

Motion of Trapped Protons in the Earth's Magnetic Field

Introduction

The Earth has a substantial magnetic field that extends outward for many thousands of kilometers. It is thought that this magnetic field is generated by circulating currents within a spinning metal liquid core — the dynamo effect. This magnetic field closely resembles a dipole field; however, there is a tilt between the spin axis of the Earth and that of the magnetic dipole. There are also other asymmetries in the Earth's magnetic field that require the use of software models to describe fully. These models are constructed using data obtained from orbiting spacecraft.

Having an accurate model of the Earth's magnetic field is very important in studies of the Earth's deep interior, its crust, and its ionosphere and magnetosphere. Therefore, a standard model is maintained by the International Association of Geomagnetism and Aeronomy (IAGA). This model is referred to as the International Geomagnetic Reference Field (IGRF). The IGRF model is updated every few years and is currently on its 12th generation (Ref. 1).

The influence of the solar wind means that the full extended shape of the Earth's magnetosphere is very different from a dipole. However, within a few Earth radii, simpler models of the Earth's magnetic field are sufficient.

Charged particles in the Earth's magnetic field travel in helical paths around the field lines. The angle between the direction of the magnetic field and a particle's trajectory is referred to as the pitch angle. As particles move along the magnetic field lines, they approach a pole in the magnetic field. Near the poles, the particles get closer to earth, causing the magnetic field to increase in magnitude and thus causing the pitch angle to increase. If the pitch angle reaches 90 degrees while the particle is still outside of the atmosphere (>100 km), the particle motion reverses direction back along the field line. The point at which this occurs is referred to as a mirror, or bounce, point. The particle similarly bounces off the other mirror point in the other hemisphere.

Particles are considered trapped if they are not lost to the Earth's atmosphere during this motion. Particles can also be liberated due to scattering by electromagnetic waves.

The latitude of a mirror point (λ_m) is related to the particle's pitch angle at the equatorial plane, or its equatorial pitch angle (α_{eq}) . As the equatorial pitch angle increases, the mirror point latitude decreases. Any particles that have equatorial pitch angles that give mirror points outside the atmosphere are said to be outside the atmospheric loss cone and are trapped.

Particles that bounce from mirror point to mirror point also exhibit a drift motion around the Earth, switching from field line to field line. This is due to the fact that the magnetic field magnitude is increasing as the particle moves toward the Earth, so that its gyration is not circular but has in fact a smaller radius on the side closer to the Earth.

As electrons and protons drift in opposite directions around the Earth, an electric current (ring current) is set up. The magnitude of the ring current increases during solar storms, and its effect can be measured on the ground as a weakening of the measured magnetic field. The disturbance storm time index, **Dst**, is one such measure of this ring current and is used to assess the severity of magnetic storms from the Sun.

These motions of trapped charge particles lead to "belts" of energetic charged particles in the near Earth environment and are referred to as the Van Allen radiation belts. They extend from about 1000 to 60,000 km (~0.15 to ~10 Earth radii) above the Earth's surface and therefore pose a real threat to space-based microelectronics.

Model Definition

Mathematically, the IGRF model consists of the Gauss coefficients, which define a spherical harmonic expansion of the magnetic scalar potential:

$$V(r, \Phi, \theta) = a \sum_{l=1}^{L} \sum_{m=-l}^{l} \left(\frac{R_{\rm e}}{r}\right)^{l-1} (g_l^m \cos m\Phi + h_l^m \sin m\Phi) P_l^m(\cos\theta)$$

where *r* is the radial distance from the Earth's center, *L* is the maximum degree of the expansion, Φ is East longitude, θ is colatitude (polar angle), R_e is the Earth's radius, g_l^m and h_l^m are Gauss coefficients, and $P_l^m \cos\theta$ are the Schmidt normalized associated Legendre functions of degree *l* and order *m*.

The model uses a simple sphere of radius R_e to represent the Earth within a larger spherical simulation domain of radius $5R_e$ where this particle trajectories are computed. The geometry is shown in Figure 1.

