Created in COMSOL Multiphysics 5.6

Rössler Attractor

Introduction

A Rössler attractor is a system of three nonlinear ordinary differential equations. The Rössler attractor is similar in nature to the Lorenz attractor. The coupled nonlinear equations can be solved using the Massless formulation available in the Mathematical Particle Tracing interface.

Model Definition

The governing differential equations for a Rössler attractor are

$$\frac{dx}{dt} = -y - z$$

$$\frac{dy}{dt} = x + ay$$

$$\frac{dz}{dt} = b + z(x - c)$$

where a, b, and c are constants. This model uses the original parameters used by Rössler: $a = 0.2$, $b = 0.2$ and $c = 5.7$. The particles are released from an initial grid in the y direction at $x = 0$ and $z = 0$. In total, 31 particles are released uniformly between $y = 3$ and $y = 8$.

In this example, **None** is selected from the **Unit system** list, and so the coordinates are dimensionless.

Results and Discussion

After the initial release of the particles, they travel counterclockwise in the xy-plane at $z = 0$. Once they cross to the positive side of the yz-plane the particles rise very sharply in the z direction. The particles with the largest radial coordinate in the xy-plane acquire the highest velocity and thus reach the highest point the z direction; see Figure 1. The region where the particles reach high elevations in a short period of time is called the upswing region. The particles with the outermost radial coordinate entering the upswing region end up at the innermost radial coordinate after the downswing region.

The average particle velocity is plotted in Figure 2. There is a double-peak in maximum velocity, corresponding to the upswing and downswing regions. A Poincaré map showing the intersection of the trajectories with the xz-plane is included in Figure 3. For $x < 0$ in this plane the particle z-coordinate is very small, while for $x > 0$ it can vary significantly.

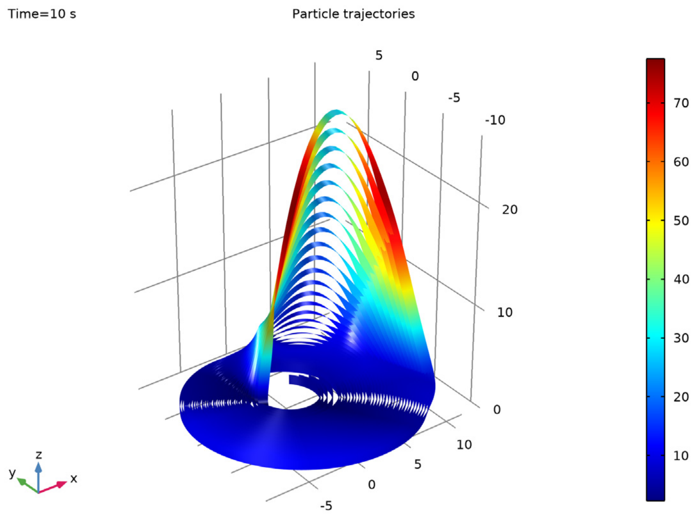

Figure 1: Plot of the Rössler attractor after 10 seconds.

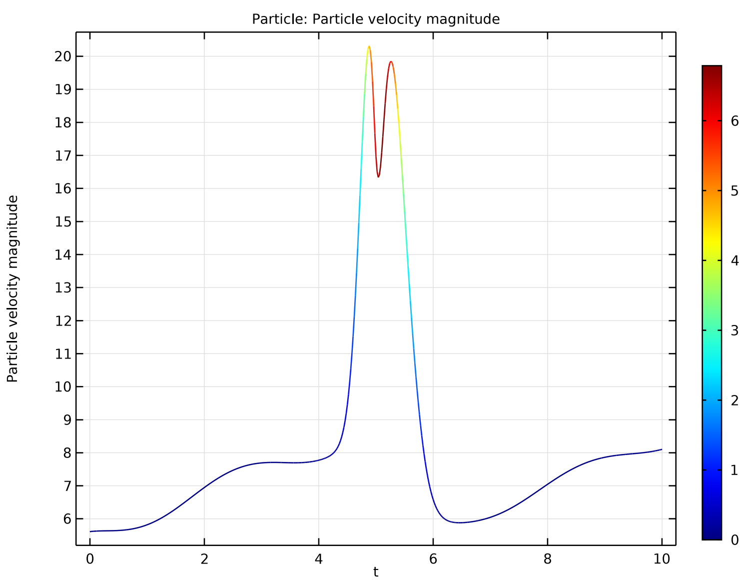

Figure 2: Plot of the average particle velocity versus time.

