Created in COMSOL Multiphysics 5.6

Radar Cross Section

Introduction

This tutorial model demonstrates the use of a background field in an electromagnetic scattering problem. Although this example is a boat hit by a radar, this same technique can be used in any situation where an isolated object meets electromagnetic waves from a distant source. For example, several orders of magnitude smaller, an equally common application is plasmon resonant nanoparticles. Besides setting up the background field and sweeping it over a range of angles of incidence, this example also shows you how to compute the far-field and the radar cross section (RCS).

Model Definition

This example computes the interaction between a boat and the incident field from a radar transmitter. The transmitter is considered to be distant enough that this field can be treated as a plane wave. This makes it possible to exclude the transmitter from the model geometry and look only at the boat and its immediate surroundings.

Although the modeling procedure is similar in 3D, this example is in 2D to quickly set up and solve. In order to focus on the concepts, the geometry is intentionally kept very simple (see Figure 1).

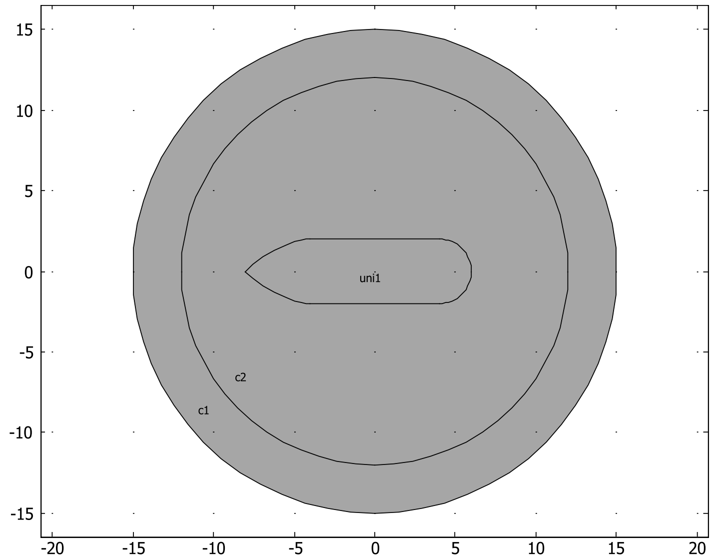


Figure 1: The model geometry. The boat has a length of 14 m and is surrounded by air - the cut plane lies above the water surface.

The inner circle in the geometry represents air surrounding the boat. The outer circle is a perfectly matched layer (PML), which minimizes unphysical reflections of the scattered wave as it leaves the model domain. The radius of the inner circle is an important model consideration. To get good results, the inner circle needs to completely surround the boat. It also must extend to a considerable fraction of the wavelength, as well as the characteristic length of the evanescent wave outside it. However, in practice, and in order to minimize time and memory usage, do not make the circle too large. For the purpose of this example, the inner circle is bigger than necessary to provide a good view of the near field. The radius of the outer circle does not matter as long as it allows for the meshing needs of the PML, which is 5–6 mesh elements across.

The background electromagnetic field from the radar is described by its out-of-plane electric field component:

$$\mathbf{E}_b = \exp(jk_0(x\cos\phi + y\sin\phi))\mathbf{e}_z$$

In this equation:

- j is the imaginary unit;

- $k_0 = 2\pi f/c$ is the wave number in vacuum;
- $c = 3\cdot 10^8$ m/s is the speed of light;
- $f = 100$ MHz is the frequency;
- ϕ is the angle of incidence; and
- $\phi = 0$ corresponds to a wave incident from the positive x direction (and hence propagating in the negative x direction, from the right to the left in the model).

The time-harmonic wave equation is then solved for the relative field, $\mathbf{E}_{\mathrm{rel}} = \mathbf{E} - \mathbf{E}_b$, where $\mathbf{E}$ is the total, measurable field.

$$\nabla \times (\mu_r^{-1} \nabla \times \mathbf{E}_{\mathrm{rel}}) - \left(\varepsilon_r - j\frac{\sigma}{\omega\varepsilon_0}\right) k_0^2 \mathbf{E}_{\mathrm{rel}} = 0$$

The relative field is the difference between the measured field caused by the presence of the boat and the background field. It is utilized to describe how detectable the boat is with the radar — its RCS. The RCS of a 3D scatterer is defined as

$$\sigma_{3\mathrm{D}} = \lim_{r \to \infty} 4\pi r^2 \frac{|\mathbf{E}_{\mathrm{rel}}|^2}{|\mathbf{E}_b|^2}$$