The Magnetic Force feature in the Charged Particle Tracing interface and the Particle Tracing for Fluid Flow interface includes a built-in option to compute magnetic fields using the data from the IGRF model. To access this data, in a 3D model select **Earth's magnetic field** from the **Magnetic flux density** list in the settings window for the Magnetic Force feature.

Figure 2 shows the IGRF magnetic field lines. The difference from a simple dipole is not really evident in this figure, but asymmetries exist in both latitude and longitude.



Figure 1: Geometry of the simulation domain, which extends from radius Re to 5Re.



Time=3 s Streamline: Magnetic flux density, Earth (rotated) Surface: Magnetic flux density norm (nT)

Figure 2: IGRF magnetic field lines.

The model uses the Charged Particle Tracing interface only, together with a Time Dependent study.

Results and Discussion

A single 10 MeV proton is released from the equatorial plane at a distance of $2R_e$ from the center of the Earth. It is released with an equatorial pitch angle of 30 degrees. The three components of its motion (gyration, bounce, and drift) are clearly visible in Figure 3. The timescales for the drift motion are much longer than that of the bounce motion, which in turn is much longer than the gyration period.



Figure 3: Particle trajectory of a single 10 MeV proton in Earth's magnetic field. Particle gyration, bounce and drift motion are clearly visible.

Due to the large changes in the magnitude of the magnetic field in different parts of the simulation domain, care has to be taken to ensure an appropriate time step is used for the particles. An good indication of how well the time steps are resolving the gyration of the particles is to plot the particle energy as a function of time and observe little or no change. Figure 4 plots the relative change in the particle kinetic energy as a function of time. This relative error is of the order 10^{-3} and so you can be certain you are resolving most of the particle motion.



Figure 4: Relative error in particle kinetic energy.

There are many aspects of the particle motion that can be investigated using this model, but the scope of this tutorial is limited to investigating how the mirror point latitude is affected by the equatorial pitch angle of the released protons.

As mentioned above, as the equatorial pitch angle of a particle decreases, the mirror point latitude is expected to increase. At the limits, a particle with an equatorial pitch angle of 90 degrees would remain in the equatorial plane whereas one with a pitch angle of zero degrees would travel directly along the field line without bouncing.

Figure 6 shows the mirror point latitude versus equatorial pitch angle for 15 particles with equatorial pitch angles ranging from 5 degrees to 80 degrees.



Figure 5: Trajectories of several trapped protons. The color expression corresponds to the equatorial pitch angle of each proton.



Figure 6: Mirror point latitude (λ_m) against particle equatorial pitch angle (Ea).

References

1. International Geomagnetic Reference Field website, http://www.ngdc.noaa.gov/IAGA/vmod/igrf.html.

Application Library path: Particle_Tracing_Module/ Charged_Particle_Tracing/trapped_protons

Modeling Instructions

From the File menu, choose New.

NEW

In the New window, click Model Wizard.

MODEL WIZARD

- I In the Model Wizard window, click 间 3D.
- 2 In the Select Physics tree, select AC/DC>Particle Tracing>Charged Particle Tracing (cpt).
- 3 Click Add.
- 4 Click \bigcirc Study.
- 5 In the Select Study tree, select General Studies>Time Dependent.
- 6 Click 🗹 Done.

GLOBAL DEFINITIONS

Parameters 1

- I In the Model Builder window, under Global Definitions click Parameters I.
- 2 In the Settings window for Parameters, locate the Parameters section.
- **3** In the table, enter the following settings:

Name	Expression	Value	Description
Re	6371.2[km]	6.3712E6 m	Radius of the Earth
E0	10[MeV]	1.6022E-12 J	Initial particle energy
alpha	30[deg]	0.5236 rad	Equatorial pitch angle

GEOMETRY I

Add a sphere with radius Re within a larger simulation domain of radius 5*Re.

- I In the Model Builder window, under Component I (compl) click Geometry I.
- 2 In the Settings window for Geometry, locate the Units section.
- 3 From the Length unit list, choose Mm.

