Created in COMSOL Multiphysics 5.6

Pierce Electron Gun

Introduction

An electron gun must be able to draw a sufficient current and accelerate the electrons to the desired speed. First the electrons are drawn from an electron source, such as a metal cathode or plasma. Then the electrons are accelerated using electric and magnetic forces.

The first part of an electron gun geometry, where electrons are first extracted, presents unique design challenges because the emitted electron speeds are usually lowest there, and therefore the space charge density usually reaches a maximum in this region. The high space charge density can cause an electron beam to spread out.

The Pierce electron gun design uses electrodes with a particular shape to counteract the Coulomb repulsion between electrons in the beam. As a result, the electrons in the beam propagate in straight lines. The emitted electrons at the cathode are assumed to be space charge limited; the initial thermal distribution of electron velocities is neglected.

Model Definition

The Pierce electron gun design uses electrodes of a specific shape, so that the outward component of the electric field at the edge of the beam is zero. This requires the electrodes to perfectly counterbalance the Coulomb repulsion between the beam electrons.

Inside the beam, both the cathode and anode are flat edges. Here the anode is understood to be a metal grid so that the electrons may pass through. At higher current it may be necessary to shoot the electron beam through a gap in the anode.

Outside the beam, the bottom electrode makes an angle of 67.5 degrees with the beam propagation direction. The anode is shaped so that

$$\left(\frac{r}{d}\right)^{4/3}\cos\frac{4\theta}{3} = 1$$

where r (SI unit: m) is the distance from the point where the flat part of the cathode meets the incline, d (SI unit: m) is the gap thickness between the cathode and anode at the beam center, and θ (SI unit: rad) is the angle that a line from this point makes with the beam propagation direction.

The determination of these electrode shapes is sometimes called the Pierce method of gun design. The full derivation is given in Appendix: Derivation of the Cathode Shape.

The gun geometry is shown in Figure 1. The interior edge separating the two domains is only kept for meshing purposes, since the space charge density distribution within the beam can be calculated more accurately using a finer mesh as shown in Figure 2.

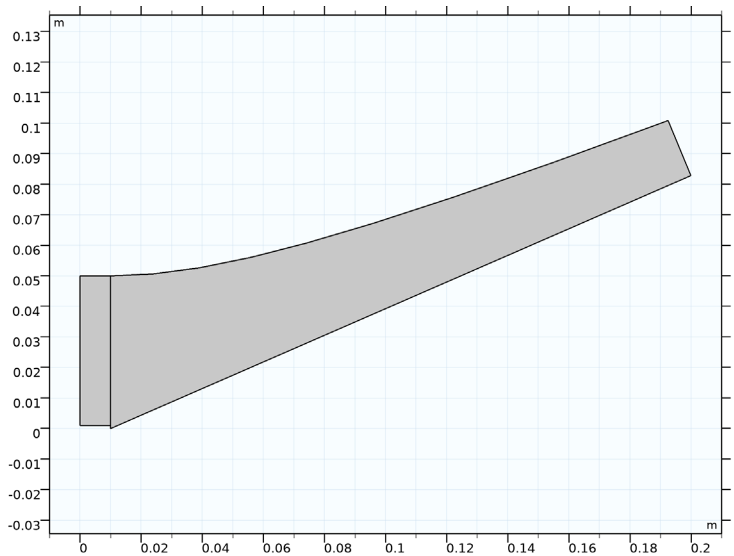

Figure 1: Geometry of the Pierce electron gun.

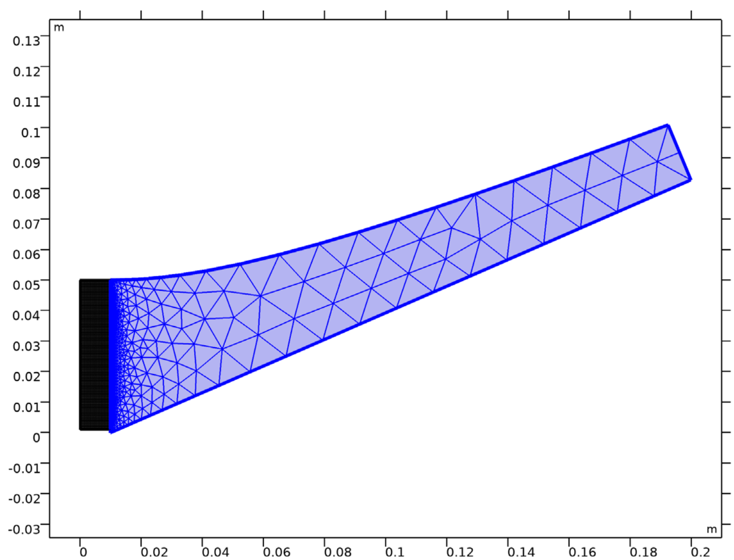

Figure 2: A fine mapped mesh is used in the beam path. The mesh coarsens as the distance from the beam is increased.

