Hexagonal Grating

Introduction

A hexagonal grating is an infinite structure that is periodic with hexagonal (or rhomboid unit) cells. Figure 1 shows the hexagonal domain used for this model. The reflecting perfectly conducting surface consists of regularly spaced protruding semispheres.

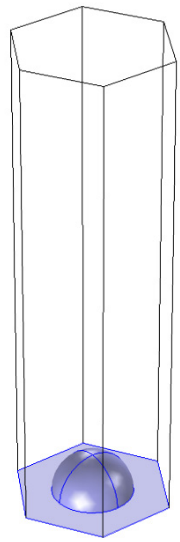


Figure 1: The hexagonal domain, used for computing the diffraction from the hexagonal grating.

As shown in Figure 2, for a hexagonal cell of side length a, the corresponding unit cell is a rhomboid with side length $\sqrt{3}a$. In Figure 2, the side vectors for the hexagonal cell starts from the point P and are denoted $\mathbf{a}_1$ and $\mathbf{a}_2$. The angle between $\mathbf{a}_1$ and $\mathbf{a}_2$ is 120 degrees. Similarly, for the rhomboid unit cell, the primitive vectors are denoted $\mathbf{u}_1$

and $\mathbf{u}_2$ and starts from the hexagon center point Q. The angle between the two primitive vectors is also 120 degrees.

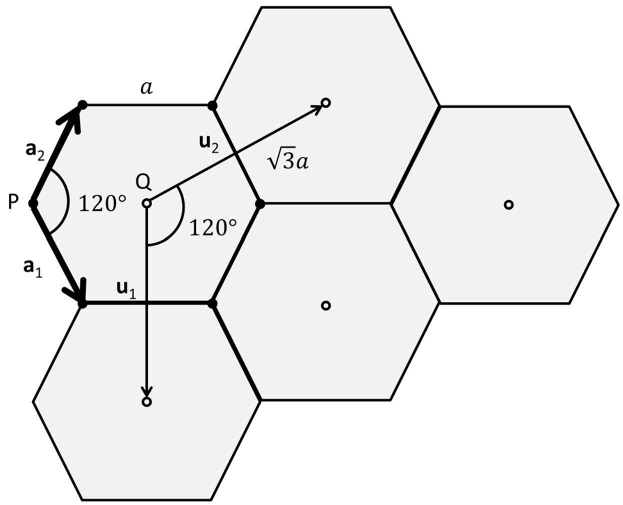


Figure 2: Schematic showing the hexagonal cells with side length a *and side vectors* $\mathbf{a}_1$ *and* $\mathbf{a}_2$. *The primitive cells are defined by the primitive vectors* $\mathbf{u}_1$ *and* $\mathbf{u}_2$.

If the incident plane wave have a wave vector defined by

$$\mathbf{k} = \mathbf{k}_{||} + \mathbf{k}_{\perp}, \tag{1}$$

where $\mathbf{k}_{||}$ is the wave vector component parallel to the periodic boundary and $\mathbf{k}\perp$ is the component orthogonal to the periodic boundary, the in-plane wave vector component for diffraction order mn is given by

$$\mathbf{k}_{||mn} = \mathbf{k}_{||} + m\mathbf{G}_1 + n\mathbf{G}_2, \tag{2}$$

where the reciprocal lattice vectors $\mathbf{G}_1$ and $\mathbf{G}_2$ are defined from the primitive vectors $\mathbf{u}_1$ and $\mathbf{u}_2$ as

$$\mathbf{G}_1 = 2\pi\frac{\mathbf{u}_2 \times \mathbf{n}}{\mathbf{u}_1 \cdot (\mathbf{u}_2 \times \mathbf{n})} \tag{3}$$

and

$$\mathbf{G}_2 = 2\pi\frac{\mathbf{n}\times\mathbf{u}_1}{\mathbf{u}_2\cdot(\mathbf{n}\times\mathbf{u}_1)}, \tag{4}$$

where $\mathbf{n}$ is the normal vector (length 1) to the periodic boundary.

Since the out-of-plane wave vector component for mode mn is defined by

$$k_{\perp mn} = \sqrt{k^2 - k_{||mn}^2}, \tag{5}$$

it is clear that for propagating modes, where the out-of-plane wave vector component above must be real, the length of the in-plane wave vector component must be smaller than the material wave number k. Figure 3 shows that it is only the modes inside the circle with radius k that will be propagating. In the example shown in Figure 3, there are five modes that will be propagating, in this case the modes $m = n = 0$ (the reflected wave), $m = -1$, $n = 0$, $m = 0$, $n = -1$, $m = -1$, $n = -1$, and $m = -2$, $n = -1$. All other modes will be evanescent and damped out.