Sphere I (sph1)

- I In the **Geometry** toolbar, click \bigoplus Sphere.
- 2 In the Settings window for Sphere, locate the Size section.
- 3 In the Radius text field, type Re.

Sphere 2 (sph2)

- I In the **Geometry** toolbar, click \bigoplus Sphere.
- 2 In the Settings window for Sphere, locate the Size section.
- 3 In the Radius text field, type 5*Re.

Difference I (dif1)

- I In the Geometry toolbar, click 📕 Booleans and Partitions and choose Difference.
- 2 Select the object **sph2** only.
- 3 In the Settings window for Difference, locate the Difference section.
- 4 Find the Objects to subtract subsection. Select the 🔲 Activate Selection toggle button.
- **5** Click the **Transparency** button in the **Graphics** toolbar.
- 6 Select the object **sph1** only.
- 7 Click 🟢 Build All Objects.
- **8** Click the 4 **Zoom Extents** button in the **Graphics** toolbar.
- 9 Click the Transparency button in the Graphics toolbar.

DEFINITIONS

Hide the outermost boundary to facilitate setup of the Charged Particle Tracing interface.

Hide for Geometry I

- I In the Model Builder window, expand the Definitions node.
- 2 Right-click View I and choose Hide for Geometry.
- 3 In the Settings window for Hide for Geometry, locate the Selection section.
- **4** From the **Geometric entity level** list, choose **Boundary**.
- 5 On the object difl, select Boundaries 1–4, 9, 10, 13, and 16 only.

Ball I

Add a Ball selection containing all boundaries on the Earth's surface.

- I In the Definitions toolbar, click hall/Disk.
- 2 In the Settings window for Ball, locate the Geometric Entity Level section.
- **3** From the Level list, choose **Boundary**.
- 4 Locate the Ball Radius section. In the Radius text field, type Re.

CHARGED PARTICLE TRACING (CPT)

- I In the Model Builder window, under Component I (comp1) click Charged Particle Tracing (cpt).
- 2 In the Settings window for Charged Particle Tracing, locate the Particle Release and Propagation section.
- 3 From the Formulation list, choose Newtonian, first order.

In this example, the particles are relativistic protons.

4 Select the **Relativistic correction** check box.

Particle Properties 1

- I In the Model Builder window, under Component I (compl)>Charged Particle Tracing (cpt) click Particle Properties I.
- 2 In the Settings window for Particle Properties, locate the Particle Species section.
- **3** From the **Particle species** list, choose **Proton**.

Magnetic Force 1

Exert a magnetic force on the particles using the Earth's magnetic field.

- I In the Physics toolbar, click 🔚 Domains and choose Magnetic Force.
- 2 In the Settings window for Magnetic Force, locate the Domain Selection section.
- **3** From the Selection list, choose All domains.
- 4 Locate the Magnetic Force section. From the B list, choose Earth's magnetic field.

Release from Grid I

Set up the **Release from Grid** feature to release a single 10 MeV proton at a distance of 2 Earth radii along the *x*-axis. Set its velocity *x*-component to zero, and its *y*- and *z*- components using the equatorial pitch angle.

- I In the Physics toolbar, click 💥 Global and choose Release from Grid.
- 2 In the Settings window for Release from Grid, locate the Initial Coordinates section.

- **3** In the $q_{x,0}$ text field, type 2*Re.
- 4 Locate the Initial Velocity section. From the Initial velocity list, choose Kinetic energy and direction.
- **5** In the E_0 text field, type E0.
- **6** Specify the **L**₀ vector as

0	x
<pre>sin(alpha)</pre>	у
cos(alpha)	z

Auxiliary Dependent Variable 1

Define two auxiliary dependent variables for the mirror point latitude and equatorial pitch angle. These auxiliary dependent variables will be disabled in Study 1 but will be needed in Study 2.

- I In the Physics toolbar, click 🖗 Global and choose Auxiliary Dependent Variable.
- 2 In the Settings window for Auxiliary Dependent Variable, locate the Auxiliary Dependent Variable section.
- 3 In the Field variable name text field, type Lm.

