

Differential Gear Mechanism

This model simulates the mechanism of a differential gear used in cars and other wheeled vehicles. The model is built using the gears functionality in the Multibody Dynamics Module in the COMSOL Multiphysics.

A differential allows the outer drive wheel to rotate faster than the inner drive wheel during a turn. This is necessary when a vehicle turns in order to allow the wheel that is traveling along the outside of the turning curve to roll faster and to cover greater distance than the wheel on the inside of the turning curve. The average of the rotational speed of the two driving wheels is simply the input rotational speed of the drive shaft. An increase in the speed of one wheel is balanced by a decrease in the speed of the other.

Model Definition

The geometry of a differential is shown in Figure 1.

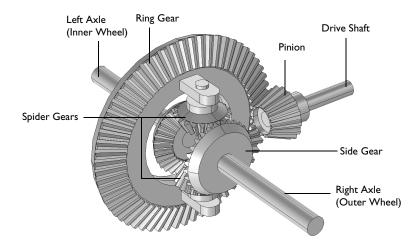


Figure 1: Geometry of a differential mechanism.

GEOMETRY AND CONNECTIONS

A differential has the following components:

- A drive shaft with pinion
- · A ring gear

- Two spider gears
- Left and right axles with side gears

A drive shaft is connected to the ring gear through a pinion. Two spider gears are mounted on the ring gear in such a way that they are free to rotate about their own axis. These spider gears are connected to side gears that are mounted on the inner end of the two axles.

The wheels, which are not modeled, are supposed to be mounted on the outer ends of the two axles. In this model, it is assumed that the wheel mounted on the right axle is on the outer side while vehicle takes a turn.

WORKING OF A DIFFERENTIAL

A differential performs the following tasks:

- Allows the left and the right axle to rotate at different speeds.
- Transfers power at 90°.
- Reduces speed of wheels compared to the drive shaft.

Under normal conditions, with small tire slip, the ratio of the speeds of the two driving wheels is determined by the track-width of the vehicle (the distance between the driving wheels) and the radius of the turn.

Straight Path

When a vehicle moves on a straight path, both wheels of the vehicle roll with the same speed. In this situation, both axles rotate with the same speed while the spider gears follow a planetary motion and thus do not spin about their own axes.

Curved Path

In case of a curved path, the outer wheel has to cover a longer distance. It must therefore roll faster than the inner wheel in order to avoid any slip. If the differential was absent in the vehicle, both wheels would have to rotate with the same speed causing slip between the wheels and the ground.

A differential allows the two wheels of the vehicle to rotate with different speeds. In this situation the spider gears start spinning together with their planetary motion. Spinning of the spider gears creates a difference in the speed of both wheels.

MODEL PARAMETERS

- The gear ratio between the pinion and the ring gear is 4.
- The gear ratio between the spider gear and the side gear is 1.5.

- To avoid any slip on a curved path, speed of the outer wheel is supposed to be 6 times higher than that of the inner wheel.
- The vehicle enters a curved path after one revolution of the drive shaft and returns back to a straight path after three revolutions of the drive shaft.

CONSTRAINTS AND LOADS

- The drive shaft is rotating about the x-axis with an angular velocity of 200 rad/s.
- The ring gear is hinged about the y-axis.
- The side gears (or axles) are hinged about the *y*-axis.

An external torque is applied on both axles while vehicle is taking a turn. This torque, which represents a friction between the wheels and the ground, forces the two wheels to rotate in the desired velocity ratio.

This external torque (T) is to be defined with

$$\begin{split} T &= c_f(\omega_o - \omega_r \omega_i) \\ \omega_r &= 1 + 5 \cdot \text{rect1}(\omega t) \end{split} \tag{1}$$

where

- c_f is the frictional damping coefficient,
- ω_0 and ω_i are the angular velocities of the outer and the inner wheels,
- ω_r is the velocity ratio between the outer and the inner wheels (set to 1 on a straight path and assumed to be 6 on a curved path), and
- ω is the angular velocity of the drive shaft.

Results and Discussion

Figure 2 shows a velocity magnitude of the different components in a differential at t = 0.125 sec. At this time instance the vehicle is moving along a straight path. This is also predicted by the model since both axles rotate with the same speed.