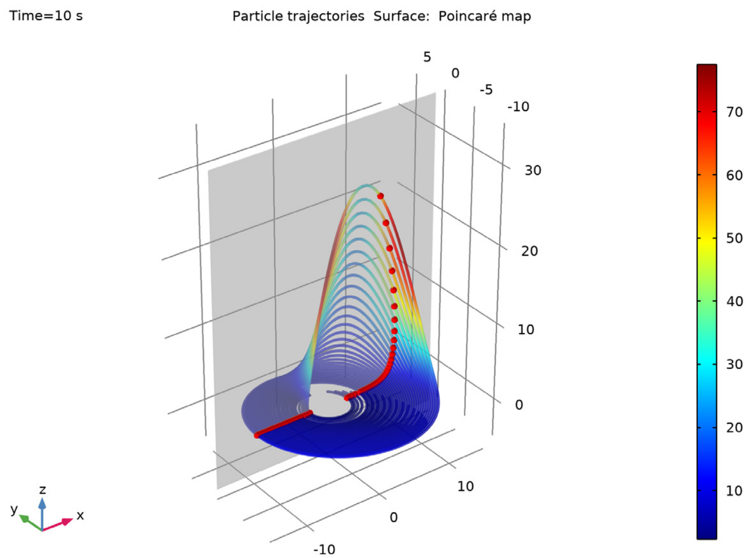

Figure 3: Particle trajectories with a Poincaré map superimposed (red dots) and Poincaré section shown (gray cut plane).

Reference

1. Wikipedia, http://en.wikipedia.org/wiki/R%C3%B6ssler_attractor

Application Library path: `Particle_Tracing_Module/Tutorials/rossler_attractor`

Modeling Instructions

From the **File** menu, choose **New**.

NEW

In the **New** window, click **Model Wizard**.

MODEL WIZARD

1 In the **Model Wizard** window, click **3D**.

2 In the **Select Physics** tree, select **Mathematics>Mathematical Particle Tracing (pt)**.

3 Click **Add**.

4 Click **Study**.

5 In the **Select Study** tree, select **General Studies>Time Dependent**.

6 Click **Done**.

GEOMETRY 1

The geometry is designated to be large enough so that the particle trajectories do not make contact with the walls.

Cylinder 1 (cyl1)

1 In the **Geometry** toolbar, click **Cylinder**.

2 In the **Settings** window for **Cylinder**, locate the **Size and Shape** section.

3 In the **Radius** text field, type `15`.

4 In the **Height** text field, type `40`.

5 Locate the **Position** section. In the **z** text field, type `-5`.

6 In the **Geometry** toolbar, click **Build All**.

COMPONENT 1 (COMP1)

1 In the **Model Builder** window, click **Component 1 (comp1)**.

2 In the **Settings** window for **Component**, locate the **General** section.

3 From the **Unit system** list, choose **None**.

GLOBAL DEFINITIONS

Define the constants a, b and c as parameters. This means they could, in principle, be varied as part of a **Parametric Sweep** when solving.

Parameters 1

1 In the **Model Builder** window, under **Global Definitions** click **Parameters 1**.

2 In the **Settings** window for **Parameters**, locate the **Parameters** section.

3 In the table, enter the following settings:

Name	Expression	Value	Description
a	0.2	0.2	Model parameter
b	0.2	0.2	Model parameter
c	5.7	5.7	Model parameter

MATHEMATICAL PARTICLE TRACING (PT)

The **Massless** formulation allows a system of first order ordinary differential equations to be solved.

1 In the **Model Builder** window, under **Component 1 (comp1)** click **Mathematical Particle Tracing (pt)**.
2 In the **Settings** window for **Mathematical Particle Tracing**, locate the **Particle Release and Propagation** section.
3 From the **Formulation** list, choose **Massless**.

Particle Properties 1

1 In the **Model Builder** window, under **Component 1 (comp1)> Mathematical Particle Tracing (pt)** click **Particle Properties 1**.
2 In the **Settings** window for **Particle Properties**, locate the **Particle Velocity** section.
3 Specify the **v** vector as

`-y-z`	x
`x+a*y`	y
`b+z*(x-c)`	z

Release from Grid 1

1 In the **Physics** toolbar, click **Global** and choose **Release from Grid**.
2 In the **Settings** window for **Release from Grid**, locate the **Initial Coordinates** section.
3 In the $q_{y,0}$ text field, type `range(3,5/30,8)`.

STUDY 1

Step 1: Time Dependent

1 In the **Model Builder** window, under **Study 1** click **Step 1: Time Dependent**.
2 In the **Settings** window for **Time Dependent**, locate the **Study Settings** section.
3 In the **Output times** text field, type `range(0,0.02,10)`.
4 In the **Home** toolbar, click **Compute**.

RESULTS

Particle Trajectories 1

Render the particle trajectories as ribbons. The default ribbon orientation is in the direction of the unit binormal, or the direction out of the plane tangent to the curved trajectory.

