Created in COMSOL Multiphysics 5.6

Child's Law Benchmark

Introduction

Space charge limited emission is a phenomenon that restricts the number of electrons or other charged particles that can be released from a surface. As the current of electrons released by a cathode increases, so does the charge density in the immediate vicinity of the cathode. This distribution of charge density exerts an electric force on the emitted electrons, directed toward the cathode. The space charge limited current is the maximum current that can be released such that the emitted particles are not repelled back toward the cathode.

In this example, the space charge limited current in a plane-parallel vacuum diode is computed using the dedicated **Space Charge Limited Emission** particle release feature. A bidirectional coupling is set up in which the electrons produce a distribution of space charge density that contributes to the electric potential being solved for. The resulting electric potential distribution and current are compared with the analytical solution given by Child's law.

The Application Library example *Thermionic Emission in a Planar Diode* expands upon this example by employing the more detailed Langmuir-Fry model, in which the initial thermal distribution of electron velocity at the cathode is also taken into account.

Model Definition

The model geometry is two-dimensional but can be thought of as quasi-1D because the current and electric potential are uniform in the y-direction. Electrons are emitted from a grounded cathode located at position $x = 0$ and propagate toward an extraction grid at a fixed potential V_0 (SI unit: V) located at $x = d$, where $d > 0$. Space charge limited emission occurs when any further increase in current would repel electrons back toward the cathode; this is equivalent to enforcing the boundary condition

$$\mathbf{n} \cdot \nabla V = 0$$

on the cathode surface, in addition to the usual boundary condition

$$V = 0$$

for a grounded surface.

ENSURING NUMERICAL CONVERGENCE

The space charge limited current is computed using an iterative process in which the particle trajectories and electric potential are computed in alternating steps. The values of variables not solved for in one step are always taken from the most recent iteration of the other. In this way, successively better approximations of the current can be computed. However, if the space charge limited current is significantly overestimated during any iteration, then all of the electrons would immediately be repelled back toward the cathode and the charge density would become saturated in the mesh elements adjacent to the cathode, causing the model to fail. To avoid this overestimation of the space charge limited current, only a small fraction of the space charge density is used to calculate the electric potential during early iterations. This fraction of the space charge density is gradually increased until it reaches unity in the **Number of iterations** specified in the Settings window for the **Electric Particle Field Interaction** node.

To further improve the stability and robustness of the model, the electrons are not released precisely at the cathode, but rather a short distance away from it where the gradient of the electric potential is slightly greater than zero. To do so, the **Space Charge Limited Emission** feature is applied to the boundary at $x = 0$. Instead of representing the cathode, the left side of the geometry represents an emission surface a short distance d_{buf} away from the cathode. The electric potential is computed in the narrow region between the cathode and the emission surface, allowing the initial particle velocity at the emission surface to be defined. The electric potential to the right of the emission surface can then be computed using the space charge density computed by the **Electric Particle Field Interaction** node.

The **Bidirectionally Coupled Particle Tracing** study step sets up a solver loop in which the particle trajectories are computed using a time-dependent solver, while the electric potential is computed using a stationary solver. The solver sequence alternates between these two solvers for a specified number of iterations, always using the previous solution for the variables not solved for. In this way, a self-consistent solution, in which the particle trajectories and the electric potential affect each other, is obtained.

Results and Discussion

For space charge limited emission (Ref. 1), the electric potential varies with position as

$$V(x) = V_0\left(\frac{x + d_{\mathrm{buf}}}{d + d_{\mathrm{buf}}}\right)^{4/3} \tag{1}$$

where d is the distance from the emission surface to the anode and x is the distance from the emission surface to a given point. The space charge limited current is given by Child's law,

$$J = \frac{4\varepsilon_0}{9}\sqrt{\frac{2e}{m_{\mathrm{e}}}}\frac{V^{3/2}}{d^2} \tag{2}$$

Where

- J (SI unit: A/m^2) is the norm of the current density,
- $\varepsilon_0 = 8.854187817 \times 10^{-12}$ F/m is the vacuum permittivity,
- $e = 1.602176634 \times 10^{-19}$ C is the elementary charge,
- $m_e = 9.10938291 \times 10^{-31}$ kg is the electron mass,
- V (SI unit: V) is the potential at the anode, and
- d (SI unit: m) is the distance between the cathode and the anode.

The comparison between the analytical potential distribution given by Equation 1 and the computed result is shown in Figure 1. The relative error is shown in Figure 2.

The model also includes **Global Evaluation** nodes to compare the total emitted current to the analytical solution given by Equation 2. The relative error in the space charge limited current is about 2%.