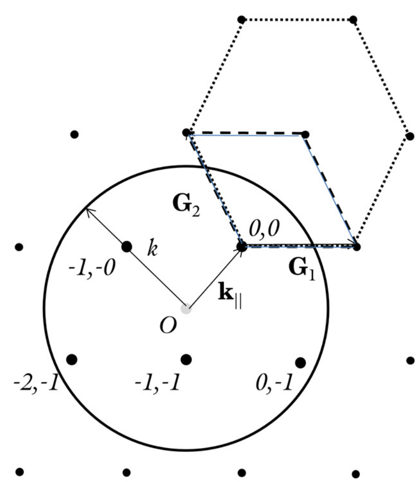


Figure 3: The reciprocal lattice, showing the reciprocal lattice vectors $\mathbf{G}_1$ *and* $\mathbf{G}_2$*, the in-plane wave vector component* $\mathbf{k}_{||}$*, and the circle with radius* k *(the material wave number) enclosing the propagating mode points (larger dots. The dotted hexagon indicates that also the reciprocal lattice is a hexagonal point lattice. The dashed rhomboid indicates the unit cell spanned by the reciprocal lattice vectors.*

Model Definition

In this model, the unit cell is small compared to the wavelength, so there will only be two modes that are propagating, the modes $m = 0$, $n = -1$ and $m = -1$, $n = -1$. For wavelengths longer than approximately 1.01 µm (the critical wavelength), the mode $m = 0$, $n = -1$ will be evanescent.

First a wavelength sweep will be made for an incident field having the polarization perpendicular to the plane of incidence (spanned by the wave vector for the incident wave and the normal to the periodic boundary) (so called s-polarization). Thereafter another wavelength sweep is made, but now with the polarization in the plane of incidence (p-polarization).

Results and Discussion

Figure 4 shows the electric field norm and the propagation directions for the incident, the reflected and the diffracted waves. Notice that the diffracted waves come in pairs (both have the same mode numbers), one wave having the polarization in the plane-of-diffraction and the other wave have orthogonal polarization to the plane-of-diffraction. The plane-of-diffraction is spanned by the wave vector for the diffracted wave and the normal to the periodic boundary. The wavelength is close the critical wavelength for the

$m = 0$, $n = -1$ mode. This is evident from the plot, as the wave vector for that mode (the yellow arrows) is almost parallel to the periodic boundary.

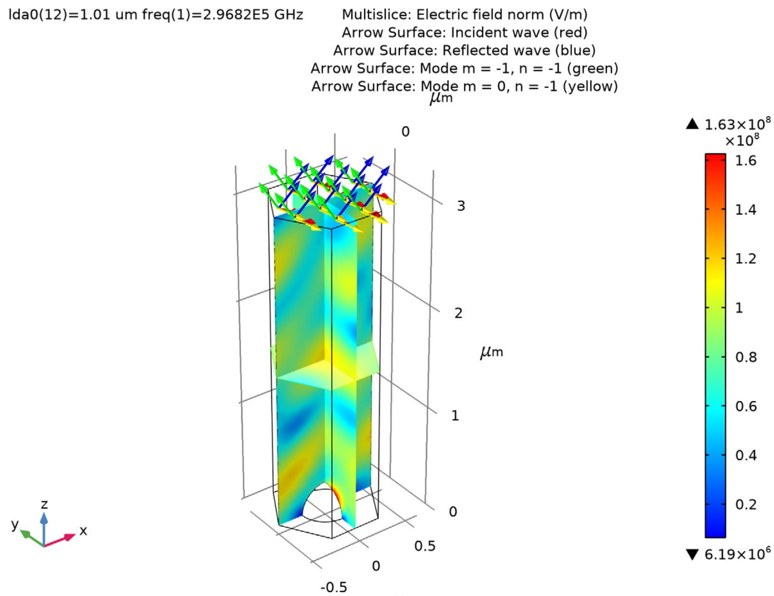


Figure 4: The electric field norm and the propagation directions for the incident wave (red arrows), the reflected wave (blue arrows) and the two diffraction orders (green and yellow arrows). The wavelength is 1.01 µ*m, which is close to the critical wavelength for the mode $m = 0$, $n = -1$, and the polarization of the incident wave is perpendicular to the plane of incidence.*

Figure 5 shows the reflectance (for mode $m = n = 0$) and the diffraction efficiencies for the diffracted waves. Notice that both the reflectance and the diffraction efficiency for the in-plane-polarized m = -1, n = -1 mode show resonances (peaks or dips) close to the critical wavelength for the m = 0, n = -1 modes.