Note that the lower end of the rectangular domain and the inclined electrode do not meet at a common vertex; the rectangle is a bit higher. This is because the electrons are released using the **Space Charge Limited Emission** feature. This feature emits model particles from a virtual cathode a short distance away from the real cathode. This is to avoid the infinite space charge density that occurs at the real cathode when the thermal distribution of released electron velocities is not taken into account.

Results and Discussion

The electric potential and some electric field streamlines are shown in Figure 3. Ideally the electric field streamlines within the beam path and the electron trajectories should be perfectly vertical.

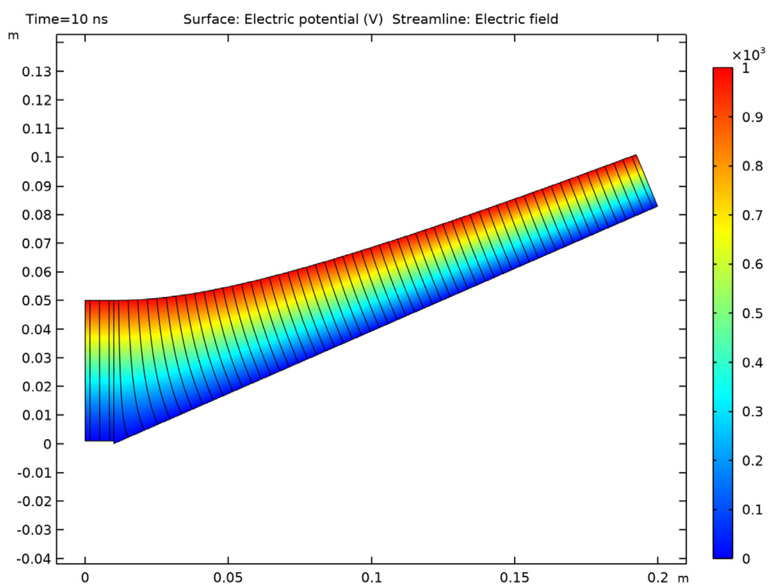

Figure 3: Surface plot of the electric potential. Electric field streamlines are shown in black.

The electron trajectories are shown in Figure 4. The beam does become about 3% thinner by the time it reaches the anode, but a small deviation can be expected because this finite-size geometry is actually a truncated version of the ideal Pierce design. For more details, see Appendix: Derivation of the Cathode Shape. To better visualize the shape of the electric potential distribution in the beam, a contour plot with electric field streamlines is shown in Figure 5. A **Filter** node was used to hide part of the geometry.

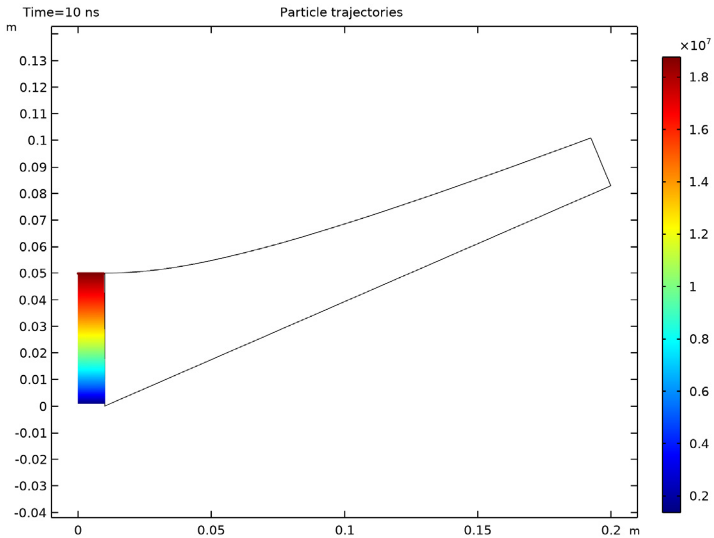

Figure 4: Particle trajectories are shown as lines. The color expression is the particle speed.

