Created in COMSOL Multiphysics 5.6

Luneburg Lens

Introduction

In this tutorial the Geometrical Optics interface is used to compute ray trajectories through a graded-index medium. The Luneburg lens has a graded refractive index which leads to special focusing properties.

A Luneburg lens, in the most general sense, is a spherically symmetric thick lens with a variable-index refracting structure that forms perfect geometrical images of two concentric spheres onto each other. The Luneburg lens has a gradient of isotropic refractive index n radially out from its center. In the generalized Luneburg lens, there is a pair of conjugate foci outside the lens.

In the limiting case where one of the foci tends to infinity and the other one is located on the lens surface, the analytical solution for the index profile takes a very simple form. This is what is usually meant by "Luneburg lens" in the narrow sense. Such a lens focuses a parallel beam to a perfect point in the geometrical optics limit. The location of the focus is on the rim of the lens directly opposite to the incidence direction.

Unlike a conventional, constant-index lens, a Luneburg lens works perfectly for wide ray bundles, and not only for paraxial beams. Thus, the lens is said to have a 180 degree field of view and zero f-number (Ref. 1). These properties of a lens can only be achieved using gradient-index optics.

Luneburg lenses can be made from transparent dielectric media for essentially any wavelength of interest, and they can be extremely broadband (Ref. 1). At microwave frequencies, they are used as small form-factor focusing devices for high-fidelity satellite antennas. Unlike a parabolic reflector dish, a Luneburg lens can focus satellite signal arriving from any position in the sky, which makes it more suitable for satellite antennas mounted on moving objects, such as trains and ships.

Model Definition

The refractive index of a general Luneburg lens takes the form:

$$n = \frac{1}{f}\sqrt{1+f^2-\left(\frac{r}{R}\right)^2}$$

where r is the radial coordinate from the center of the lens and R is the radius of the lens. The dimensionless parameter f determines whether rays are focused inside or outside the lens. For $f = 1$ the focal point lies on the surface of the lens.

Results and Discussion

A collimated array of rays is released into the sphere.The focal parameter can be adjusted to change the refractive index profile within the lens. For this simulation, the parameter is set to $f = 1$.

The Luneburg Lens is shown in Figure 1 together with the ray trajectories. The curved path of the rays within the lens can be clearly seen. In this figure the rays have been colored according to their radial location at the best focus position.

The image quality at the focal point can be seen in Figure 2. As expected, with $f = 1$, the best focus is exactly on the surface of the sphere opposite the direction of the incoming rays.

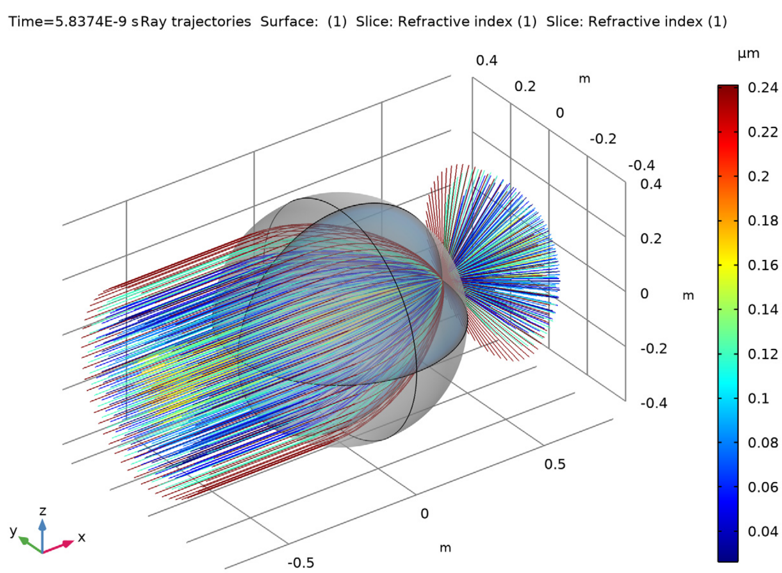


Figure 1: Ray trajectories in a Luneburg lens. The color indicates the radial distance of each ray from the centroid of the ray bundle as it exits the sphere.