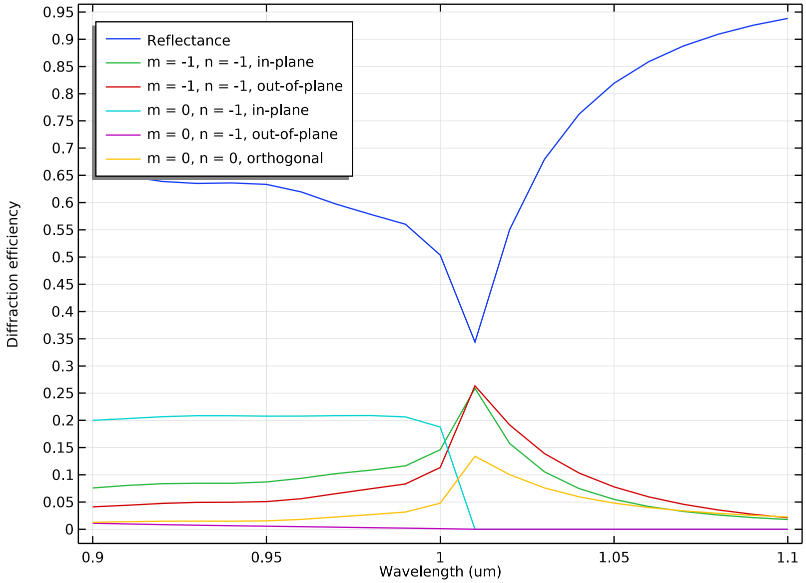


Figure 5: Diffraction efficiencies for the reflected wave and the diffracted waves. The polarization of the incident wave is perpendicular to the plane of incidence.

Figure 6 shows a similar plot as Figure 4, but here the polarization of the incident wave is parallel with the plane of incidence.

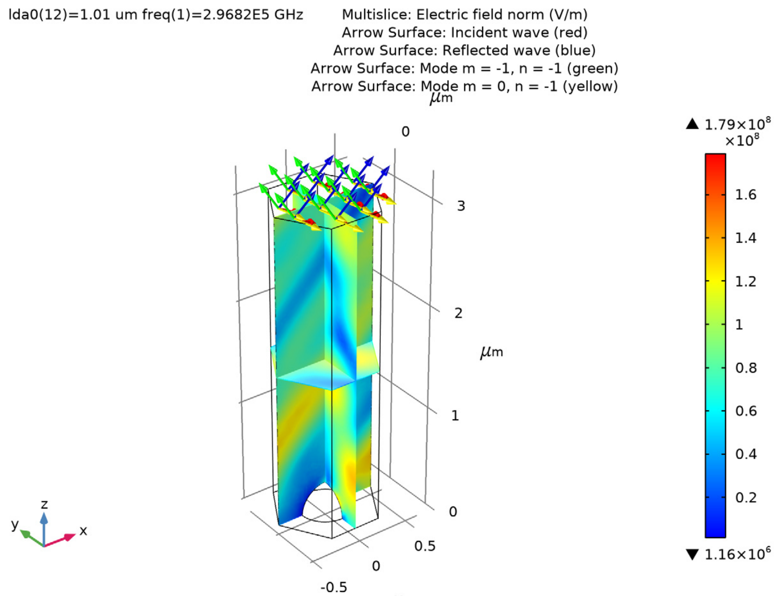


Figure 6: Similar plot as in Figure 4*, but here the polarization of the incident wave is parallel to the plane of incidence.*

[Figure 7](#) shows that for p-polarization both the reflected wave and the two m = -1, n = -1 modes show resonances close to the critical wavelength for the m = 0, n = -1 mode.

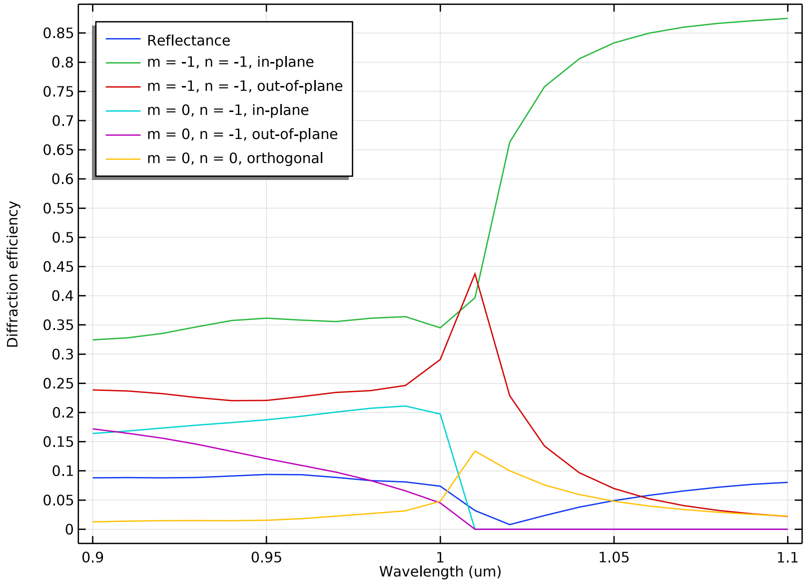


Figure 7: Similar plot as in Figure 5, but here the polarization of the incident wave is parallel to the plane of incidence.

Application Library path: `RF_Module/Tutorials/hexagonal_grating`

Modeling Instructions

From the **File** menu, choose **New**.

NEW

In the **New** window, click  **Model Wizard**.