Auxiliary Dependent Variable 2

- I In the Physics toolbar, click 🖗 Global and choose Auxiliary Dependent Variable.
- 2 In the Settings window for Auxiliary Dependent Variable, locate the Auxiliary Dependent Variable section.
- 3 In the Field variable name text field, type Ea.

Velocity Reinitialization 1

The **Velocity Reinitialization** feature can be used to detect when a particle bounces, and set the auxiliary dependent variable Lm to the instantaneous value of the latitude at that point.

- I In the Physics toolbar, click 🔚 Domains and choose Velocity Reinitialization.
- 2 Select Domain 1 only.
- **3** In the **Settings** window for **Velocity Reinitialization**, locate the **Velocity Reinitialization** section.
- 4 In the e text field, type (cpt.vx*cpt.mf1.Berx+cpt.vy*cpt.mf1.Bery+cpt.vz* cpt.mf1.Berz)<0 && Lm==0.</p>

This expression is true for the first time step at which the angle between the particle velocity and the Earth's magnetic field is greater than 90 degrees.

- 5 From the Effect on primary particle list, choose None.
- 6 Click to expand the New Value of Auxiliary Dependent Variables section. Select the Assign new value to auxiliary variable : Lm check box.
- 7 In the Lm_{new} text field, type (-acos(qz/sqrt(qx^2+qy^2+qz^2))+pi/2)*(180/pi).

Release from Grid 1

Duplicate the release feature to create a separate release feature for the second study.

Release from Grid 2

- I In the Model Builder window, under Component I (compl)>Charged Particle Tracing (cpt) right-click Release from Grid I and choose Duplicate.
- 2 In the Settings window for Release from Grid, locate the Initial Velocity section.
- **3** Specify the **L**₀ vector as

0	x
sin(Ea)	у
cos(Ea)	z

The second **Release from Grid** feature stores the initial equatorial pitch angle of each particle.

- **4** Locate the **Initial Value of Auxiliary Dependent Variables** section. From the **Distribution function** list, choose **List of values**.
- 5 Click Range.
- 6 In the Range dialog box, type 10[deg] in the Start text field.
- 7 In the **Step** text field, type 5[deg].
- 8 In the Stop text field, type 80[deg].
- 9 Click Replace.

Since the initial direction will be defined in terms of the equatorial pitch angle, specify that this auxiliary variable must be initialized before the particle momentum.

- **10** In the Settings window for Release from Grid, locate the **Initial Value of Auxiliary Dependent Variables** section.
- **II** Select the second **Initialize before particle momentum** check box, which corresponds to the variable for equatorial pitch angle Ea.

STUDY I

Step 1: Time Dependent

Disable the features that are not needed for the first study.

- I In the Model Builder window, under Study I click Step I: Time Dependent.
- 2 In the Settings window for Time Dependent, locate the Physics and Variables Selection section.
- **3** Select the Modify model configuration for study step check box.
- 4 In the Physics and variables selection tree, select Component I (compl)> Charged Particle Tracing (cpt)>Auxiliary Dependent Variable I.
- 5 Click 📿 Disable.
- 6 In the Physics and variables selection tree, select Component I (compl)> Charged Particle Tracing (cpt)>Auxiliary Dependent Variable 2.
- 7 Click 📿 Disable.
- 8 In the Physics and variables selection tree, select Component I (compl)> Charged Particle Tracing (cpt)>Velocity Reinitialization I.
- 9 Click 🕢 Disable.
- 10 In the Physics and variables selection tree, select Component I (compl)> Charged Particle Tracing (cpt)>Release from Grid 2.
- II Click 🕖 Disable.
- 12 Locate the Study Settings section. Click Range.
- **I3** In the **Range** dialog box, type 0.005 in the **Step** text field.
- **I4** In the **Stop** text field, type **3**.
- 15 Click Replace.
- **I6** In the **Home** toolbar, click **= Compute**.

RESULTS

Particle Trajectories (cpt)

- I In the Settings window for 3D Plot Group, locate the Plot Settings section.
- 2 Clear the **Plot dataset edges** check box.

Particle Trajectories 1

I In the Model Builder window, expand the Particle Trajectories (cpt) node, then click Particle Trajectories I.