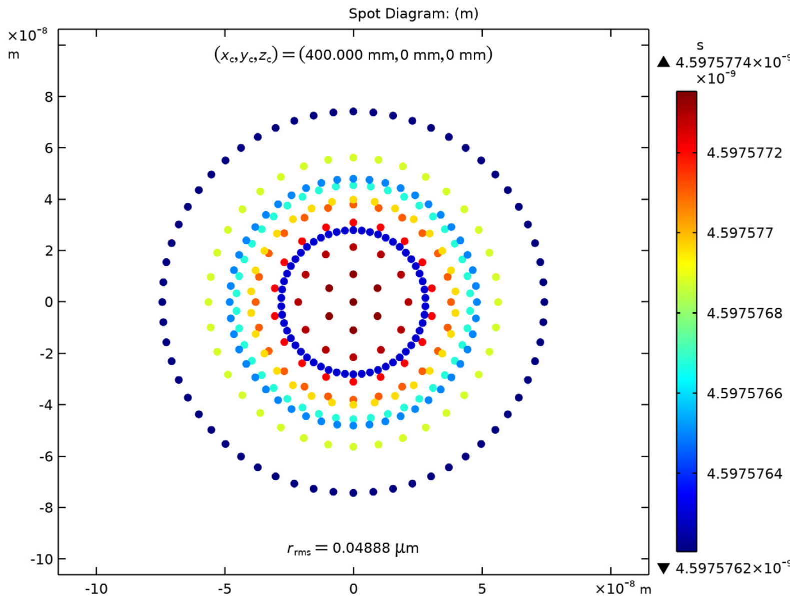


Figure 2: A spot diagram for the Luneburg lens. The image quality is essentially perfect.

References

1. N. Kundtz and D.R. Smith, *Extreme-Angle Broadband Metamaterial Lens*, Nature Materials Letters, 2009.

2. L.D. Landau and E.M. Lifshitz, *The Classical Theory of Fields*, 4th ed., Butterworth-Heinemann, Oxford, 1975.

Application Library path: `Ray_Optics_Module/Lenses_Cameras_and_Telescopes/luneburg_lens`

Modeling Instructions

From the **File** menu, choose **New**.

NEW

In the **New** window, click **Model Wizard**.

MODEL WIZARD

1 In the **Model Wizard** window, click **3D**.

2 In the **Select Physics** tree, select **Optics>Ray Optics>Geometrical Optics (gop)**.

3 Click **Add**.

4 Click **Study**.

5 In the **Select Study** tree, select **Preset Studies for Selected Physics Interfaces>Ray Tracing**.

6 Click **Done**.

GLOBAL DEFINITIONS

Add some parameters for the geometry dimensions.

Parameters 1

1 In the **Model Builder** window, under **Global Definitions** click **Parameters 1**.

2 In the **Settings** window for **Parameters**, locate the **Parameters** section.

3 In the table, enter the following settings:

Name	Expression	Value	Description
L	1[m]	1 m	Box length
R	0.4[m]	0.4 m	Outer radius

GEOMETRY 1

The Luneburg lens is simply a sphere containing a graded-index medium.

Sphere 1 (sph1)

1 In the **Geometry** toolbar, click **Sphere**.

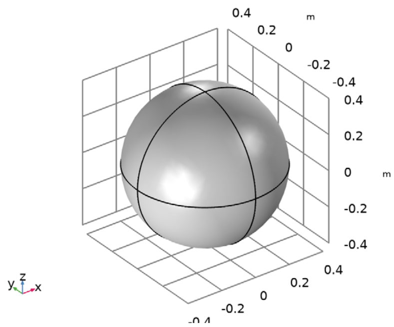
2 In the **Settings** window for **Sphere**, locate the **Size** section.