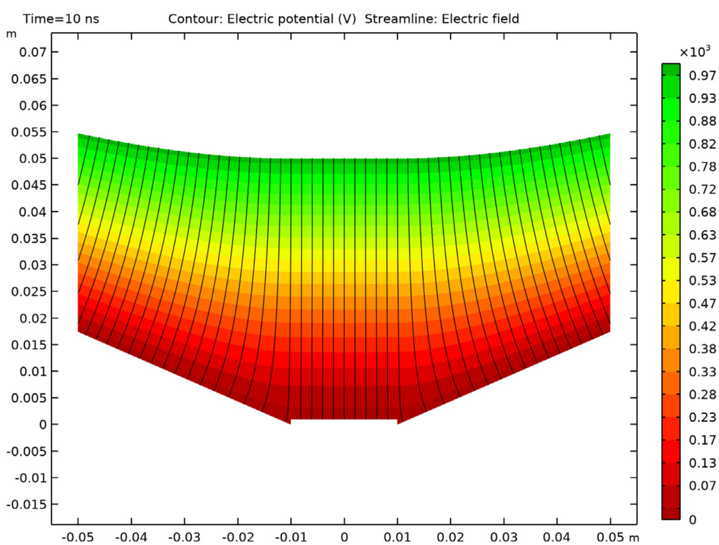

Figure 5: Contour plot of the electric potential, Electric field streamlines are shown in black.

Reference

1. S. Humphries, *Charged Particle Beams*, Dover Publications, New York, 2013.

2. J. R. Pierce, "Rectilinear electron flow in beams", *Journal of Applied Physics*, vol. 11, no. 8, pp. 548-554, 1940.

Application Library path: `Particle_Tracing_Module/Charged_Particle_Tracing/pierce_electron_gun`

Modeling Instructions

From the **File** menu, choose **New**.

NEW

In the **New** window, click **Model Wizard**.

MODEL WIZARD

1 In the **Model Wizard** window, click **2D**.

2 In the **Select Physics** tree, select **AC/DC>Particle Tracing>Particle Field Interaction, Non-Relativistic**.

3 Click **Add**.

4 Click **Study**.

5 In the **Select Study** tree, select **Preset Studies for Selected Physics Interfaces>Charged Particle Tracing>Bidirectionally Coupled Particle Tracing**.

6 Click **Done**.

GLOBAL DEFINITIONS

Parameters 1

1 In the **Model Builder** window, under **Global Definitions** click **Parameters 1**.

2 In the **Settings** window for **Parameters**, locate the **Parameters** section.

3 In the table, enter the following settings:

Name	Expression	Value	Description
d	5[cm]	0.05 m	Gap thickness
db	1[mm]	0.001 m	Virtual cathode height
w1	1[cm]	0.01 m	Cathode width
w2	20[cm]	0.2 m	Focusing electrode width
alpha	22.5[deg]	0.3927 rad	Focusing electrode angle
V0	1[kV]	1000 V	Anode potential

GEOMETRY 1

Rectangle 1 (r1)

1 In the **Geometry** toolbar, click **Rectangle**.

2 In the **Settings** window for **Rectangle**, locate the **Size and Shape** section.

3 In the **Width** text field, type `w1`.

4 In the **Height** text field, type `d-db`.

5 Locate the **Position** section. In the **y** text field, type `db`.

The bottom edge of the rectangle is the flat virtual cathode. The top edge is a flat anode grid that the released electrons can pass through. At high current it may be necessary to replace the grid with a gap in the anode so that the beam does not melt the grid.

Next add the curved part of the anode.

Parametric Curve 1 (pc1)

1 In the **Geometry** toolbar, click **More Primitives** and choose **Parametric Curve**.

2 In the **Settings** window for **Parametric Curve**, locate the **Parameter** section.

3 In the **Name** text field, type `theta`.

4 In the **Maximum** text field, type `62*pi/180`.

This maximum parameter value is somewhat arbitrary but must be less than 67.5 degrees.

5 Locate the **Expressions** section. In the **x** text field, type `w1+d*sec(4*theta/3)^0.75*sin(theta)`.

6 In the **y** text field, type `d*sec(4*theta/3)^0.75*cos(theta)`.

Add a **Polygon** to define the focusing electrode. An ideal Pierce gun extends infinitely, but here the polygon is cut off at a sufficient distance from the beam.

