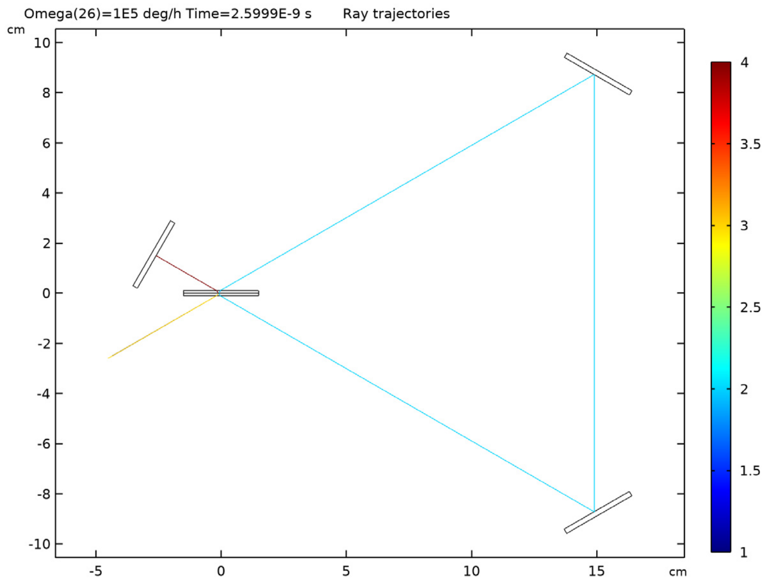

Figure 3: Ray propagation in a Sagnac interferometer consisting of two mirrors and a beam splitter. The entire device rotates counterclockwise, resulting in a small phase difference between the rays.

In Figure 4 the beat frequency is given as a function of the angular velocity. The beat frequency is on the order of 1 Hz even for the smallest angular velocity shown, wheres the smallest optical path difference would have been on the femtometer scale.

The slope of this line, the scale factor, is shown in Figure 5, where it is compared to the analytic result from a simple geometric analysis of the interferometer. At lower angular velocity values, the computed scale factor is noisy and inaccurate because of cancellation error; at the lowest value, the path difference can only be known to one digit of precision.

At higher values of Ω, the two lines differ, but for a different, more physical reason. The analytic expression for scale factor has been written assuming that optical path within the triangle is equivalent to distance. However, for a short interval when entering and leaving the beam splitter, the refractive index is not unity. Ref. 1 provides more detailed expressions for the optical path difference and transit time difference when the interferometer contains a co-moving medium. That this is a real, physical effect can also be demonstrated by re-running the model with an extremely thin beam splitter; then the two lines show better agreement at larger values of Ω.

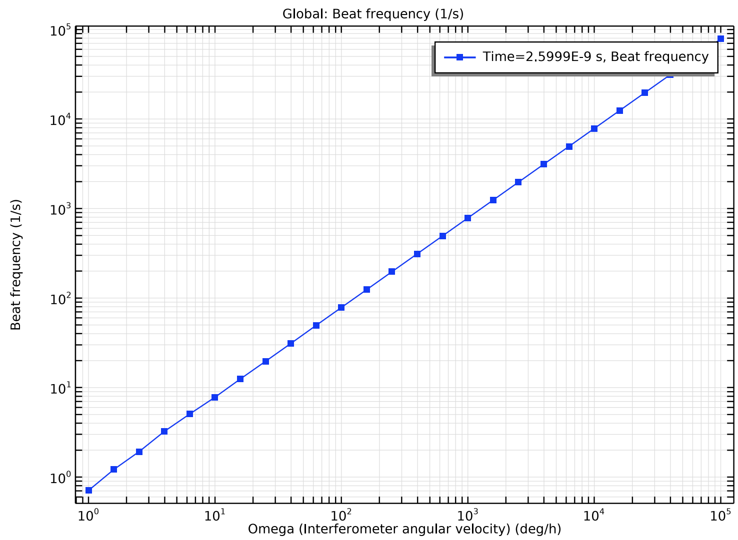

Figure 4: Beat frequency in the Sagnac interferometer as a function of angular velocity. The relationship is linear; the slope is the scale factor of the interferometer.