For the 2D scatterer in this example, the RCS per unit length is used to address its monostatic scattering characteristics at the angle where the incident wave comes from, which is given by

$$\sigma_{2\mathrm{D}} = \lim_{r \to \infty} 2\pi r \frac{|\mathbf{E}_{\mathrm{rel}}|^2}{|\mathbf{E}_b|^2}$$

where the relative field as a function of radius is calculated with the help of the far-field computation, $\mathbf{E}_{\mathrm{far}}$. The RCS of a 3D model which has a constant geometrical cross-section of a 2D model can also be estimated from the RCS per unit length by

$$\sigma_{3\mathrm{D}} = \sigma_{2\mathrm{D}} \frac{2l^2}{\lambda}$$

where l is the length of a scatterer and λ is the wavelength.

Results and Discussion

Using a parametric solver, the results are for the full range of angles of incidence, from 0 to 359 degrees in 1-degree steps. Figure 2 shows the norm of the total field caused by a background plane wave incident from a 30-degree angle. The reflections on the boat create a standing wave pattern. The boat casts a shadow on the lower-left side.

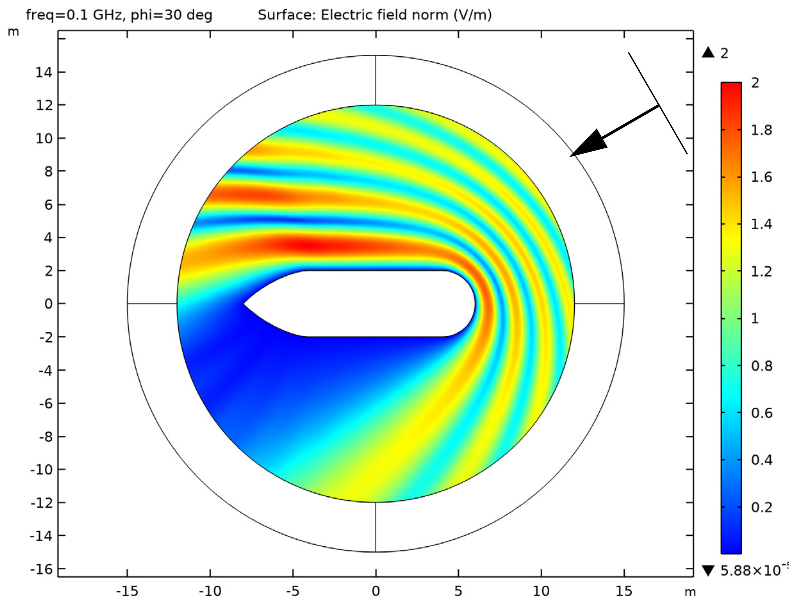


Figure 2: The total field norm for a 30 degree angle of incidence. The arrow represents the propagation direction of the incident background field.

You can also visualize the relative field sent out from the boat. Figure 3 shows both its instantaneous value at a zero phase and its norm. Note that the lack of standing waves in the latter indicates that the PML does its job of absorbing the outgoing field without any reflections.