MODEL WIZARD

1 In the **Model Wizard** window, click  **3D**.

2 In the **Select Physics** tree, select **Radio Frequency>Electromagnetic Waves, Frequency Domain (emw)**.

3 Click **Add**.

4 Click **Study**.

5 In the **Select Study** tree, select **General Studies>Frequency Domain**.

6 Click **Done**.

GLOBAL DEFINITIONS

First add some parameters that defines the geometry and the incident electric field.

Parameters 1

1 In the **Model Builder** window, under **Global Definitions** click **Parameters 1**.

2 In the **Settings** window for **Parameters**, locate the **Parameters** section.

3 In the table, enter the following settings:

Name	**Expression**	**Value**	**Description**
wl0	1[um]	1E-6 m	Center wavelength
lda0	wl0	1E-6 m	Wavelength
f0	c_const/lda0	2.9979E14 1/s	Frequency
a	wl0/2	5E-7 m	Hexagon side length
h0	3*wl0	3E-6 m	Air height
a1	a/2	2.5E-7 m	Sphere radius
theta	pi/3	1.0472	Elevation angle
phi	12[deg]	0.20944 rad	Azimuth angle
E0	1[V/m]	1 V/m	Electric field amplitude
H0	1[A/m]	1 A/m	Magnetic field amplitude

Notice that the azimuth angle `phi` above measures the angle for the wave vector of the incident wave from the x-axis.

Here, `c_const` is a predefined COMSOL constant for the speed of light in vacuum.

STUDY 1

Step 1: Frequency Domain

1 In the **Model Builder** window, under **Study 1** click **Step 1: Frequency Domain**.

2 In the **Settings** window for **Frequency Domain**, locate the **Study Settings** section.

3 In the **Frequencies** text field, type `f0`.

GEOMETRY 1

1 In the **Model Builder** window, under **Component 1 (comp1)** click **Geometry 1**.

2 In the **Settings** window for **Geometry**, locate the **Units** section.

3 From the **Length unit** list, choose **µm**.

The geometry consists of an extruded hexagon, with a semisphere removed from it at the bottom.

Work Plane 1 (wp1)

1 In the **Geometry** toolbar, click **Work Plane**.

2 In the **Settings** window for **Work Plane**, click **Show Work Plane**.

Work Plane 1 (wp1)>Plane Geometry

In the **Model Builder** window, click **Plane Geometry**.

Work Plane 1 (wp1)>Polygon 1 (pol1)

1 In the **Work Plane** toolbar, click **Polygon**.

2 In the **Settings** window for **Polygon**, locate the **Coordinates** section.

3 In the table, enter the following settings:

xw (µm)	yw (µm)
a	0
a/2	sqrt(3)/2*a
-a/2	sqrt(3)/2*a
-a	0
-a/2	-sqrt(3)/2*a
a/2	-sqrt(3)/2*a

Extrude 1 (ext1)

1 In the **Model Builder** window, right-click **Geometry 1** and choose **Extrude**.

2 In the **Settings** window for **Extrude**, locate the **Distances** section.

3 In the table, enter the following settings:

Distances (µm)
h0

Sphere 1 (sph1)

1 In the **Geometry** toolbar, click **Sphere**.

2 In the **Settings** window for **Sphere**, locate the **Size** section.

3 In the **Radius** text field, type `a1`.

Difference 1 (dif1)

1 In the **Geometry** toolbar, click **Booleans and Partitions** and choose **Difference**.

2 Select the object **ext1** only.

3 In the **Settings** window for **Difference**, locate the **Difference** section.

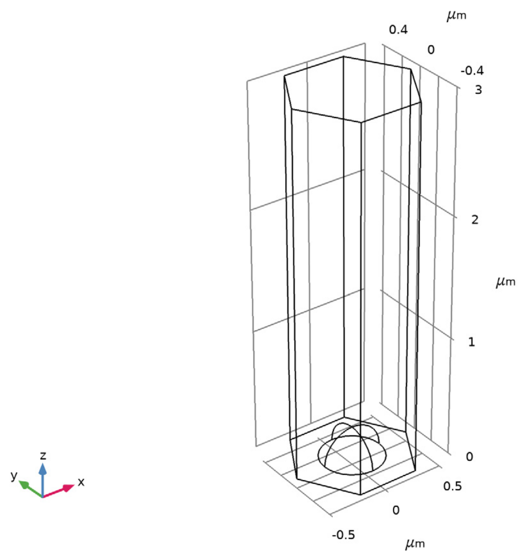
4 Find the **Objects to subtract** subsection. Select the **Activate Selection** toggle button.

5 Select the object **sph1** only.

6 Click **Build All Objects**.

7 Click the **Go to Default View** button in the **Graphics** toolbar.

8 Click the **Wireframe Rendering** button in the **Graphics** toolbar.



ADD MATERIAL

1 In the **Home** toolbar, click **Add Material** to open the **Add Material** window.

2 Go to the **Add Material** window.

3 In the tree, select **Built-in>Air**.

4 Click **Add to Component** in the window toolbar.

5 In the **Home** toolbar, click **Add Material** to close the **Add Material** window.

ELECTROMAGNETIC WAVES, FREQUENCY DOMAIN (EMW)

1 In the **Model Builder** window, under **Component 1 (comp1)** click **Electromagnetic Waves, Frequency Domain (emw)**.