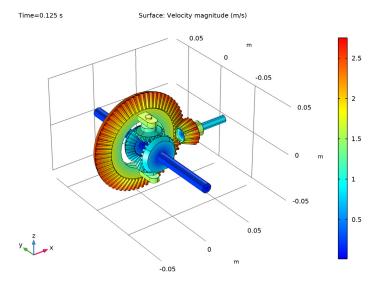


Figure 2: Speed of the different components in a differential after 0.125 s.

Figure 3 shows the velocity y-component of different differential parts at t = 0.08 sec. At this time instance, the vehicle is moving on a curved path. This is also predicted by the model since the spider gears have a nonzero velocity along the y-axis, which indicates that besides a planetary motion they are also spinning. The velocity y-component of the remaining parts is zero as they all rotate about the y-axis.

The spinning of the spider gears when the vehicle travels on a curved path can be visualized by plotting the relative velocity of the spider gears with respect to the ring gear as shown in Figure 4.

Figure 5 shows the angular velocity of the inner and outer wheels for the two cases when a vehicle moves on a straight path and on a curved path. It can be seen that as soon as the vehicle starts taking a turn, the outer wheel velocity increases and the inner wheel velocity decreases. It happens in such a way that the average is kept constant while the ratio between the two velocity components changes to 6, which is the desired ratio for no slip. Once the vehicle comes back on a straight path, both wheels start rotating with the same speed.

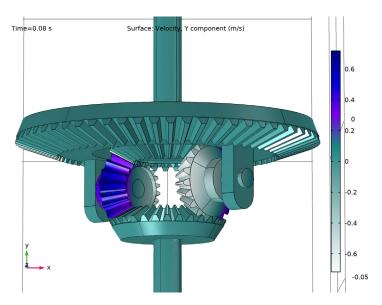


Figure 3: Velocity y-component in different parts of a differential after 0.08 s.

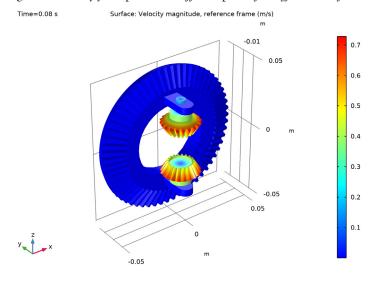


Figure 4: Relative velocity of spider gears with respect to ring gear at 0.08 s.

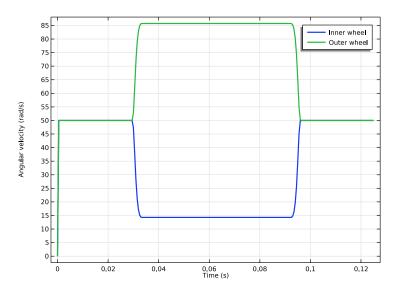


Figure 5: Time variation of the angular velocity of an inner and an outer wheel.

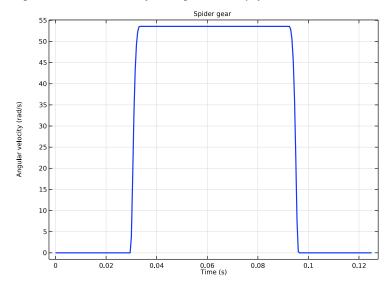


Figure 6: Time variation of the spinning velocity of a spider gears.

Figure 6 shows the spinning velocity of the spider gears. It can be seen that as soon as the vehicle starts taking a turn, the spider gears start spinning. Through this motion it transfers

speed from inner wheel to the outer wheel in such a way that both wheels roll without slip. The spider gears stops spinning, once the vehicle comes back on a straight path.

Notes About the COMSOL Implementation

- To build a gear geometry, you can import a gear part from the Parts Library and
 customize it by changing its input parameters. Alternatively, you can also create an
 equivalent disc or cone to represent the gear.
- All the gears are assumed rigid. The elasticity of a gear mesh can be included on Gear
 Pair nodes using the Gear Elasticity subnode.
- All the Gear Pair nodes are assumed ideal and frictionless. You can add Transmission Error, Backlash, or Friction subnodes when required.
- To constraint gear motion, you can use Prescribed Displacement/Rotation or Fixed
 Constraint subnodes. Alternatively, you can mount gears on a shaft or on the ground through various Joint nodes.
- The contact force on a Gear Pair is computed using Weak constraints or Penalty method.
 By default, the contact force computation is turned off. Use the weak constraints
 method for more accurate contact forces. However you can switch to the penalty
 method for large rigid body systems.