Polygon 1 (pol1)

1 In the **Geometry** toolbar, click **Polygon**.

2 In the **Settings** window for **Polygon**, locate the **Object Type** section.

3 From the **Type** list, choose **Open curve**.

4 Locate the **Coordinates** section. In the table, enter the following settings:

x (m)	y (m)
w1	d
w1	0
w2	w2*tan(alpha)
w2-d*tan(alpha)	w2*tan(alpha)+d

Convert to Solid 1 (csol1)

1 In the **Geometry** toolbar, click **Conversions** and choose **Convert to Solid**.

2 Select the objects **pc1** and **pol1** only.

3 In the **Settings** window for **Convert to Solid**, click **Build All Objects**.

4 Click the **Zoom Extents** button in the **Graphics** toolbar. The plot should look like Figure 1.

DEFINITIONS

Integration 1 (intop1)

1 In the **Definitions** toolbar, click **Nonlocal Couplings** and choose **Integration**.

2 In the **Settings** window for **Integration**, locate the **Source Selection** section.

3 From the **Geometric entity level** list, choose **Point**.

4 Select Point 4 only.

This **Integration** coupling will be used to evaluate the electric potential at the end of the virtual cathode, in order to avoid discontinuities in the potential.

MATERIALS

Material 1 (mat1)

1 In the **Model Builder** window, under **Component 1 (comp1)** right-click **Materials** and choose **Blank Material**.

2 In the **Settings** window for **Material**, locate the **Material Contents** section.

3 In the table, enter the following settings:

Property	**Variable**	**Value**	**Unit**	**Property group**
Relative permittivity	epsilonr_iso ; epsilonrii = epsilonr_iso, epsilonrij = 0	1	1	Basic

CHARGED PARTICLE TRACING (CPT)

Particle Properties 1

1 In the **Model Builder** window, under **Component 1 (comp1)>Charged Particle Tracing (cpt)** click **Particle Properties 1**.

2 In the **Settings** window for **Particle Properties**, locate the **Particle Species** section.

3 From the **Particle species** list, choose **Electron**.

Electric Force 1

1 In the **Model Builder** window, click **Electric Force 1**.

2 In the **Settings** window for **Electric Force**, locate the **Electric Force** section.

3 From the $\mathbf{E}$ list, choose **Electric field (es/ccn1)**.

MULTIPHYSICS

Space Charge Limited Emission 1 (scle1)

1 In the **Physics** toolbar, click **Multiphysics Couplings** and choose **Boundary> Space Charge Limited Emission**.

2 Select Boundary 2 only.

3 In the **Settings** window for **Space Charge Limited Emission**, locate the **Space Charge Limited Emission** section.

4 In the o_s text field, type `db`.

The **Space Charge Limited Emission** node has two purposes: to release electrons and to determine the electric potential at the virtual cathode.

Electric Particle Field Interaction 1 (epfi1)

1 In the **Model Builder** window, click **Electric Particle Field Interaction 1 (epfi1)**.

2 In the **Settings** window for **Electric Particle Field Interaction**, locate the **Continuation Settings** section.

3 Select the **Use cumulative space charge density** check box.

4 In the β text field, type `10`.

When **Use cumulative space charge density** is selected, the space charge density of the electrons is gradually increased over the first few iterations of the study. This prevents the space charge density from being overestimated, which could possibly cause the initial particle velocity to point in the wrong direction.

Set up the boundary conditions for the Electrostatics interface. Specify the potentials on the anode and the focusing electrode. The left boundary is a symmetry plane so the default **Zero Charge** condition may be used.

ELECTROSTATICS (ES)

In the **Model Builder** window, under **Component 1 (comp1)** click **Electrostatics (es)**.

Anode

1. In the **Physics** toolbar, click **Boundaries** and choose **Electric Potential**.
2. In the **Settings** window for **Electric Potential**, type `Anode` in the **Label** text field.
3. Select Boundaries 3 and 8 only.
4. Locate the **Electric Potential** section. In the V_0 text field, type `V0`.

Focusing Electrode

1. In the **Physics** toolbar, click **Boundaries** and choose **Electric Potential**.
2. In the **Settings** window for **Electric Potential**, type `Focusing Electrode` in the **Label** text field.
3. Select Boundary 5 only.

Adjacent to Virtual Cathode

1. In the **Physics** toolbar, click **Boundaries** and choose **Electric Potential**.
2. In the **Settings** window for **Electric Potential**, type `Adjacent to Virtual Cathode` in the **Label** text field.
3. Select Boundary 4 only.
4. Locate the **Electric Potential** section. In the V_0 text field, type `intop1(scle1.V0)*(y/db)^(4/3)`.