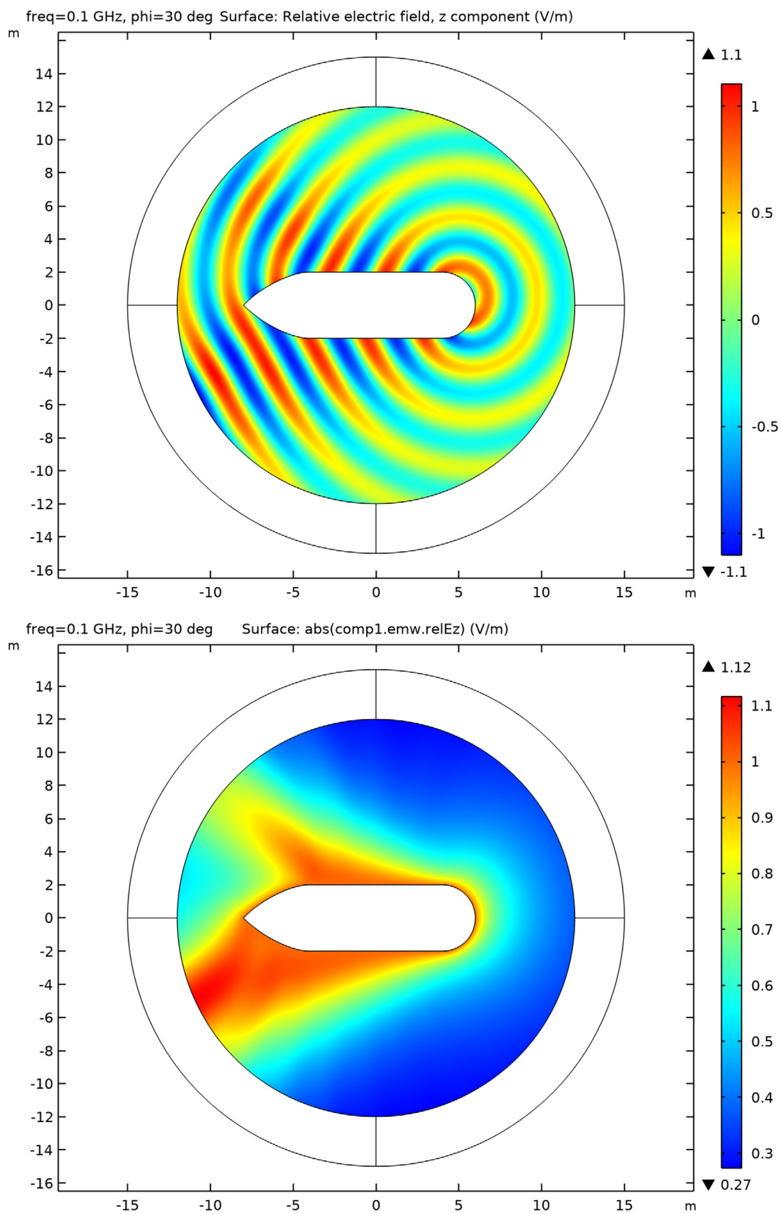


Figure 3: The instantaneous value (top) and magnitude (bottom) of the relative electric field sent out by the boat for a 30-degree angle of incidence.

As shown in these near field plots, you can guess that a distant observer would see peaks in the relative field centered around 150 and 210 degrees. This is confirmed by the far-field plot in Figure 4. The far-field computation uses the Stratton–Chu formula (see *Far-Field Calculations Theory* in the *RF Module User's Guide*) with the relative electric field on the boundaries of the boat as the input. Note that the relative field is the only component for which it makes sense to evaluate the far field. While the relative field falls off with the distance from the boat, the incident field amplitude remains constant. Hence the total field is non-trivial only at finite distances from the boat.

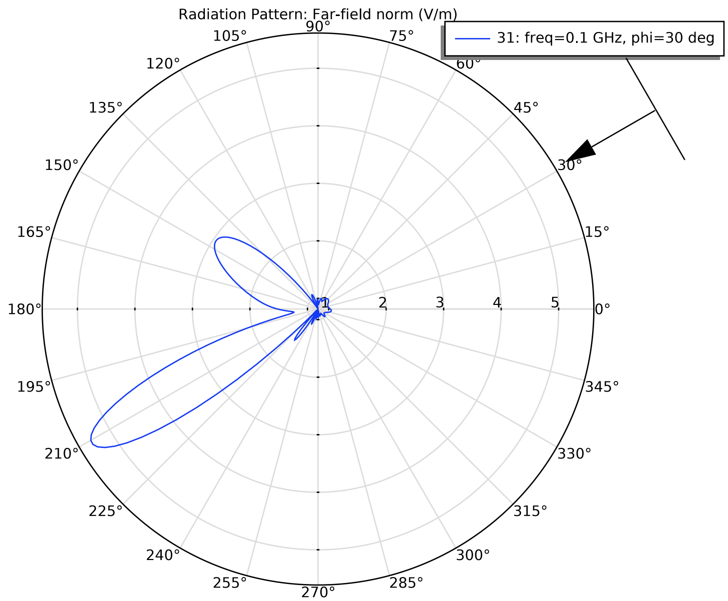


Figure 4: Far-field radiation plot for a 30-degree angle of incidence. The distance to the center represents the far field in dB.