Application Library path: Multibody_Dynamics_Module/ Automotive and Aerospace/differential gear

Modeling Instructions

From the File menu, choose New.

NEW

In the New window, click Model Wizard.

MODEL WIZARD

- I In the Model Wizard window, click **3D**.
- 2 In the Select Physics tree, select Structural Mechanics>Multibody Dynamics (mbd).
- 3 Click Add.
- 4 Click Study.

- 5 In the Select Study tree, select General Studies>Time Dependent.
- 6 Click M Done.

GLOBAL DEFINITIONS

Parameters 1

- I In the Model Builder window, under Global Definitions click Parameters I.
- 2 In the Settings window for Parameters, locate the Parameters section.
- 3 Click Load from File.
- **4** Browse to the model's Application Libraries folder and double-click the file differential gear parameters.txt.

GEOMETRY I

Import I (impl)

- I In the Home toolbar, click Import.
- 2 In the Settings window for Import, locate the Import section.
- 3 Click Browse.
- **4** Browse to the model's Application Libraries folder and double-click the file differential gear.mphbin.
- 5 Click Import.

Form Union (fin)

- I In the Model Builder window, under Component I (compl)>Geometry I click Form Union (fin).
- 2 In the Settings window for Form Union/Assembly, locate the Form Union/Assembly section.
- 3 From the Action list, choose Form an assembly.
- 4 Clear the Create pairs check box.
- 5 In the Home toolbar, click **Build All**.

ADD MATERIAL

- I In the Home toolbar, click **Add Material** to open the Add Material window.
- 2 Go to the Add Material window.
- 3 In the tree, select Built-in>Structural steel.
- 4 Click Add to Component in the window toolbar.
- 5 In the Home toolbar, click Radd Material to close the Add Material window.

MULTIBODY DYNAMICS (MBD)

Add a **Bevel Gear** node and specify its properties.

Bevel Gear: Pinion

- I In the Model Builder window, under Component I (compl) right-click Multibody Dynamics (mbd) and choose Gears>Bevel Gear.
- 2 In the Settings window for Bevel Gear, type Bevel Gear: Pinion in the Label text field.
- **3** Select Domain 6 only.
- **4** Locate the **Gear Properties** section. In the n text field, type n_pn.
- **5** In the d_p text field, type dp_pn.
- **6** In the α text field, type alpha.
- 7 In the γ text field, type gamma_pn.
- $\boldsymbol{8}$ Locate the \boldsymbol{Gear} \boldsymbol{Axis} section. Specify the \boldsymbol{e}_g vector as

-1	x
0	у
0	z

- 9 Locate the Center of Rotation section. From the list, choose User defined.
- **IO** Specify the \mathbf{X}_c vector as

dp_rg/2	x
-dp_pn/2	у
0	z

Bevel Gears

Similarly add 5 more **Bevel Gears** using the information given in the following table:

Name	Selection (Domains)	Number of Teeth	Pitch Diameter	Pressure Angle	Cone Angle
Bevel Gear: Ring Gear	1	n_rg	dp_rg	alpha	gamma_rg
Bevel Gear: Spider Gear I	5	n_sp	dp_sp	alpha	gamma_sp
Bevel Gear: Spider Gear 2	4	n_sp	dp_sp	alpha	gamma_sp

Name	Selection (Domains)	Number of Teeth	Pitch Diameter	Pressure Angle	Cone Angle
Bevel Gear: Side Gear I	3	n_sd	dp_sd	alpha	gamma_sd
Bevel Gear: Side Gear 2	2	n_sd	dp_sd	alpha	gamma_sd

Bevel Gears

I Use the axis and center information given in the following table:

Name	Gear Axis	Center of Rotation
Bevel Gear: Ring Gear	(0,-1,0)	(0,0,0)
Bevel Gear: Spider Gear I	(0,0,-1)	(0,-d_sp,dp_sd/2)
Bevel Gear: Spider Gear 2	(0,0,1)	(0,-d_sp,-dp_sd/2)
Bevel Gear: Side Gear I	(0,-1,0)	(0,-d_sp+dp_sp/2,0)
Bevel Gear: Side Gear 2	(0,1,0)	(0,-d_sp-dp_sp/2,0)

2 In the Model Builder window, under Component I (compl)>Multibody Dynamics (mbd) click Bevel Gear: Pinion.

Prescribed Displacement/Rotation I

- I In the Physics toolbar, click 🦳 Attributes and choose Prescribed Displacement/Rotation.
- 2 In the Settings window for Prescribed Displacement/Rotation, locate the Prescribed Displacement at Center of Rotation section.
- 3 Select the Prescribed in x direction check box.
- 4 Select the Prescribed in y direction check box.
- **5** Select the **Prescribed in z direction** check box.
- 6 Locate the Prescribed Rotation section. From the By list, choose Prescribed rotation.
- **7** Specify the Ω vector as

- 1	x
0	у
0	z

8 In the ϕ_0 text field, type omega*t.

Bevel Gear: Ring Gear

In the Model Builder window, click Bevel Gear: Ring Gear.

Prescribed Displacement/Rotation I

- I In the Physics toolbar, click 🕞 Attributes and choose Prescribed Displacement/Rotation.
- 2 In the Settings window for Prescribed Displacement/Rotation, locate the Prescribed Displacement at Center of Rotation section.
- 3 Select the Prescribed in x direction check box.
- 4 Select the Prescribed in y direction check box.
- 5 Select the Prescribed in z direction check box.
- 6 Locate the Prescribed Rotation section. From the By list, choose Constrained rotation.
- 7 Select the Constrain rotation around x-axis check box.
- 8 Select the Constrain rotation around z-axis check box.

Gear Pair I

- I In the Physics toolbar, click A Global and choose Gear Pair.
- 2 In the Settings window for Gear Pair, locate the Gear Selection section.
- 3 From the Wheel list, choose Bevel Gear: Pinion.
- 4 From the Pinion list, choose Bevel Gear: Ring Gear.

Gear Pairs

Similarly add 4 more Gear Pairs using the information given in the following table:

Name	Wheel	Pinion
Gear Pair 2	Bevel Gear: Spider Gear I	Bevel Gear: Side Gear I
Gear Pair 3	Bevel Gear: Spider Gear I	Bevel Gear: Side Gear 2
Gear Pair 4	Bevel Gear: Spider Gear 2	Bevel Gear: Side Gear I
Gear Pair 5	Bevel Gear: Spider Gear 2	Bevel Gear: Side Gear 2

Hinge Joint I

- I In the Physics toolbar, click Global and choose Hinge Joint.
- 2 In the Settings window for Hinge Joint, locate the Attachment Selection section.
- 3 From the Source list, choose Bevel Gear: Ring Gear.
- 4 From the Destination list, choose Bevel Gear: Spider Gear 1.
- 5 Locate the Center of Joint section. From the list, choose Centroid of selected entities.
- 6 From the Entity level list, choose Point.

Center of Joint: Point I

- I In the Model Builder window, expand the Hinge Joint I node, then click Center of Joint: Point 1.
- 2 In the Settings window for Center of Joint: Point, locate the Point Selection section.
- 3 Click Paste Selection.
- 4 In the Paste Selection dialog box, type 1528, 1531 in the Selection text field.
- 5 Click OK.

Hinge Joint 1

- I In the Model Builder window, click Hinge Joint I.
- 2 In the Settings window for Hinge Joint, locate the Axis of Joint section.
- **3** Specify the \mathbf{e}_0 vector as

0	x
0	у
1	z

Hinge Joints

Similarly add 3 more **Hinge Joints** using the information given in the following table:

Name	Source	Destination	Center of Joint (Points)	Axis of Joint
Hinge Joint 2	Bevel Gear: Ring Gear	Bevel Gear: Spider Gear 2	1282, 1341	(0,0,1)
Hinge Joint 3	Fixed	Bevel Gear: Side Gear I	1060, 1061	(0,1,0)
Hinge Joint 4	Fixed	Bevel Gear: Side Gear 2	748, 749	(0,1,0)

Add a damping force on both side gears (or axles) to prevent the slip on a curved path.