1 In the **Model Builder** window, expand the **Particle Trajectories (pt)** node, then click **Particle Trajectories 1**.
2 In the **Settings** window for **Particle Trajectories**, locate the **Coloring and Style** section.
3 Find the **Line style** subsection. From the **Type** list, choose **Ribbon**.
4 Select the **Width scale factor** check box.
5 In the associated text field, type `0.5`.
6 Find the **Point style** subsection. From the **Type** list, choose **None**.

Particle Trajectories (pt)

1 In the **Model Builder** window, click **Particle Trajectories (pt)**.
2 In the **Settings** window for **3D Plot Group**, locate the **Plot Settings** section.
3 Clear the **Plot dataset edges** check box.
4 In the **Particle Trajectories (pt)** toolbar, click **Plot**.
5 Click the **Go to Default View** button in the **Graphics** toolbar. The plot should look like Figure 1.

Now plot the average particle velocity using the **1D Plot Group** and a **Particle** plot type.

Average Particle Velocity Magnitude

1 In the **Home** toolbar, click **Add Plot Group** and choose **1D Plot Group**.
2 In the **Settings** window for **1D Plot Group**, type `Average Particle Velocity Magnitude` in the **Label** text field.
3 Locate the **Data** section. From the **Dataset** list, choose **Particle 1**.

Particle 1

1 In the **Average Particle Velocity Magnitude** toolbar, click **More Plots** and choose **Particle**.
2 In the **Settings** window for **Particle**, click **Replace Expression** in the upper-right corner of the **y-Axis Data** section. From the menu, choose **Component 1 (comp1)>Mathematical Particle Tracing>Velocity and energy>pt.V - Particle velocity magnitude - m/s**.
3 Locate the **Data Series Operation** section. From the **Operation** list, choose **Average**.

Color Expression 1

1 Right-click **Particle 1** and choose **Color Expression**.

2 In the **Settings** window for **Color Expression**, click **Replace Expression** in the upper-right corner of the **Expression** section. From the menu, choose **Component 1 (comp1)>Mathematical Particle Tracing>Particle position>qz - Particle position, z component - m**.

3 In the **Average Particle Velocity Magnitude** toolbar, click **Plot**. The plot should look like Figure 2.

A **Cut Plane** dataset can be used to visualize how the particles cross a specific area of interest in the modeling domain. A **Poincaré map** is created using the intersection points of the trajectories with the plane.

Cut Plane 1

1 In the **Results** toolbar, click **Cut Plane**.

2 In the **Settings** window for **Cut Plane**, locate the **Plane Data** section.

3 From the **Plane** list, choose **ZX-planes**.

4 Locate the **Data** section. From the **Dataset** list, choose **Particle 1**.

Particle Trajectories and Poincaré Map

1 In the **Model Builder** window, right-click **Particle Trajectories (pt)** and choose **Duplicate**.

2 In the **Settings** window for **3D Plot Group**, type `Particle Trajectories and Poincaré Map` in the **Label** text field.

Particle Trajectories 1

1 In the **Model Builder** window, expand the **Particle Trajectories and Poincaré Map** node, then click **Particle Trajectories 1**.

2 In the **Settings** window for **Particle Trajectories**, locate the **Coloring and Style** section.

3 Find the **Line style** subsection. From the **Type** list, choose **Tube**.

4 In the **Tube radius expression** text field, type `10`.

Surface 1

1 In the **Model Builder** window, right-click **Particle Trajectories and Poincaré Map** and choose **Surface**.

2 In the **Settings** window for **Surface**, locate the **Data** section.

3 From the **Dataset** list, choose **Cut Plane 1**.

4 Locate the **Coloring and Style** section. From the **Coloring** list, choose **Uniform**.

5 From the **Color** list, choose **Gray**.

6 Click the **Transparency** button in the **Graphics** toolbar.

Particle Trajectories and Poincaré Map

In the **Model Builder** window, click **Particle Trajectories and Poincaré Map**.

Poincaré Map 1

1 In the **Particle Trajectories and Poincaré Map** toolbar, click **More Plots** and choose **Poincaré Map**.

2 In the **Settings** window for **Poincaré Map**, locate the **Data** section.

3 From the **Cut plane** list, choose **Cut Plane 1**.

4 In the **Particle Trajectories and Poincaré Map** toolbar, click **Plot**.

5 Locate the **Coloring and Style** section. Select the **Radius scale factor** check box.

6 In the associated text field, type 1.

7 In the **Particle Trajectories and Poincaré Map** toolbar, click **Plot**.

8 Click the **Zoom Extents** button in the **Graphics** toolbar. The plot should look like Figure 3.