- 2 In the Settings window for Particle Trajectories, locate the Coloring and Style section.
- 3 Find the Line style subsection. From the Type list, choose Tube.
- 4 Select the Radius scale factor check box.
- **5** In the associated text field, type **0.01**.
- 6 From the Interpolation list, choose Uniform.
- 7 In the Number of interpolated times text field, type 2000.
- 8 Find the Point style subsection. From the Type list, choose None.

Color Expression 1

- I In the Model Builder window, expand the Particle Trajectories I node, then click Color Expression I.
- 2 In the Settings window for Color Expression, click Replace Expression in the upper-right corner of the Expression section. From the menu, choose Component I (compl)> Charged Particle Tracing>Fields>cpt.mfl.normB Magnetic flux density norm T.
- 3 In the Particle Trajectories (cpt) toolbar, click 🗿 Plot.
- 4 Click the ^{∠zx} Go to ZX View button in the Graphics toolbar. Compare the resulting plot with Figure 3.

Create some additional datasets to visualize the magnetic field using a Streamline plot.

Cut Plane 1

- I In the **Results** toolbar, click **Cut Plane**.
- 2 In the Settings window for Cut Plane, locate the Plane Data section.
- 3 From the Plane list, choose XY-planes.

Study I/Solution I (2) (soll)

In the Model Builder window, under Results>Datasets right-click Study I/Solution I (soll) and choose Duplicate.

Selection

- I In the Results toolbar, click 🐐 Attributes and choose Selection.
- 2 In the Settings window for Selection, locate the Geometric Entity Selection section.
- **3** From the **Geometric entity level** list, choose **Boundary**.
- 4 From the Selection list, choose Ball I.

Magnetic Flux Density

I In the **Results** toolbar, click **I 3D Plot Group**.

- 2 In the Settings window for 3D Plot Group, type Magnetic Flux Density in the Label text field.
- 3 Locate the Plot Settings section. Clear the Plot dataset edges check box.

Streamline 1

- I Right-click Magnetic Flux Density and choose Streamline.
- In the Settings window for Streamline, click Replace Expression in the upper-right corner of the Expression section. From the menu, choose Component I (compl)>
 Charged Particle Tracing>Fields>cpt.mfl.Berx,...,cpt.mfl.Berz Magnetic flux density, Earth (rotated).
- **3** Locate the **Streamline Positioning** section. From the **Positioning** list, choose **Startingpoint controlled**.
- 4 In the **Points** text field, type 50.
- 5 From the Along curve or surface list, choose Cut Plane I.
- 6 Locate the Coloring and Style section. Find the Line style subsection. From the Type list, choose Tube.

Color Expression 1

- I Right-click Streamline I and choose Color Expression.
- 2 In the Settings window for Color Expression, click Replace Expression in the upper-right corner of the Expression section. From the menu, choose Component I (compl)> Charged Particle Tracing>Fields>cpt.mfl.normB Magnetic flux density norm T.
- 3 Locate the Expression section. From the Unit list, choose nT.

Surface 1

- I In the Model Builder window, right-click Magnetic Flux Density and choose Surface.
- 2 In the Settings window for Surface, locate the Data section.
- 3 From the Dataset list, choose Study I/Solution I (2) (soll).
- 4 Click Replace Expression in the upper-right corner of the Expression section. From the menu, choose Component I (compl)>Charged Particle Tracing>Fields>cpt.mfl.normB Magnetic flux density norm T.
- 5 Locate the Expression section. From the Unit list, choose nT.
- 6 Click to expand the Inherit Style section. From the Plot list, choose Streamline I.
- 7 In the Magnetic Flux Density toolbar, click 🗿 Plot.
- 8 Click the YZ View button in the Graphics toolbar. Compare the resulting plot with Figure 2.

Energy Loss

- I In the Home toolbar, click 📠 Add Plot Group and choose ID Plot Group.
- 2 In the Settings window for ID Plot Group, type Energy Loss in the Label text field.
- 3 Locate the Data section. From the Dataset list, choose Particle I.