Figure 5 shows the RCS per unit length. Compared to the other plots, which show various aspects of the fields for a specific angle of incidence, this output is generally only possible by solving for the complete range of angles, from 0 to 360 degrees. In this example, you could avoid solving for half of this range by noting that because of the geometry symmetry, the results in the upper and lower half plane are identical.

Except for the symmetry, a main feature of the RCS per unit length plot is the prominent peak at 90 degrees, due to the flat side of the boat. If the radar is in this direction, much of the field that hits the boat reflects back toward it. The peak around 135 degrees is explained similarly, but the side of the bow replaces the side of the boat. This peak is lower and wider because the boundary is shorter and slightly bent. There is a dip at 180 degrees because most of the field that hits the bow from a straight angle reflects to the sides.

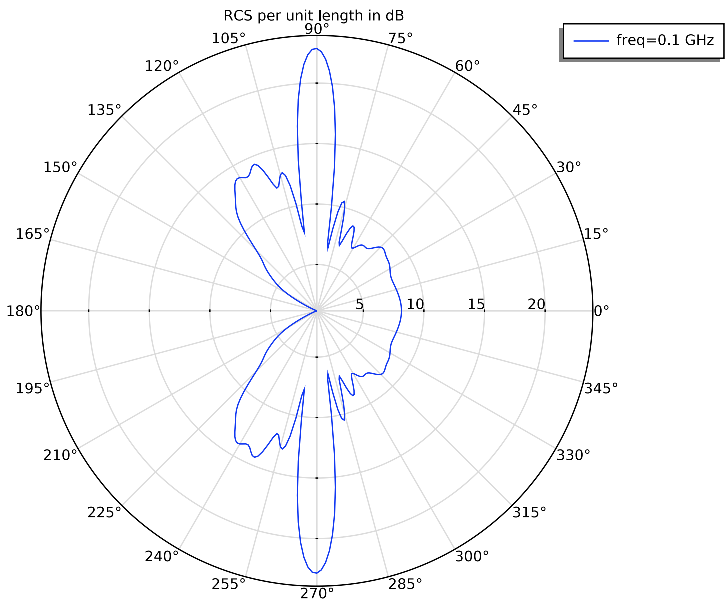


Figure 5: Polar plot of the RCS per unit length as a function of the angle of incidence.

Application Library path: `RF_Module/Scattering_and_RCS/radar_cross_section`

Modeling Instructions

From the **File** menu, choose **New**.

NEW

In the **New** window, click **Model Wizard**.

MODEL WIZARD

1 In the **Model Wizard** window, click **2D**.

2 In the **Select Physics** tree, select **Radio Frequency>Electromagnetic Waves, Frequency Domain (emw)**.

3 Click **Add**.

4 Click **Study**.

5 In the **Select Study** tree, select **General Studies>Frequency Domain**.

6 Click **Done**.

STUDY 1

Step 1: Frequency Domain

1 In the **Model Builder** window, under **Study 1** click **Step 1: Frequency Domain**.

2 In the **Settings** window for **Frequency Domain**, locate the **Study Settings** section.

3 In the **Frequencies** text field, type `100[MHz]`.

GLOBAL DEFINITIONS

Parameters 1

1 In the **Model Builder** window, under **Global Definitions** click **Parameters 1**.

2 In the **Settings** window for **Parameters**, locate the **Parameters** section.

3 In the table, enter the following settings:

Name	Expression	Value	Description
`phi`	`0[deg]`	0 rad	`Angle of incidence, degrees`
`r0`	`12[m]`	12 m	`Radius, air domain`

GEOMETRY 1

Circle 1 (c1)

1 In the **Geometry** toolbar, click **Circle**.

2 In the **Settings** window for **Circle**, locate the **Size and Shape** section.

3 In the **Radius** text field, type `r0+3[m]`.

4 Click to expand the **Layers** section. In the table, enter the following settings:

Layer name	Thickness (m)
`Layer 1`	`3[m]`

Circular Arc 1 (ca1)

1 In the **Geometry** toolbar, click **More Primitives** and choose **Circular Arc**.

2 In the **Settings** window for **Circular Arc**, locate the **Properties** section.

3 From the **Specify** list, choose **Endpoints and start angle**.

4 Locate the **Starting Point** section. In the **x** text field, type `6`.

5 Locate the **Endpoint** section. In the **x** text field, type 4.

6 In the **y** text field, type 2.

Circular Arc 2 (ca2)