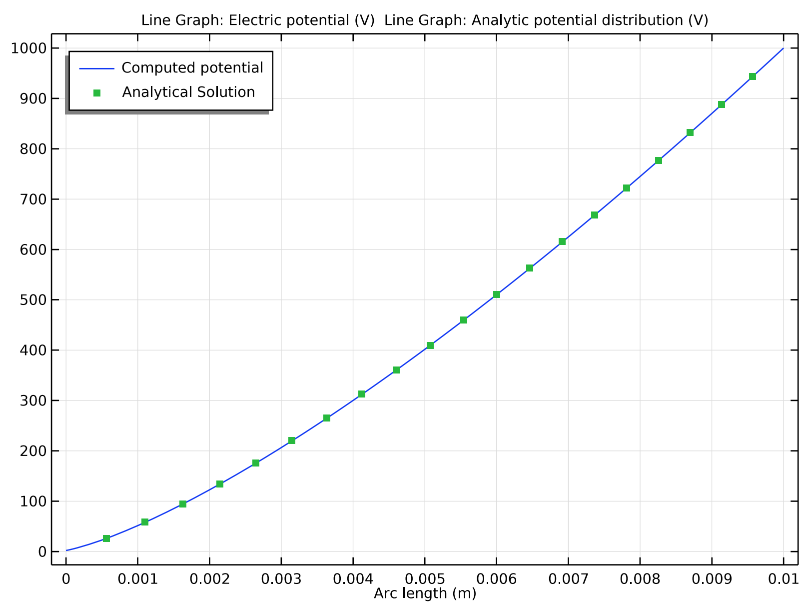

Figure 1: Comparison of the computed electric potential and the analytic solution given by Child's law.

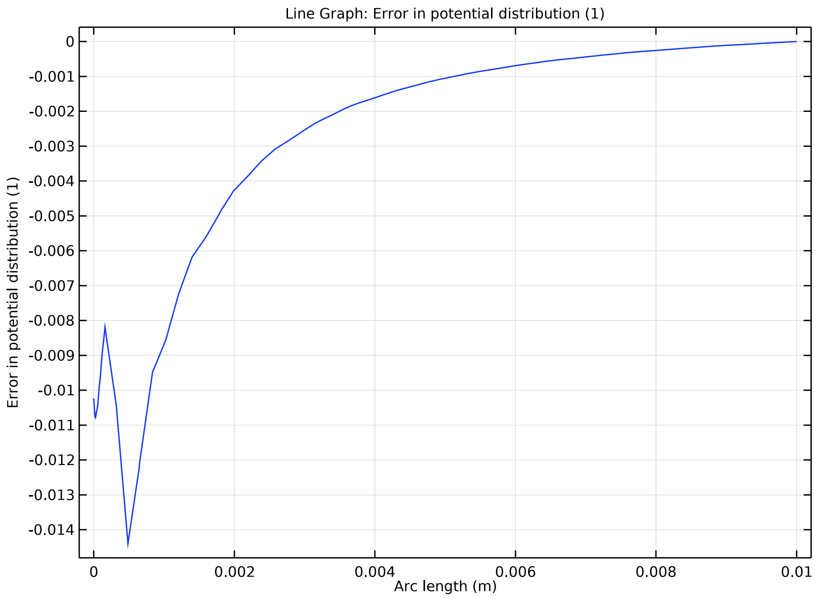

Figure 2: Relative error between the computed electric potential and the analytic solution given by Child's law.

Reference

1. S. Humphries, *Charged Particle Beams*, Dover Publications, New York, 2013.

Application Library path: `Particle_Tracing_Module/Charged_Particle_Tracing/childs_law_benchmark`

Modeling Instructions

From the **File** menu, choose **New**.

NEW

In the **New** window, click **Model Wizard**.

MODEL WIZARD

1 In the **Model Wizard** window, click **2D**.
2 In the **Select Physics** tree, select **AC/DC>Particle Tracing>Particle Field Interaction, Non-Relativistic**.
3 Click **Add**.
4 Click **Study**.
5 In the **Select Study** tree, select **Preset Studies for Selected Physics Interfaces> Charged Particle Tracing>Bidirectionally Coupled Particle Tracing**.
6 Click **Done**.