3 In the **Radius** text field, type R.

4 Click **Build All Objects**.

5 Click the **Orthographic Projection** button in the **Graphics** toolbar.

6 Click the **Go to Default View** button in the **Graphics** toolbar.



COMPONENT 1 (COMP1)

1. In the **Model Builder** window, click **Component 1 (comp1)**.
2. In the **Settings** window for **Component**, locate the **General** section.
3. Find the **Mesh frame coordinates** subsection. From the **Geometry shape function** list, choose **Cubic Lagrange**. The ray tracing algorithm used by the Geometrical Optics interface computes the refracted ray direction based on a discretized geometry via the underlying finite element mesh. A cubic geometry shape order usually introduces less discretization error compared to the default, which uses linear and quadratic polynomials.

DEFINITIONS

Add some expressions for the radius from the center of the lens. This will be used to define the refractive index later.

Variables 1

1. In the **Model Builder** window, under **Component 1 (comp1)** right-click **Definitions** and choose **Variables**.
2. In the **Settings** window for **Variables**, locate the **Variables** section.
3. In the table, enter the following settings:

Name	Expression	Unit	Description
r	sqrt(x^2+y^2+z^2+eps)	m	Radial coordinate
f	1.0		Focal shift parameter
n	sqrt(1+f^2-(r/R)^2)/f		Refractive index

MATERIALS

Material 1 (mat1)

1 In the **Model Builder** window, under **Component 1 (comp1)** right-click **Materials** and choose **Blank Material**.

2 In the **Settings** window for **Material**, locate the **Material Contents** section.

3 In the table, enter the following settings:

Property	Variable	Value	Unit	Property group
Refractive index, real part	n_iso ; nii = n_iso, nij = 0	n	1	Refractive index

GEOMETRICAL OPTICS (GOP)

1 In the **Model Builder** window, under **Component 1 (comp1)** click **Geometrical Optics (gop)**.

2 In the **Settings** window for **Geometrical Optics**, locate the **Ray Release and Propagation** section.

3 In the **Maximum number of secondary rays** text field, type 0. This model is only concerned with the transmitted (refracted) rays and not the reflected ones, so set the maximum number of secondary rays to 0.

4 Select the **Use geometry normals for ray-boundary interactions** check box. In this simulation, the geometry normals are used to apply the boundary conditions on all refracting surfaces. This is appropriate for the highest accuracy ray traces in single-physics simulations, where the geometry is not deformed.

Material Discontinuity 1

1 In the **Model Builder** window, under **Component 1 (comp1)>Geometrical Optics (gop)** click **Material Discontinuity 1**.

2 In the **Settings** window for **Material Discontinuity**, locate the **Rays to Release** section.

3 From the **Release reflected rays** list, choose **Never**.

Release the rays in the x direction in collimated hexapolar grid.

Release from Grid 1

1 In the **Physics** toolbar, click **Global** and choose **Release from Grid**.

2 In the **Settings** window for **Release from Grid**, locate the **Initial Coordinates** section.

3 From the **Grid type** list, choose **Hexapolar**.

4 Specify the $\mathbf{q}_c$ vector as

-0.75	x

5 In the R_c text field, type 0.35.

6 In the N_c text field, type 10.

7 Locate the **Ray Direction Vector** section. Specify the $\mathbf{L}_0$ vector as

1	x
0	y
0	z

The default Physics-controlled mesh can be used for this simulation.

MESH 1

In the **Model Builder** window, under **Component 1 (comp1)** right-click **Mesh 1** and choose **Build All**.

STUDY 1

Step 1: Ray Tracing

1 In the **Model Builder** window, under **Study 1** click **Step 1: Ray Tracing**.

2 In the **Settings** window for **Ray Tracing**, locate the **Study Settings** section.

3 From the **Time-step specification** list, choose **Specify maximum path length**.

4 In the **Lengths** text field, type range(0,0.05,1.75).

In the following steps, adjust the solver settings to ensure that sufficiently small time steps are taken through the graded index medium.