   This expression uses the **Integration** coupling defined earlier. It ensures continuity between the zero potential at the focusing electrode and the nonzero potential on the edge of the virtual cathode.

MESH 1

Define a very fine mapped mesh where the electron beam will propagate, to accurately accumulate the space charge density. The mesh is allowed to become coarser in the charge-free region away from the beam.

Mapped 1

1 In the **Mesh** toolbar, click **Mapped**.
2 In the **Settings** window for **Mapped**, locate the **Domain Selection** section.
3 From the **Geometric entity level** list, choose **Domain**.
4 Select Domain 1 only.

Distribution 1

1 Right-click **Mapped 1** and choose **Distribution**.
2 Select Boundaries 2 and 3 only.
3 In the **Settings** window for **Distribution**, locate the **Distribution** section.
4 In the **Number of elements** text field, type 20.

Distribution 2

1 In the **Model Builder** window, right-click **Mapped 1** and choose **Distribution**.
2 Select Boundaries 1 and 6 only.
3 In the **Settings** window for **Distribution**, locate the **Distribution** section.
4 In the **Number of elements** text field, type 100.

Free Triangular 1

1 In the **Mesh** toolbar, click **Free Triangular**.
2 In the **Settings** window for **Free Triangular**, click **Build All**. The plot should look like Figure 2.

STUDY 1

The **Bidirectionally Coupled Particle Tracing** study step alternates between stationary calculations of the electric potential and transient calculations of the particle trajectories, allowing the particles and the electric potential to influence each other.

Step 1: Bidirectionally Coupled Particle Tracing

1 In the **Model Builder** window, under **Study 1** click **Step 1: Bidirectionally Coupled Particle Tracing**.
2 In the **Settings** window for **Bidirectionally Coupled Particle Tracing**, locate the **Study Settings** section.

3 From the **Time unit** list, choose **ns**.

4 In the **Output times** text field, type `range(0,1,10)`.

5 Locate the **Iterations** section. In the **Number of iterations** text field, type `20`.

6 In the **Home** toolbar, click **Compute**.

RESULTS

Streamline 1

The default plot shows the electric potential in the domain. First add some electric field lines. In order to show that the focusing electrode offsets the space charge of the beam, the electric field lines in the beam should be vertical.

1 Right-click **Electric Potential (es)** and choose **Streamline**.

2 Select Boundaries 3 and 8 only.

3 In the **Settings** window for **Streamline**, locate the **Streamline Positioning** section.

4 In the **Number** text field, type `60`.

5 In the **Electric Potential (es)** toolbar, click **Plot**. The plot should look like Figure 3.

Particle Trajectories 1

1 In the **Model Builder** window, expand the **Particle Trajectories (cpt)** node, then click **Particle Trajectories 1**.

2 In the **Settings** window for **Particle Trajectories**, locate the **Coloring and Style** section.

3 Find the **Line style** subsection. From the **Type** list, choose **Line**.

4 In the **Particle Trajectories (cpt)** toolbar, click **Plot**. The plot should look like Figure 4.

To better visualize the electric potential, add a **Mirror** dataset to fill in the left half of the geometry.

Mirror 2D 1

In the **Results** toolbar, click **More Datasets** and choose **Mirror 2D**.

Using this new **Mirror** dataset, create a contour plot of the electric potential.

Contours with Field Lines

1 In the **Results** toolbar, click **2D Plot Group**.

2 In the **Settings** window for **2D Plot Group**, type `Contours with Field Lines` in the **Label** text field.

3 Locate the **Data** section. From the **Dataset** list, choose **Mirror 2D 1**.

4 Locate the **Plot Settings** section. Clear the **Plot dataset edges** check box.

Contour 1

1 Right-click **Contours with Field Lines** and choose **Contour**.

2 In the **Settings** window for **Contour**, locate the **Coloring and Style** section.

3 From the **Contour type** list, choose **Filled**.

4 From the **Color table** list, choose **Traffic**.

5 Select the **Reverse color table** check box.

Filter 1

1 Right-click **Contour 1** and choose **Filter**.

2 In the **Settings** window for **Filter**, locate the **Element Selection** section.

3 In the **Logical expression for inclusion** text field, type `x<5[cm]`.

Streamline 1

1 In the **Model Builder** window, right-click **Contours with Field Lines** and choose **Streamline**.

2 In the **Settings** window for **Streamline**, locate the **Streamline Positioning** section.

3 From the **Entry method** list, choose **Coordinates**.

4 In the **x** text field, type `range(-0.05,0.002,0.05)`.

5 In the **y** text field, type `0.045`.

Filter 1

1 Right-click **Streamline 1** and choose **Filter**.

2 In the **Settings** window for **Filter**, locate the **Element Selection** section.

3 In the **Logical expression for inclusion** text field, type `x<5[cm]`.

4 In the **Contours with Field Lines** toolbar, click **Plot**.

5 Click the **Zoom Extents** button in the **Graphics** toolbar. The plot should look like Figure 5.

Appendix: Derivation of the Cathode Shape

In a charge-free region of space, the electric potential V (SI unit: V) potential must satisfy Poisson's equation,