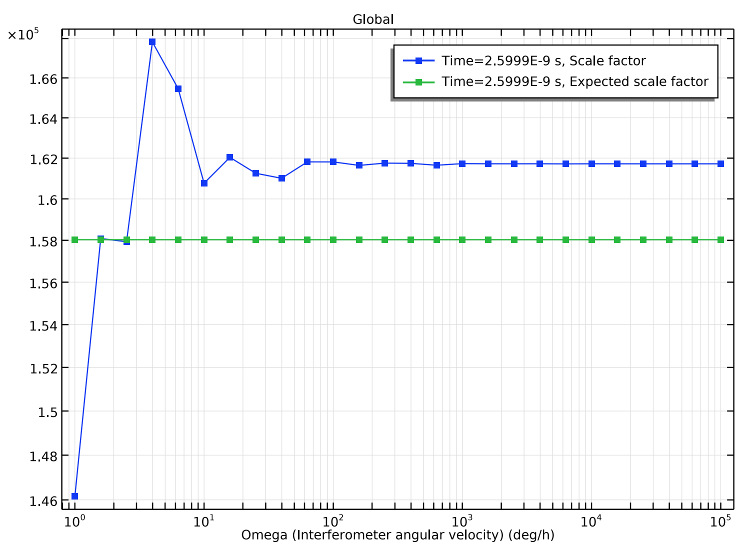

Figure 5: Scale factor of the Sagnac interferometer as a function of angular velocity.

Reference

1. E.J. Post, "Sagnac effect," *Reviews of Modern Physics*, vol. 39, no. 2, pp. 475–493, 1967.

Application Library path: `Ray_Optics_Module/Spectrometers_and_Monochromators/sagnac_interferometer`

Modeling Instructions

From the **File** menu, choose **New**.

NEW

In the **New** window, click **Model Wizard**.

MODEL WIZARD

1 In the **Model Wizard** window, click **2D**.

2 In the **Select Physics** tree, select **Optics>Ray Optics>Geometrical Optics (gop)**.

3 Click **Add**.

4 Click **Study**.

5 In the **Select Study** tree, select **Preset Studies for Selected Physics Interfaces>Ray Tracing**.

6 Click **Done**.

GLOBAL DEFINITIONS

Parameters 1

Load the global parameters for the interferometer from a text file.

1 In the **Model Builder** window, under **Global Definitions** click **Parameters 1**.

2 In the **Settings** window for **Parameters**, locate the **Parameters** section.

3 Click **Load from File**.

4 Browse to the model's Application Libraries folder and double-click the file `sagnac_interferometer_parameters.txt`.

GEOMETRY 1

1 In the **Model Builder** window, under **Component 1 (comp1)** click **Geometry 1**.

2 In the **Settings** window for **Geometry**, locate the **Units** section.

3 From the **Length unit** list, choose **cm**. This is more convenient than the SI unit, given the dimensions of the geometry.

The interferometer geometry is a triangular arrangement of two mirrors and a beam splitter. The geometry also includes an obstruction to absorb the outgoing rays.

Bottom-Right Mirror

1 In the **Geometry** toolbar, click **Rectangle**.

2 In the **Settings** window for **Rectangle**, type `Bottom-Right Mirror` in the **Label** text field.

3 Locate the **Size and Shape** section. In the **Width** text field, type `wm`.

4 In the **Height** text field, type `hm`.

5 Locate the **Position** section. From the **Base** list, choose **Center**.

6 In the **x** text field, type `xm1`.

7 In the **y** text field, type `ym1`.

8 Locate the **Rotation Angle** section. In the **Rotation** text field, type `30`.

Top-Right Mirror

1 In the **Geometry** toolbar, click **Rectangle**.

2 In the **Settings** window for **Rectangle**, type `Top-Right Mirror` in the **Label** text field.

3 Locate the **Size and Shape** section. In the **Width** text field, type `wm`.

4 In the **Height** text field, type `hm`.

5 Locate the **Position** section. From the **Base** list, choose **Center**.

6 In the **x** text field, type `xm2`.

7 In the **y** text field, type `ym2`.

8 Locate the **Rotation Angle** section. In the **Rotation** text field, type `150`.

Beam Splitter

1 In the **Geometry** toolbar, click **Rectangle**.

2 In the **Settings** window for **Rectangle**, type `Beam Splitter` in the **Label** text field.

3 Locate the **Size and Shape** section. In the **Width** text field, type `ws`.

4 In the **Height** text field, type `hs`.

5 Locate the **Position** section. From the **Base** list, choose **Center**.

6 Click to expand the **Layers** section. In the table, enter the following settings:

Layer name	Thickness (cm)
`Layer 1`	`hs/2`

The interior boundary splits the incoming ray into reflected and refracted rays. The outer surfaces of the glass will be assigned antireflective coatings.