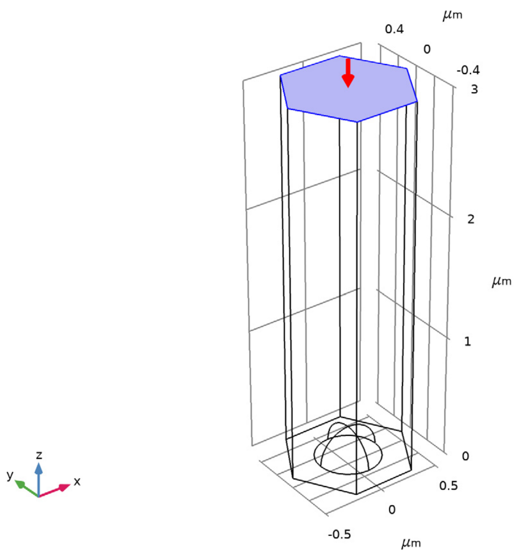
2 In the **Settings** window for **Electromagnetic Waves, Frequency Domain**, locate the **Analysis Methodology** section.

3 From the **Methodology options** list, choose **Robust**.

Port 1

1 In the **Physics** toolbar, click **Boundaries** and choose **Port**.

2 Select Boundary 4 only.



3 In the **Settings** window for **Port**, locate the **Port Properties** section.

4 From the **Type of port** list, choose **Periodic**.

For the first port, wave excitation is **on** by default.

5 Locate the **Port Mode Settings** section. Specify the $\mathbf{E}_0$ vector as

`-E0*sin(phi)`	x
`E0*cos(phi)`	y
`0`	z

6 In the α_1 text field, type `theta`.

7 In the α_2 text field, type `phi+pi/3`, as this angle is measured from the first side vector of the port (not the x-axis).

Periodic Port Reference Point 1

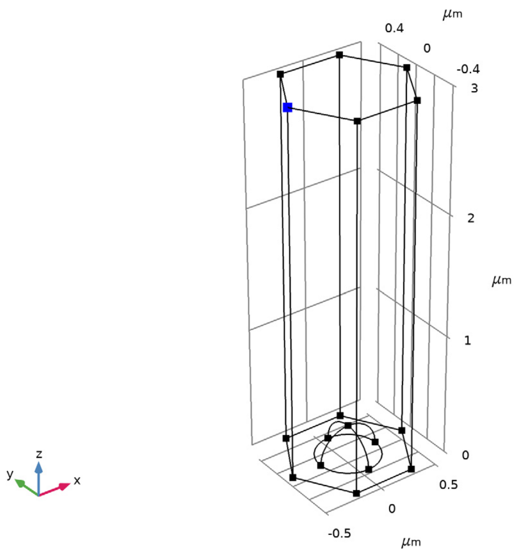
Before creating the diffraction orders ports, a reference point must be defined on the periodic port.

1 In the **Physics** toolbar, click **Attributes** and choose **Periodic Port Reference Point**.

2 In the **Settings** window for **Periodic Port Reference Point**, locate the **Point Selection** section.

3 Click **Clear Selection**.

4 Select Point 2 only. This point selection makes the angle previously provided for `alpha_2` consistent with the intended angle of incidence for the incident wave.



Port 1

1 In the **Model Builder** window, click **Port 1**.

2 In the **Settings** window for **Port**, locate the **Automatic Diffraction Order Calculation** section.

3 Click **Add Diffraction Orders**.

Periodic Condition 1

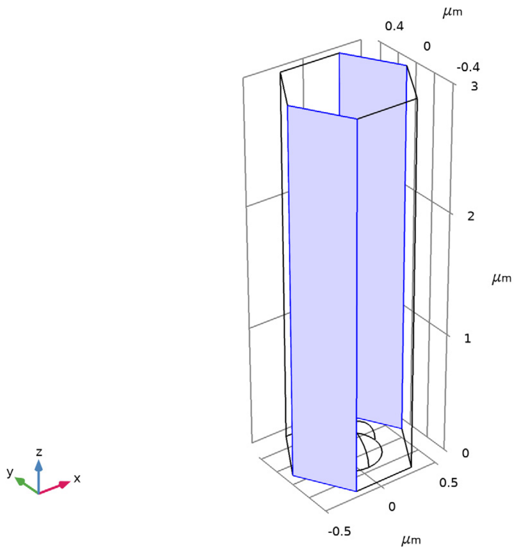
1 In the **Physics** toolbar, click **Boundaries** and choose **Periodic Condition**.