GLOBAL DEFINITIONS

Parameters 1

1 In the **Model Builder** window, under **Global Definitions** click **Parameters 1**.
2 In the **Settings** window for **Parameters**, locate the **Parameters** section.
3 In the table, enter the following settings:

Name	Expression	Value	Description
V0	1000[V]	1000 V	Potential difference across gap
d	1[cm]	0.01 m	Length of the modeling domain
dbuf	0.01[cm]	1E-4 m	Length of the buffer region
H	2[cm]	0.02 m	Height of the modeling domain
Depth	1[m]	1 m	Depth into the modeling domain
Jan	(4*epsilon0_const/9)*sqrt(2*V0*e_const/me_const)*V0/d^2	738.06 A/m²	Analytic current density
Ian	Jan*H*Depth	14.761 A	Analytic total current

GEOMETRY 1

Rectangle 1 (r1)

1 In the **Geometry** toolbar, click **Rectangle**.
2 In the **Settings** window for **Rectangle**, locate the **Size and Shape** section.
3 In the **Width** text field, type d.

4 In the **Height** text field, type H.

DEFINITIONS

Variables 1

1 In the **Model Builder** window, under **Component 1 (comp1)** right-click **Definitions** and choose **Variables**.

2 In the **Settings** window for **Variables**, locate the **Variables** section.

3 In the table, enter the following settings:

Name	Expression	Unit	Description
Van	V0*((x+dbuf)/(d+dbuf))^(4/3)	V	Analytic potential distribution
rel_err	(V-Van)/Van		Error in potential distribution

MATERIALS

Material 1 (mat1)

1 In the **Model Builder** window, under **Component 1 (comp1)** right-click **Materials** and choose **Blank Material**.

2 In the **Settings** window for **Material**, locate the **Material Contents** section.

3 In the table, enter the following settings:

Property	Variable	Value	Unit	Property group
Relative permittivity	epsilonr_iso ; epsilonrii = epsilonr_iso, epsilonrij = 0	1	1	Basic

CHARGED PARTICLE TRACING (CPT)

Particle Properties 1

1 In the **Model Builder** window, under **Component 1 (comp1)>Charged Particle Tracing (cpt)** click **Particle Properties 1**.

2 In the **Settings** window for **Particle Properties**, locate the **Particle Species** section.

3 From the **Particle species** list, choose **Electron**.

Electric Force 1

1 In the **Model Builder** window, click **Electric Force 1**.

2 In the **Settings** window for **Electric Force**, locate the **Electric Force** section.

3 From the $\mathbf{E}$ list, choose **Electric field (es/ccn1)**.

ELECTROSTATICS (ES)

In the **Model Builder** window, under **Component 1 (comp1)** click **Electrostatics (es)**.

Electric Potential 1

1 In the **Physics** toolbar, click **Boundaries** and choose **Electric Potential**.

2 Select Boundary 4 only.

3 In the **Settings** window for **Electric Potential**, locate the **Electric Potential** section.

4 In the V_0 text field, type `V0`.

MULTIPHYSICS

Set up space charge limited emission of electrons at the left boundary.

Space Charge Limited Emission 1 (scle1)

1 In the **Physics** toolbar, click **Multiphysics Couplings** and choose **Boundary>Space Charge Limited Emission**.

2 Select Boundary 1 only.

3 In the **Settings** window for **Space Charge Limited Emission**, locate the **Space Charge Limited Emission** section.

4 In the o_s text field, type `dbuf`.

The iterative scheme employed when solving can be made more stable by ramping up the computed space charge density over a number of iterations. The rate at which the space charge density is ramped up is controlled by the **Number of iterations** check box in the settings window for the **Electric Particle Field Interaction** node. Further stability can be achieved by selecting the **Use cumulative space charge density** check box, which treats the charge density as the average over successive iterations. The number of iterations should typically be around 15, depending on the size of the buffer zone specified in the **Space Charge Limited Emission** feature. The smaller the buffer zone, the larger the number of iterations should be. In addition, the number of iterations over which to ramp up to full space charge density should be 10-15 less than the number specified in the **Bidirectionally Coupled Particle Tracing** study settings.

Electric Particle Field Interaction 1 (epfi1)

1 In the **Model Builder** window, click **Electric Particle Field Interaction 1 (epfi1)**.

2 In the **Settings** window for **Electric Particle Field Interaction**, locate the **Continuation Settings** section.

3 Select the **Use cumulative space charge density** check box.

4 In the β text field, type 15.

MESH 1

Use a boundary layer mesh immediately in front of the emission surface for higher accuracy.

Free Triangular 1

In the **Mesh** toolbar, click **Free Triangular**.