Obstruction

1 In the **Geometry** toolbar, click **Rectangle**.

2 In the **Settings** window for **Rectangle**, type `Obstruction` in the **Label** text field.

3 Locate the **Size and Shape** section. In the **Width** text field, type `wo`.

4 In the **Height** text field, type `ho`.

5 Locate the **Position** section. From the **Base** list, choose **Center**.

6 In the **x** text field, type `xo`.

7 In the **y** text field, type `yo`.

8 Locate the **Rotation Angle** section. In the **Rotation** text field, type `60`.

9 Click **Build All Objects**.

10 Click the **Zoom Extents** button in the **Graphics** toolbar. Compare the geometry to Figure 1.

DEFINITIONS

Rotating Domain 1

1 In the **Definitions** toolbar, click **Moving Mesh** and choose **Rotating Domain**.

2 In the **Settings** window for **Rotating Domain**, locate the **Domain Selection** section.

3 From the **Selection** list, choose **All domains**.

4 Locate the **Rotation** section. From the **Rotation type** list, choose **Specified rotational velocity**.

5 In the ω text field, type `Omega`.

6 Locate the **Axis** section. Specify the $\mathbf{r}_{ax}$ vector as

`xc`	X
`yc`	Y

Load some local variable definitions. These will be used later to interpret the results.

Variables 1

1 In the **Model Builder** window, right-click **Definitions** and choose **Variables**.

Load the variable definitions from a text file.

2 In the **Settings** window for **Variables**, locate the **Variables** section.

3 Click **Load from File**.

4 Browse to the model's Application Libraries folder and double-click the file `sagnac_interferometer_variables.txt`.

GEOMETRICAL OPTICS (GOP)

1 In the **Model Builder** window, under **Component 1 (comp1)** click **Geometrical Optics (gop)**.

2 In the **Settings** window for **Geometrical Optics**, locate the **Ray Release and Propagation** section.

3 In the **Maximum number of secondary rays** text field, type `3`. The Geometrical Optics interface will apply deterministic ray splitting at the beam splitter boundary. A total of three secondary rays will split off from the released ray.

4 Locate the **Intensity Computation** section. From the **Intensity computation** list, choose **Compute intensity**.

5 Locate the **Additional Variables** section. Select the **Compute optical path length** check box. The optical path difference between two of the rays will be used to compute the beat frequency of the output.

Ray Properties 1

1 In the **Model Builder** window, under **Component 1 (comp1)>Geometrical Optics (gop)** click **Ray Properties 1**.

2 In the **Settings** window for **Ray Properties**, locate the **Ray Properties** section.

3 In the λ_0 text field, type `lam0`.

Define some boundary conditions for the beam splitter, mirrors, and obstruction.

ARC

1 In the **Physics** toolbar, click **Boundaries** and choose **Material Discontinuity**.

2 Select Boundaries 6 and 9 only.

3 In the **Settings** window for **Material Discontinuity**, locate the **Rays to Release** section.

4 From the **Release reflected rays** list, choose **Never**.

5 In the **Label** text field, type `ARC`.

Beam Splitter

1 In the **Physics** toolbar, click **Boundaries** and choose **Material Discontinuity**.

2 Select Boundary 8 only.

3 In the **Settings** window for **Material Discontinuity**, type `Beam Splitter` in the **Label** text field.

4 Locate the **Coatings** section. From the **Thin dielectric films on boundary** list, choose **Specify reflectance**.

5 In the R text field, type `0.5`.

Mirrors

1 In the **Physics** toolbar, click **Boundaries** and choose **Mirror**.

2 Select Boundaries 13 and 15 only.

3 In the **Settings** window for **Mirror**, type `Mirrors` in the **Label** text field.

Obstruction

1 In the **Physics** toolbar, click **Boundaries** and choose **Wall**.

2 Select Boundary 3 only.

3 In the **Settings** window for **Wall**, type `Obstruction` in the **Label** text field.

Release from Grid 1

1 In the **Physics** toolbar, click **Global** and choose **Release from Grid**.

2 In the **Settings** window for **Release from Grid**, locate the **Initial Coordinates** section.

3 In the $q_{x,0}$ text field, type `q0x`.

4 In the $q_{y,0}$ text field, type `q0y`.

5 Locate the **Ray Direction Vector** section. Specify the $\mathbf{L}_0$ vector as

L0x	x
L0y	y

Use the **Ray Termination** feature to delete any rays that escape from the geometry.