Particle 1

I In the Energy Loss toolbar, click \sim More Plots and choose Particle.

Verify that energy is conserved throughout the study.

- 2 In the Settings window for Particle, locate the y-Axis Data section.
- 3 In the Expression text field, type 1-(cpt.Ep)/at(0,cpt.Ep).
- **4** In the **Energy Loss** toolbar, click **I Plot**. Compare the resulting plot with Figure 4.

ADD STUDY

- I In the Home toolbar, click \sim Add Study to open the Add Study window.
- 2 Go to the Add Study window.
- 3 Find the Studies subsection. In the Select Study tree, select General Studies> Time Dependent.
- 4 Click Add Study in the window toolbar.
- 5 In the Home toolbar, click ~ 1 Add Study to close the Add Study window.

STUDY 2

Step 1: Time Dependent

- I In the Settings window for Time Dependent, locate the Physics and Variables Selection section.
- 2 Select the Modify model configuration for study step check box.
- 3 In the Physics and variables selection tree, select Component I (compl)> Charged Particle Tracing (cpt)>Release from Grid I.
- 4 Click 🖉 Disable.
- 5 Locate the Study Settings section. Click Range.
- 6 In the Range dialog box, type 0.001 in the Step text field.
- 7 In the **Stop** text field, type 0.7.
- 8 Click Replace.
- **9** In the **Home** toolbar, click **= Compute**.

RESULTS

Particle Trajectories (cpt) 1

- I In the Settings window for 3D Plot Group, locate the Plot Settings section.
- 2 Clear the **Plot dataset edges** check box.

Particle Trajectories 1

- I In the Model Builder window, expand the Particle Trajectories (cpt) I node, then click Particle Trajectories I.
- 2 In the Settings window for Particle Trajectories, locate the Coloring and Style section.
- **3** Find the Line style subsection. From the Type list, choose Line.
- 4 Find the **Point style** subsection. From the **Type** list, choose **None**.

Color Expression 1

- I In the Model Builder window, expand the Particle Trajectories I node, then click Color Expression I.
- 2 In the Settings window for Color Expression, locate the Expression section.
- **3** In the **Expression** text field, type Ea*180/pi.

Surface 1

- I In the Model Builder window, right-click Particle Trajectories (cpt) I and choose Surface.
- 2 In the Settings window for Surface, locate the Data section.
- 3 From the Dataset list, choose Study I/Solution I (2) (sol1).
- 4 Locate the Coloring and Style section. From the Coloring list, choose Uniform.
- 5 From the Color list, choose Gray.
- 6 In the Particle Trajectories (cpt) I toolbar, click 🗿 Plot.
- 7 Click the Go to Default View button in the Graphics toolbar. Compare the resulting plot with Figure 5.

Mirror Point Latitude

- I In the Home toolbar, click 🚛 Add Plot Group and choose ID Plot Group.
- 2 In the Settings window for ID Plot Group, type Mirror Point Latitude in the Label text field.

Plot the particle mirror point latitude against equatorial pitch angle.

- 3 Locate the Data section. From the Dataset list, choose Particle 2.
- **4** From the **Time selection** list, choose **Last**.
- 5 Locate the Plot Settings section. Select the x-axis label check box.

- 6 In the associated text field, type Equatorial pitch angle (deg).
- 7 Select the y-axis label check box.
- 8 In the associated text field, type Mirror point latitude (deg).

Particle 1

- I In the Mirror Point Latitude toolbar, click \sim More Plots and choose Particle.
- In the Settings window for Particle, click Replace Expression in the upper-right corner of the y-Axis Data section. From the menu, choose Component I (compl)>
 Charged Particle Tracing>Auxiliary dependent variables>Lm Auxiliary dependent variable Lm.
- 3 Locate the x-Axis Data section. From the Parameter list, choose Expression.
- **4** In the **Expression** text field, type Ea*180/pi.
- 5 In the Mirror Point Latitude toolbar, click 🗿 Plot.
- 6 Click the Zoom Extents button in the Graphics toolbarCompare the resulting plot with Figure 6.