2 In the **Settings** window for **Periodic Condition**, locate the **Periodicity Settings** section.

3 From the **Type of periodicity** list, choose **Floquet periodicity**.

4 From the **k-vector for Floquet periodicity** list, choose **From periodic port**.

5 Select Boundaries 1 and 12 only.



Periodic Condition 2

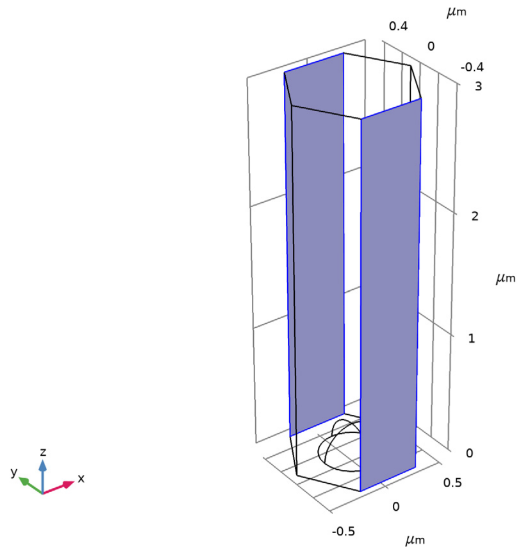
1 In the **Physics** toolbar, click **Boundaries** and choose **Periodic Condition**.

2 In the **Settings** window for **Periodic Condition**, locate the **Periodicity Settings** section.

3 From the **Type of periodicity** list, choose **Floquet periodicity**.

4 From the **k-vector for Floquet periodicity** list, choose **From periodic port**.

5 Select Boundaries 5 and 8 only.



Periodic Condition 3

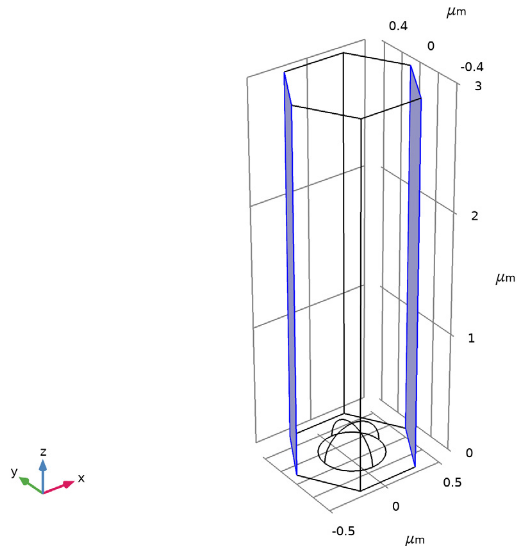
1 In the **Physics** toolbar, click **Boundaries** and choose **Periodic Condition**.

2 In the **Settings** window for **Periodic Condition**, locate the **Periodicity Settings** section.

3 From the **Type of periodicity** list, choose **Floquet periodicity**.

4 From the **k-vector for Floquet periodicity** list, choose **From periodic port**.

5 Select Boundaries 2 and 11 only.



MESH 1

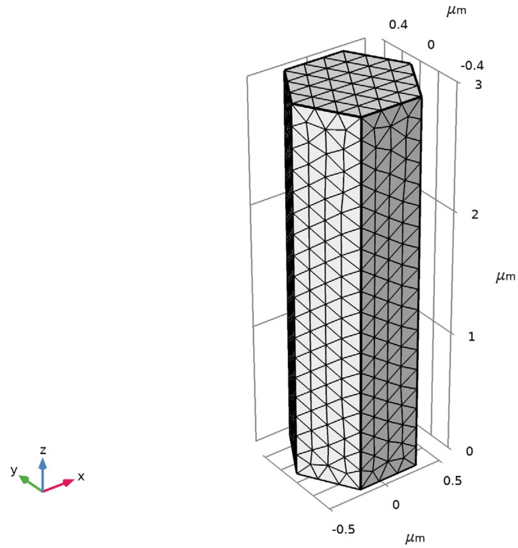
1 In the **Model Builder** window, under **Component 1 (comp1)** click **Mesh 1**.

2 In the **Settings** window for **Mesh**, locate the **Electromagnetic Waves, Frequency Domain (emw)** section.

3 From the **Maximum mesh element size control parameter** list, choose **User defined**.

4 In the **Maximum element size in free space** text field, type `wl0/6`.

5 Click **Build All**.



STUDY 1

Parametric Sweep

1 In the **Study** toolbar, click **Parametric Sweep**.

2 In the **Settings** window for **Parametric Sweep**, locate the **Study Settings** section.

3 Click **Add**.

4 From the list in the **Parameter name** column, choose **lda0 (Wavelength)**.

5 Click **Range**.

6 In the **Range** dialog box, type `0.9[um]` in the **Start** text field.

7 In the **Step** text field, type `0.01[um]`.

8 In the **Stop** text field, type `1.1[um]`.

9 Click **Replace**.

10 In the **Settings** window for **Parametric Sweep**, locate the **Study Settings** section.

11 In the table, enter the following settings:

Parameter name	Parameter value list	Parameter unit
lda0 (Wavelength)	`range(0.9[um],0.01[um],1.1[um])`	um

Step 1: Frequency Domain

In the **Study** toolbar, click **Compute**.