Ray Termination 1

1 In the **Physics** toolbar, click **Global** and choose **Ray Termination**.

2 In the **Settings** window for **Ray Termination**, locate the **Termination Criteria** section.

3 From the **Spatial extents of ray propagation** list, choose **Bounding box, from geometry**.

MATERIALS

Specify the refractive index in the domains. The surroundings are treated as a vacuum with $n = 0$.

Material 1 (mat1)

1 In the **Model Builder** window, under **Component 1 (comp1)** right-click **Materials** and choose **Blank Material**.

2 In the **Settings** window for **Material**, locate the **Material Contents** section.

3 In the table, enter the following settings:

Property	Variable	Value	Unit	Property group
Refractive index, real part	n_iso ; nii = n_iso, nij = 0	1.5	1	Refractive index

STUDY 1

The study will include a **Parametric Sweep** over different values of the angular velocity.

Parametric Sweep

1 In the **Study** toolbar, click **Parametric Sweep**.

2 In the **Settings** window for **Parametric Sweep**, locate the **Study Settings** section.

3 Click **Add**.

4 In the table, click to select the cell at row number 1 and column number 1.

5 In the table, enter the following settings:

Parameter name	Parameter value list	Parameter unit
Omega (Interferometer angular velocity)	`10^{range(0,0.2,5)}`	`deg/h`

Step 1: Ray Tracing

1 In the **Model Builder** window, click **Step 1: Ray Tracing**.

2 In the **Settings** window for **Ray Tracing**, locate the **Study Settings** section.

3 From the **Time-step specification** list, choose **Specify maximum path length**.

4 From the **Length unit** list, choose **cm**.

5 In the **Lengths** text field, type `0 1.5*P`. Since P is the perimeter of the triangle, this is a sufficiently long optical path length for the rays to reach the obstruction.

6 In the **Study** toolbar, click **Compute**.

RESULTS

Ray Trajectories (gop)

The default plot shows the ray propagation through the interferometer. The default color expression indicates the optical path length.

1 In the **Model Builder** window, expand the **Ray Trajectories (gop)** node.

Color Expression 1

1 In the **Model Builder** window, expand the **Results>Ray Trajectories (gop)> Ray Trajectories 1** node, then click **Color Expression 1**.

2 In the **Settings** window for **Color Expression**, click **Replace Expression** in the upper-right corner of the **Expression** section. From the menu, choose **Component 1 (comp1)> Geometrical Optics>Ray statistics>gop.pidx - Ray index**.

3 In the **Ray Trajectories (gop)** toolbar, click **Plot**. Compare the resulting plot to Figure 3.

Create additional plots to analyze the beat frequency and scale factor.

Beat Frequency

1 In the **Home** toolbar, click **Add Plot Group** and choose **1D Plot Group**.

2 In the **Settings** window for **1D Plot Group**, type `Beat Frequency` in the **Label** text field.

3 Locate the **Data** section. From the **Dataset** list, choose **Ray 1**.

4 From the **Time selection** list, choose **Last**.

Global 1

1 Right-click **Beat Frequency** and choose **Global**.

2 In the **Settings** window for **Global**, click **Replace Expression** in the upper-right corner of the **y-Axis Data** section. From the menu, choose **Component 1 (comp1)>Definitions> Variables>dnu - Beat frequency - 1/s**.

3 Locate the **x-Axis Data** section. From the **Axis source data** list, choose **Outer solutions**.

4 Click to expand the **Coloring and Style** section. Find the **Line markers** subsection. From the **Marker** list, choose **Point**.

5 From the **Positioning** list, choose **In data points**.

6 In the **Beat Frequency** toolbar, click **Plot**.

7 Click the **x-Axis Log Scale** button in the **Graphics** toolbar.

8 Click the **y-Axis Log Scale** button in the **Graphics** toolbar. Compare the resulting plot to Figure 4.

Scale Factor

1 In the **Model Builder** window, right-click **Beat Frequency** and choose **Duplicate**.

2 In the **Settings** window for **1D Plot Group**, type `Scale Factor` in the **Label** text field.

Global 1

1 In the **Model Builder** window, expand the **Scale Factor** node, then click **Global 1**.

2 In the **Settings** window for **Global**, click **Replace Expression** in the upper-right corner of the **y-Axis Data** section. From the menu, choose **Component 1 (comp1)>Definitions>Variables>SF - Scale factor - rad**.

3 Click **Add Expression** in the upper-right corner of the **y-Axis Data** section. From the menu, choose **Global definitions>Parameters>SF_exp - Expected scale factor**.

4 In the **Scale Factor** toolbar, click **Plot**. Compare the resulting plot to Figure 5.