1 In the **Geometry** toolbar, click **More Primitives** and choose **Circular Arc**.

2 In the **Settings** window for **Circular Arc**, locate the **Properties** section.

3 From the **Specify** list, choose **Endpoints and start angle**.

4 Locate the **Starting Point** section. In the **x** text field, type 4.

5 In the **y** text field, type -2.

6 Locate the **Endpoint** section. In the **x** text field, type 6.

7 In the **y** text field, type 0.

8 Locate the **Angles** section. In the **Start angle** text field, type 270.

Line Segment 1 (ls1)

1 In the **Geometry** toolbar, click **More Primitives** and choose **Line Segment**.

2 In the **Settings** window for **Line Segment**, locate the **Starting Point** section.

3 From the **Specify** list, choose **Coordinates**.

4 Locate the **Endpoint** section. From the **Specify** list, choose **Coordinates**.

5 Locate the **Starting Point** section. In the **x** text field, type -4.

6 In the **y** text field, type -2.

7 Locate the **Endpoint** section. In the **x** text field, type 4.

8 In the **y** text field, type -2.

Line Segment 2 (ls2)

1 In the **Geometry** toolbar, click **More Primitives** and choose **Line Segment**.

2 In the **Settings** window for **Line Segment**, locate the **Starting Point** section.

3 From the **Specify** list, choose **Coordinates**.

4 Locate the **Endpoint** section. From the **Specify** list, choose **Coordinates**.

5 Locate the **Starting Point** section. In the **x** text field, type -4.

6 In the **y** text field, type 2.

7 Locate the **Endpoint** section. In the **x** text field, type 4.

8 In the **y** text field, type 2.

Quadratic Bézier 1 (qb1)

1 In the **Geometry** toolbar, click **More Primitives** and choose **Quadratic Bézier**.

2 In the **Settings** window for **Quadratic Bézier**, locate the **Control Points** section.

3 In row **1**, set **x** to `-4` and **y** to `2`.

4 In row **2**, set **x** to `-6` and **y** to `2`.>

5 In row **3**, set **x** to `-8`.

Quadratic Bézier 2 (qb2)

1 In the **Geometry** toolbar, click **More Primitives** and choose **Quadratic Bézier**.

2 In the **Settings** window for **Quadratic Bézier**, locate the **Control Points** section.

3 In row **1**, set **x** to `-8`.

4 In row **2**, set **x** to `-6` and **y** to `-2`.

5 In row **3**, set **x** to `-4` and **y** to `-2`.

Convert to Solid 1 (csol1)

1 In the **Geometry** toolbar, click **Conversions** and choose **Convert to Solid**.

2 Select the objects **ca1**, **ca2**, **ls1**, **ls2**, **qb1**, and **qb2** only.

DEFINITIONS

Boat Boundaries

1 In the **Definitions** toolbar, click **Explicit**.

2 In the **Settings** window for **Explicit**, type `Boat Boundaries` in the **Label** text field.

3 Select Domain 4 only.

4 Locate the **Output Entities** section. From the **Output entities** list, choose **Adjacent boundaries**.

You will select the boundaries of the boat several times throughout this tutorial. The Named Selection you just defined makes this more convenient.

Perfectly Matched Layer 1 (pml1)

1 In the **Definitions** toolbar, click **Perfectly Matched Layer**.

2 Select Domains 1, 2, 5, and 6 only.

3 In the **Settings** window for **Perfectly Matched Layer**, locate the **Geometry** section.

4 From the **Type** list, choose **Cylindrical**.

The PML will make sure that the scattered field from the boat is almost completely absorbed before what remains of it reflects on the exterior boundaries of the model. Note that the background field is not affected by any of this - it is by definition what you are setting it to be.

ADD MATERIAL

1 In the **Home** toolbar, click **Add Material** to open the **Add Material** window.

2 Go to the **Add Material** window.

3 In the tree, select **Built-in>Air**.

4 Click **Add to Component** in the window toolbar.

5 In the tree, select **Built-in>Aluminum**.

6 Click **Add to Component** in the window toolbar.

7 In the **Home** toolbar, click **Add Material** to close the **Add Material** window.

MATERIALS

Aluminum (mat2)

1 In the **Settings** window for **Material**, locate the **Geometric Entity Selection** section.

2 From the **Geometric entity level** list, choose **Boundary**.

3 From the **Selection** list, choose **Boat Boundaries**.

ELECTROMAGNETIC WAVES, FREQUENCY DOMAIN (EMW)

1 In the **Model Builder** window, under **Component 1 (comp1)** click **Electromagnetic Waves, Frequency Domain (emw)**.