RESULTS

Electric Field (emw)

Add Arrow Surface plots showing the wave vector directions for the incident field, the reflected field and the diffracted fields. Notice that the diffracted fields come in pairs, where each pair have the same wave vector. Thus, only two wave vectors for the diffraction orders need to be added in this case.

Arrow Surface 1

1 Right-click **Electric Field (emw)** and choose **Arrow Surface**.

2 In the **Settings** window for **Arrow Surface**, click **Replace Expression** in the upper-right corner of the **Expression** section. From the menu, choose **Component 1 (comp1)>Electromagnetic Waves, Frequency Domain>Ports>emw.kIncx_1,...,emw.kIncz_1 - Incident wave vector**.

3 Locate the **Expression** section. Select the **Description** check box.

4 In the associated text field, type `Incident wave (red)`.

Arrow Surface 2

1 Right-click **Arrow Surface 1** and choose **Duplicate**.

2 In the **Settings** window for **Arrow Surface**, click **Replace Expression** in the upper-right corner of the **Expression** section. From the menu, choose **Component 1 (comp1)>Electromagnetic Waves, Frequency Domain>Ports>emw.kModex_1,...,emw.kModez_1 - Port mode wave vector**.

3 Locate the **Coloring and Style** section. From the **Color** list, choose **Blue**.

4 Locate the **Expression** section. In the **Description** text field, type `Reflected wave (blue)`.

Arrow Surface 3

1 Right-click **Arrow Surface 2** and choose **Duplicate**.

2 In the **Settings** window for **Arrow Surface**, click **Replace Expression** in the upper-right corner of the **Expression** section. From the menu, choose **Component 1 (comp1)>Electromagnetic Waves, Frequency Domain>Ports>emw.kModex_2,...,emw.kModez_2 - Port mode wave vector**.

3 Locate the **Coloring and Style** section. From the **Color** list, choose **Green**.

4 Locate the **Expression** section. In the **Description** text field, type `Mode m = -1, n = -1 (green)`.

Arrow Surface 4

1 Right-click **Arrow Surface 3** and choose **Duplicate**.

2 In the **Settings** window for **Arrow Surface**, click **Replace Expression** in the upper-right corner of the **Expression** section. From the menu, choose **Component 1 (comp1)> Electromagnetic Waves, Frequency Domain>Ports>emw.kModex_4,...,emw.kModez_4 - Port mode wave vector**.

3 Locate the **Coloring and Style** section. From the **Color** list, choose **Yellow**.

4 Locate the **Expression** section. In the **Description** text field, type `Mode m = 0, n = -1 (yellow)`.

Electric Field (emw)

Select the wavelength closest to the critical wavelength for the mode $m = 0, n = -1$.

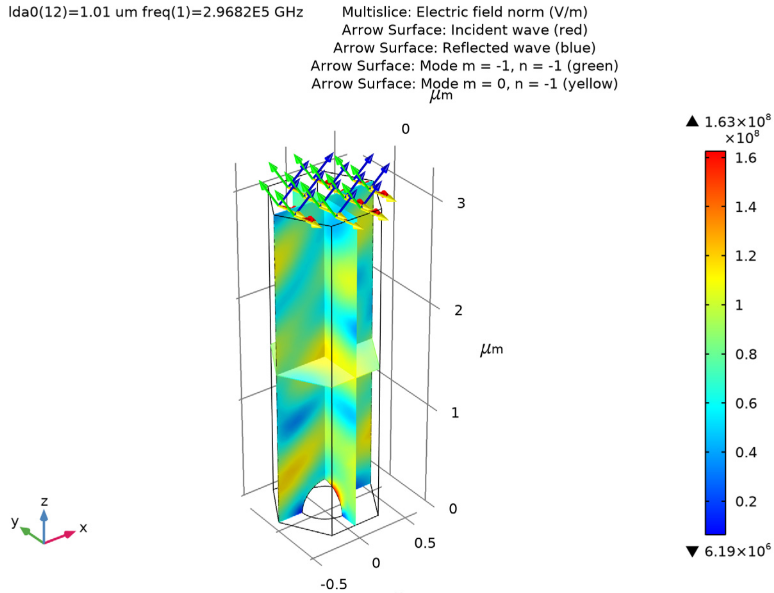
1 In the **Model Builder** window, click **Electric Field (emw)**.

2 In the **Settings** window for **3D Plot Group**, locate the **Data** section.

3 From the **Parameter value (lda0 (um))** list, choose **1.01**.

4 In the **Electric Field (emw)** toolbar, click **Plot**.

5 Click the  **Zoom Extents** button in the **Graphics** toolbar.



Your plot should look the same as Figure 4.