Solution 1 (sol1)

1 In the **Study** toolbar, click **Show Default Solver**.

2 In the **Model Builder** window, expand the **Solution 1 (sol1)** node, then click **Time-Dependent Solver 1**.

3 In the **Settings** window for **Time-Dependent Solver**, click to expand the **Time Stepping** section.

4 From the **Maximum step constraint** list, choose **Constant**.

5 In the **Maximum step** text field, type 2.5e-12. This gives around 1000 time steps within the lens.

6 Click to expand the **Output** section. In the **Study** toolbar, click **Compute**.

RESULTS

Ray Diagram 1

1 In the **Settings** window for **3D Plot Group**, type `Ray Diagram 1` in the **Label** text field.

2 Locate the **Color Legend** section. Select the **Show units** check box.

3 In the **Model Builder** window, expand the **Ray Diagram 1** node.

Color Expression 1

1 In the **Model Builder** window, expand the **Results>Ray Diagram 1>Ray Trajectories 1** node, then click **Color Expression 1**.

2 In the **Settings** window for **Color Expression**, locate the **Expression** section.

3 In the **Expression** text field, type `at(4.5975768e-9,gop.rrel)`.

4 From the **Unit** list, choose **µm**.

Surface 1

1 In the **Model Builder** window, right-click **Ray Diagram 1** and choose **Surface**.

2 In the **Settings** window for **Surface**, locate the **Coloring and Style** section.

3 From the **Coloring** list, choose **Uniform**.

4 From the **Color** list, choose **Gray**.

Transparency 1

Right-click **Surface 1** and choose **Transparency**.

Slice 1

1 In the **Model Builder** window, right-click **Ray Diagram 1** and choose **Slice**.

2 In the **Settings** window for **Slice**, locate the **Expression** section.

3 In the **Expression** text field, type `n`.

4 Locate the **Plane Data** section. From the **Plane** list, choose **xy-planes**.

5 In the **Planes** text field, type `1`.

6 Locate the **Coloring and Style** section. From the **Color table** list, choose **JupiterAuroraBorealis**.

7 Clear the **Color legend** check box.

Transparency 1

Right-click **Slice 1** and choose **Transparency**.

Slice 2

1 In the **Model Builder** window, under **Results>Ray Diagram 1** right-click **Slice 1** and choose **Duplicate**.
2 In the **Settings** window for **Slice**, locate the **Plane Data** section.
3 From the **Plane** list, choose **zx-planes**.
4 In the **Planes** text field, type `1`.
5 In the **Ray Diagram 1** toolbar, click **Plot**.
6 Click the **Zoom Extents** button in the **Graphics** toolbar. Compare the resulting plot to Figure 1.

Spot Diagram

1 In the **Home** toolbar, click **Add Plot Group** and choose **2D Plot Group**.
2 In the **Settings** window for **2D Plot Group**, type `Spot Diagram` in the **Label** text field.
3 Locate the **Color Legend** section. Select the **Show maximum and minimum values** check box.
4 Select the **Show units** check box.
5 Click to expand the **Number Format** section. Select the **Manual color legend settings** check box.
6 In the **Precision** text field, type `8`.

Spot Diagram 1

1 In the **Spot Diagram** toolbar, click **More Plots** and choose **Spot Diagram**.
2 In the **Settings** window for **Spot Diagram**, click to expand the **Focal Plane Orientation** section.
3 From the **Normal to focal plane** list, choose **User defined**.
4 In the **x** text field, type `1`.
5 In the **z** text field, type `0`.
6 Click **Create Focal Plane Dataset**.
7 Click to expand the **Annotations** section. Select the **Show spot coordinates** check box.
8 From the **Coordinate system** list, choose **Global**.
9 In the **Display precision** text field, type `6`.
10 In the **Display precision** text field, type `4`.

Color Expression 1

1 Right-click **Spot Diagram 1** and choose **Color Expression**.

2 In the **Settings** window for **Color Expression**, locate the **Expression** section.

3 In the **Expression** text field, type `t`. This shows the time at which the RMS focus is minimized. This time was used to define the color expression for the Ray Diagram shown in Figure 1.

4 In the **Spot Diagram** toolbar, click **Plot**.

5 Click the **Zoom Extents** button in the **Graphics** toolbar. Compare the resulting plot to Figure 2.