2 Select Domains 1–3, 5, and 6 only.

Because you will be using a boundary condition to represent the boat, the field inside it will be identically zero and is not necessary to solve for. Removing the boat domain this way will save time and memory.

3 In the **Settings** window for **Electromagnetic Waves, Frequency Domain**, locate the **Formulation** section.

4 From the list, choose **Scattered field**.

The option you just selected lets you enter a background field. The expression you will use, represents a plane wave coming in from an angle phi. In this expression, the wave propagation constant in vacuum, emw.k0, is automatically provided by the physics interface. Once you have solved the model, you can plot the background field to verify that you have set it up correctly.

5 Specify the $\mathbf{E}_\mathrm{b}$ vector as

0	x
0	y
1[V/m]*exp(j*emw.k0*(x*cos(phi)+y*sin(phi)))	z

6 Locate the **Components** section. From the **Electric field components solved for** list, choose **Out-of-plane vector**.

The background field has only the z-component of electric field. By choosing the **Out-of-plane vector**, the computation can be more efficient.

Impedance Boundary Condition 1

1 In the **Physics** toolbar, click **Boundaries** and choose **Impedance Boundary Condition**.

2 In the **Settings** window for **Impedance Boundary Condition**, locate the **Boundary Selection** section.

3 From the **Selection** list, choose **Boat Boundaries**.

The use of an impedance boundary condition assumes that the skin depth in the material is much less than the material thickness. At 200 MHz, the skin depth in Aluminum is of the order of microns, so it is safe to say this is the case. You could even have used a perfect electric conductor condition instead, with largely unchanged results. In the case of scattering on a nanoparticle, the skin depth is often of the same order of magnitude as the particle itself. In such situations, you should not use the impedance boundary condition but rather keep the material domain active.

Far-Field Domain 1

In the **Physics** toolbar, click **Domains** and choose **Far-Field Domain**.

MESH 1

In the **Model Builder** window, under **Component 1 (comp1)** right-click **Mesh 1** and choose **Build All**.

STUDY 1

Step 1: Frequency Domain

1 In the **Model Builder** window, under **Study 1** click **Step 1: Frequency Domain**.

2 In the **Settings** window for **Frequency Domain**, click to expand the **Study Extensions** section.

3 Select the **Auxiliary sweep** check box.

4 Click **Add**.

5 In the table, enter the following settings:

Parameter name	Parameter value list	Parameter unit
phi (Angle of incidence, degrees)	`range(0,1,360)`	`deg`

You are solving this model for incidence angles from 0 to 360 degrees, in steps of 1 degree. Smaller steps would give you more accurate RCS plots, but the total solution time as well as the size of the model file increases linearly with the number of angles.