Size

1 In the **Model Builder** window, click **Size**.

2 In the **Settings** window for **Size**, locate the **Element Size** section.

3 From the **Predefined** list, choose **Finer**.

Boundary Layers 1

1 In the **Mesh** toolbar, click **Boundary Layers**.

2 In the **Settings** window for **Boundary Layers**, click to expand the **Transition** section.

3 Clear the **Smooth transition to interior mesh** check box.

Boundary Layer Properties

1 In the **Model Builder** window, click **Boundary Layer Properties**.

2 Select Boundary 1 only.

3 In the **Settings** window for **Boundary Layer Properties**, locate the **Boundary Layer Properties** section.

4 In the **Number of boundary layers** text field, type 4.

5 Click **Build All**.

STUDY 1

Step 1: Bidirectionally Coupled Particle Tracing

1 In the **Model Builder** window, under **Study 1** click **Step 1: Bidirectionally Coupled Particle Tracing**.

2 In the **Settings** window for **Bidirectionally Coupled Particle Tracing**, locate the **Study Settings** section.

3 From the **Time unit** list, choose **ns**.

4 In the **Output times** text field, type `range(0,0.1,3)`.

5 Locate the **Iterations** section. From the **Termination method** list, choose **Convergence of global variable**.

6 In the **Global variable** text field, type `scle1.rc`.

7 In the **Maximum number of iterations** text field, type `35`.

8 In the **Home** toolbar, click **Compute**.

RESULTS

Use the **Cut Line 2D** dataset to visualize the relative error in the electric potential versus the prediction of Child's Law.

Cut Line 2D 1

1 In the **Model Builder** window, expand the **Results>Datasets** node.

2 Right-click **Results>Datasets** and choose **Cut Line 2D**.

3 In the **Settings** window for **Cut Line 2D**, locate the **Line Data** section.

4 In row **Point 1**, set **y** to `1[cm]`.

5 In row **Point 2**, set **x** to `1[cm]` and **y** to `1[cm]`.

Comparison With Child's Law

1 In the **Results** toolbar, click **1D Plot Group**.

2 In the **Settings** window for **1D Plot Group**, type `Comparison With Child's Law` in the **Label** text field.

3 Locate the **Data** section. From the **Dataset** list, choose **Cut Line 2D 1**.

4 From the **Time selection** list, choose **Last**.

5 Locate the **Legend** section. From the **Position** list, choose **Upper left**.

Line Graph 1

1 Right-click **Comparison With Child's Law** and choose **Line Graph**.

2 In the **Settings** window for **Line Graph**, click to expand the **Legends** section.

3 Select the **Show legends** check box.

4 From the **Legends** list, choose **Manual**.

5 In the table, enter the following settings:

Legends
`Computed potential`

Line Graph 2

1 In the **Model Builder** window, right-click **Comparison With Child's Law** and choose **Line Graph**.

2 In the **Settings** window for **Line Graph**, locate the **y-Axis Data** section.

3 In the **Expression** text field, type `Van`.

4 Click to expand the **Coloring and Style** section. Find the **Line style** subsection. From the **Line** list, choose **None**.

5 Find the **Line markers** subsection. From the **Marker** list, choose **Point**.

6 In the **Number** text field, type `20`.

7 Locate the **Legends** section. Select the **Show legends** check box.

8 From the **Legends** list, choose **Manual**.

9 In the table, enter the following settings:

Legends
`Analytical Solution`

10 In the **Comparison With Child's Law** toolbar, click **Plot**. The plot should look like Figure 1.

Relative Error

1 In the **Home** toolbar, click **Add Plot Group** and choose **1D Plot Group**.

2 In the **Settings** window for **1D Plot Group**, type `Relative Error` in the **Label** text field.

3 Locate the **Data** section. From the **Dataset** list, choose **Cut Line 2D 1**.

4 From the **Time selection** list, choose **Last**.

Line Graph 1

1 Right-click **Relative Error** and choose **Line Graph**.

2 In the **Settings** window for **Line Graph**, locate the **y-Axis Data** section.

3 In the **Expression** text field, type `rel_err`.

4 In the **Relative Error** toolbar, click **Plot**. The plot should look like Figure 2.

Finally, check that the space charge limited current matches the analytical solution.

Global Evaluation 1

1 In the **Results** toolbar, click **Global Evaluation**.

2 In the **Settings** window for **Global Evaluation**, locate the **Data** section.

3 From the **Time selection** list, choose **Last**.

4 Click **Replace Expression** in the upper-right corner of the **Expressions** section. From the menu, choose **Global definitions>Parameters>Ian - Analytic total current - A**.

5 Click **Evaluate**.

Global Evaluation 2

1 In the **Results** toolbar, click **Global Evaluation**.

2 In the **Settings** window for **Global Evaluation**, locate the **Data** section.

3 From the **Time selection** list, choose **Last**.

4 Click **Replace Expression** in the upper-right corner of the **Expressions** section. From the menu, choose **Component 1 (comp1)>Currents and charge>scle1.rc - Release current magnitude - A**.

5 Click the arrow next to the **Evaluate** button and click **Table 1 - Global Evaluation 1 (Ian)**.