$$\frac{\partial^2 V}{\partial x^2} + \frac{\partial^2 V}{\partial y^2} = 0$$

The Pierce method for determining the shape of the electrodes takes advantage of the observation that any function of the complex variable $y + ix$ will satisfy Poisson's equation.

Define $u = y + ix$ and consider an arbitrary function $f(y + ix)$. It can be shown by repeated applications of the chain rule,

$$\begin{aligned}\frac{\partial^2 f}{\partial x^2} + \frac{\partial^2 f}{\partial y^2} &= \frac{\partial}{\partial x}\left(\frac{\partial u}{\partial x}\frac{\partial f}{\partial u}\right) + \frac{\partial}{\partial y}\left(\frac{\partial u}{\partial y}\frac{\partial f}{\partial u}\right) \\ &= \frac{\partial}{\partial x}\left(i\frac{\partial f}{\partial u}\right) + \frac{\partial}{\partial y}\left(\frac{\partial f}{\partial u}\right) \\ &= \frac{\partial u}{\partial x}\frac{\partial}{\partial u}\left(i\frac{\partial f}{\partial u}\right) + \frac{\partial u}{\partial y}\frac{\partial}{\partial u}\left(\frac{\partial f}{\partial u}\right) \\ &= i^2\frac{\partial^2 f}{\partial u^2} + \frac{\partial^2 f}{\partial u^2} \\ &= -\frac{\partial^2 f}{\partial u^2} + \frac{\partial^2 f}{\partial u^2} \\ &= 0\end{aligned}$$

The function f must be defined so that it matches the space charge limited potential in the beam,

$$f(u) = V_{\mathrm{a}}\left(\frac{u}{d}\right)^{4/3}$$

So the electric potential is

$$V = V_{\mathrm{a}}\mathrm{Re}\left(\left(\frac{u}{d}\right)^{4/3}\right)$$

Then the cathode is the equipotential surface where $V = 0$, and the anode is the equipotential surface where $V = V_{\mathrm{a}}$. It is somewhat easier to determine the shape of these equipotential surfaces by expressing u in cylindrical polar coordinates,

$$u = re^{i\theta}$$

Thus

$$V = V_{\mathrm{a}}\mathrm{Re}\left(\left(\frac{r}{d}e^{i\theta}\right)^{4/3}\right)$$

Since r is real,

$$V = V_{\mathrm{a}}\left(\frac{r}{d}\right)^{4/3}\mathrm{Re}(e^{4i\theta/3})$$

Invoking Euler's theorem yields

$$V = V_{\mathrm{a}}\left(\frac{r}{d}\right)^{4/3}\cos\frac{4\theta}{3}$$

The solution for $V = 0$ is a straight line with

$$\frac{4\theta}{3} = \frac{\pi}{2} \tag{1}$$

That is, $\theta = 3\pi/8$ or 67.5 degrees. Recalling that the beam propagation direction is the real axis, this means the equipotential surface is a straight line that makes a 67.5 degree angle with the beam propagation direction.

The solution for $V = V_{\mathrm{a}}$ is

$$\left(\frac{r}{d}\right)^{4/3}\cos\frac{4\theta}{3} = 1$$

which can be rearranged and solved for r,

$$r = d\left(\sec\frac{4\theta}{3}\right)^{3/4} \tag{2}$$

In fact, the line given by Equation 1 is the asymptote of Equation 2 as the radial coordinate r becomes very large. The ideal Pierce electron gun therefore has a thin sliver-like gap between the electrodes that is infinitely long, but the electrodes will never touch. In this example, the electrodes are simply truncated at a sufficient distance from the beam. The line cutting off this gap is drawn perpendicular to the lower boundary because this is approximately the direction in which the electric field will point there, so the disruption from imposing the default **Zero Charge** boundary condition along such an edge will be minimized.