6 In the **Home** toolbar, click **Compute**.

RESULTS

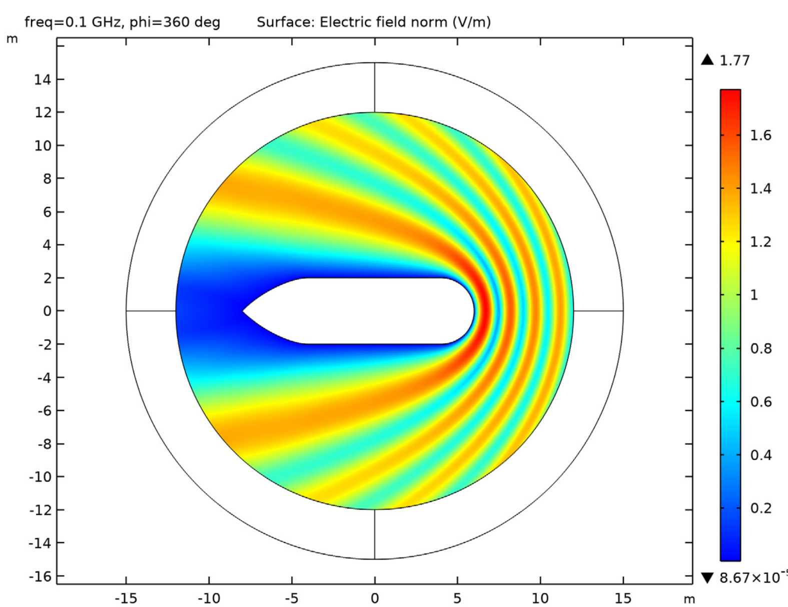
Selection 1

1 In the **Model Builder** window, expand the **Electric Field (emw)** node.

2 Right-click **Surface** and choose **Selection**.

3 Select Domain 3 only.

4 In the **Electric Field (emw)** toolbar, click **Plot**.



The default plot shows the norm of the total electric field for an angle of incidence equal to 360 degrees. The total electric field is the actual, measurable, physical field. The plot is dominated by a standing wave pattern caused by the reflections mainly on the stern and the sides of the boat. As you might expect with the wave coming in almost from the right, a wake is forming to the left of the boat, beyond the bow. To reproduce Figure 2, see what the field looks like with a 30 degree angle of incidence.

Electric Field (emw)

1 In the **Model Builder** window, click **Electric Field (emw)**.

2 In the **Settings** window for **2D Plot Group**, locate the **Data** section.

3 From the **Parameter value (phi (deg))** list, choose **30**.

4 In the **Electric Field (emw)** toolbar, click **Plot**.

At 30 degrees, the wake widens. It is also possible to discern an increased field above and to the left of the boat, due to the reflections on its upper side. Try also plotting the instantaneous value of the total and the relative field.

Surface

1 In the **Model Builder** window, click **Surface**.

2 In the **Settings** window for **Surface**, click **Replace Expression** in the upper-right corner of the **Expression** section. From the menu, choose **Component 1 (comp1)>Electromagnetic Waves, Frequency Domain>Electric>Electric field - V/m>emw.Ez - Electric field, z component**.

3 In the **Electric Field (emw)** toolbar, click **Plot**.

4 Click **Replace Expression** in the upper-right corner of the **Expression** section. From the menu, choose **Component 1 (comp1)>Electromagnetic Waves, Frequency Domain>Electric>Relative electric field - V/m>emw.relEz - Relative electric field, z component**.

5 In the **Electric Field (emw)** toolbar, click **Plot**.

Because the relative field is the difference between the observed and the background field, its magnitude will increase both in the wake and where the total field is enhanced by reflections. This trend is even clearer if you plot the absolute value of the relative field.

6 Locate the **Expression** section. In the **Expression** text field, type `abs(comp1.emw.relEz)`.

7 In the **Electric Field (emw)** toolbar, click **Plot**.

The plot should now resemble the bottom plot in Figure 3. You can get a quantitative measure of how the reflected field is radiating out in different directions from a plot of the far field as a function of the angle. A common way to do this is as a polar plot. You already have a default polar plot that you can make slight changes to.

2D Far Field (emw)

1 In the **Model Builder** window, click **2D Far Field (emw)**.

2 In the **Settings** window for **Polar Plot Group**, locate the **Data** section.

3 From the **Parameter selection (phi)** list, choose **From list**.

4 In the **Parameter values (phi (deg))** list, select **30**.

Radiation Pattern 1

1 In the **Model Builder** window, expand the **2D Far Field (emw)** node, then click **Radiation Pattern 1**.

2 In the **Settings** window for **Radiation Pattern**, locate the **Evaluation** section.

3 Find the **Angles** subsection. In the **Number of angles** text field, type `360`.

4 In the **2D Far Field (emw)** toolbar, click **Plot**.

To conclude, create a similar plot of the monostatic RCS per unit length. While the far-field plot is for one specific angle of incidence at a time, this plot will visualize the back scattering as a function of the angle of incidence.

Polar Plot Group 3

In the **Home** toolbar, click **Add Plot Group** and choose **Polar Plot Group**.

Global 1

1 Right-click **Polar Plot Group 3** and choose **Global**.

2 In the **Settings** window for **Global**, locate the **r-Axis Data** section.

3 In the table, enter the following settings:

Expression	Unit	Description
`10*log10(at2(r0*cos(phi),r0*sin(phi), emw.bRCS2D))`		

4 Locate the θ **Angle Data** section. From the **Parameter** list, choose **Expression**.

5 In the **Expression** text field, type `phi`.

6 Click to expand the **Title** section. From the **Title type** list, choose **Manual**.

7 In the **Title** text area, type `RCS per unit length in dB`.

8 In the **Polar Plot Group 3** toolbar, click **Plot**.