Global 1

1 In the **Model Builder** window, expand the **Results>S-parameter (emw)** node, then click **Global 1**.

2 In the **Settings** window for **Global**, locate the **y-Axis Data** section.

3 In the table, enter the following settings:

Expression	Unit	Description
`abs(emw.S11)^2`	1	`Reflectance`
`abs(emw.S21)^2`	1	`m = -1, n = -1, in-plane`
`abs(emw.S31)^2`	1	`m = -1, n = -1, out-of-plane`
`abs(emw.S41)^2`	1	`m = 0, n = -1, in-plane`
`abs(emw.S51)^2`	1	`m = 0, n = -1, out-of-plane`
`abs(emw.S61)^2`	1	`m = 0, n = 0, orthogonal`

4 Locate the **x-Axis Data** section. From the **Axis source data** list, choose **Outer solutions**.

5 Select the **Description** check box.

6 Click to expand the **Legends** section. From the **Legends** list, choose **Manual**.

7 In the table, enter the following settings:

Legends
`Reflectance`
`m = -1, n = -1, in-plane`
`m = -1, n = -1, out-of-plane`
`m = 0, n = -1, in-plane`
`m = 0, n = -1, out-of-plane`
`m = 0, n = 0, orthogonal`

S-parameter (emw)

1 In the **Model Builder** window, click **S-parameter (emw)**.

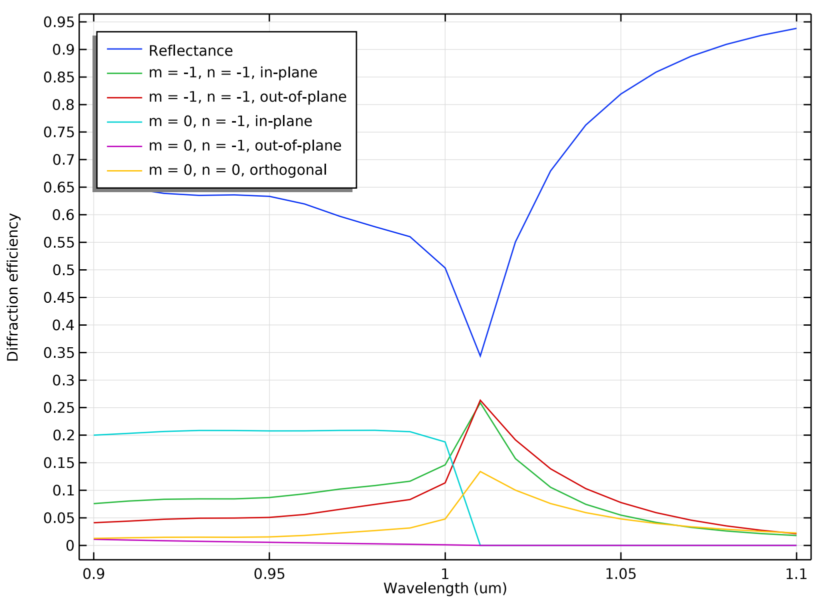
2 In the **Settings** window for **1D Plot Group**, click to expand the **Title** section.

3 From the **Title type** list, choose **None**.

4 Locate the **Plot Settings** section. In the **y-axis label** text field, type `Diffraction efficiency`.

5 Locate the **Legend** section. From the **Position** list, choose **Upper left**.

6 In the **S-parameter (emw)** toolbar, click **Plot**.



Your plot should look like Figure 5.

Polarization Plot (emw)

1 In the **Model Builder** window, click **Polarization Plot (emw)**.

2 In the **Settings** window for **1D Plot Group**, locate the **Legend** section.

3 From the **Position** list, choose **Upper left**.

4 In the **Polarization Plot (emw)** toolbar, click **Plot**.

This plot shows the polarization ellipses for the modes for TE polarization.

ELECTROMAGNETIC WAVES, FREQUENCY DOMAIN (EMW)

Port 1

Now, repeat the simulation for an incoming wave with p-polarization (the electric field polarized in the plane of incidence).

1 In the **Model Builder** window, under **Component 1 (comp1)>Electromagnetic Waves, Frequency Domain (emw)** click **Port 1**.

2 In the **Settings** window for **Port**, locate the **Port Mode Settings** section.

3 From the **Input quantity** list, choose **Magnetic field**.

4 Specify the $\mathbf{H}_0$ vector as

-H0*sin(phi)	x
H0*cos(phi)	y
0	z

5 In the **Home** toolbar, click **Compute**.

RESULTS

Electric Field (emw)

1 In the **Settings** window for **3D Plot Group**, locate the **Data** section.

2 From the **Parameter value (lda0 (um))** list, choose **1.01**.

3 Click the **Zoom Extents** button in the **Graphics** toolbar, and verify that your plot look the same as Figure 6.

S-parameter (emw)

1 In the **Model Builder** window, click **S-parameter (emw)**.

2 In the **S-parameter (emw)** toolbar, click **Plot**, and verify that your plot look the same as Figure 7.

Polarization Plot (emw)

Finally, take a look at the polarization states when the input wave is p-polarized.

1 In the **Model Builder** window, click **Polarization Plot (emw)**.
2 In the **Polarization Plot (emw)** toolbar, click **Plot**.

   Also in this case, the polarization is elliptical for the different modes.