Hinge Joint 3

In the Model Builder window, click Hinge Joint 3.

Applied Force and Moment 1

- I In the Physics toolbar, click 🕞 Attributes and choose Applied Force and Moment.
- 2 In the Settings window for Applied Force and Moment, locate the Applied On section.
- **3** From the list, choose **Joint**.
- 4 Locate the Applied Force and Moment section. In the M text field, type cf*omega_d.

Hinge Joint 4

In the Model Builder window, click Hinge Joint 4.

Applied Force and Moment 1

- I In the Physics toolbar, click 🧲 Attributes and choose Applied Force and Moment.
- 2 In the Settings window for Applied Force and Moment, locate the Applied On section.
- **3** From the list, choose **Joint**.
- 4 Locate the Applied Force and Moment section. In the M text field, type -cf*omega_d.

Define the wheel slip in terms of the difference in angular velocity between the outer and the inner wheel.

DEFINITIONS

Variables 1

- I In the Model Builder window, expand the Component I (compl)>Definitions node.
- 2 Right-click **Definitions** and choose **Variables**.
- 3 In the Settings window for Variables, locate the Variables section.
- **4** In the table, enter the following settings:

Name	Expression	Unit	Description
omega_i	mbd.hgj3.tht	rad/s	Angular velocity of inner wheel
omega_o	mbd.hgj4.tht	rad/s	Angular velocity of outer wheel
omega_r	1+5*rect1(omega*t)		Velocity ratio of outer and inner wheel
omega_d	omega_o-omega_r* omega_i		Velocity difference of outer and inner wheel

Rectangle I (rect1)

- I In the Home toolbar, click f(x) Functions and choose Global>Rectangle.
- 2 In the Settings window for Rectangle, locate the Parameters section.
- 3 In the Lower limit text field, type 2*pi.
- 4 In the Upper limit text field, type 6*pi.
- 5 Click to expand the Smoothing section. In the Size of transition zone text field, type pi/ 4.

MESH I

- I In the Model Builder window, under Component I (compl) click Mesh I.
- 2 In the Settings window for Mesh, locate the Physics-Controlled Mesh section.
- **3** From the **Element size** list, choose **Finer**.
- 4 Click Build All.

STUDY I

Step 1: Time Dependent

- I In the Model Builder window, under Study I click Step I: Time Dependent.
- 2 In the Settings window for Time Dependent, locate the Study Settings section.
- 3 In the Output times text field, type range (0,5e-4,0.125).
- 4 In the Home toolbar, click **Compute**.

RESULTS

Use the following instructions to plot the speed of the different gears in the differential as shown in Figure 2.

Velocity: Magnitude

- I Right-click Displacement (mbd) and choose Duplicate.
- 2 In the Settings window for 3D Plot Group, type Velocity: Magnitude in the Label text field.

Surface

- I In the Model Builder window, expand the Velocity: Magnitude node, then click Surface.
- 2 In the Settings window for Surface, locate the Expression section.
- 3 In the Expression text field, type mbd.vel.
- 4 In the Velocity: Magnitude toolbar, click Plot.
- **5** Click the **Zoom Extents** button in the **Graphics** toolbar.

Use the following instructions to plot the rotation of the spider gears about their own axis as shown in Figure 3.

Study 1/Solution 1 (2) (sol1)

- I In the Model Builder window, expand the Results>Datasets node.
- 2 Right-click Results>Datasets>Study I/Solution I (soll) and choose Duplicate.

Selection

I In the Results toolbar, click \(\frac{1}{2} \) Attributes and choose Selection.

- 2 In the Settings window for Selection, locate the Geometric Entity Selection section.
- 3 From the Geometric entity level list, choose Domain.
- **4** Select Domains 1–5 only.

Velocity: Y Component

- I In the Model Builder window, right-click Velocity: Magnitude and choose Duplicate.
- 2 In the Settings window for 3D Plot Group, type Velocity: Y Component in the Label text field.
- 3 Locate the Data section. From the Dataset list, choose Study I/Solution I (2) (soll).
- 4 From the Time (s) list, choose 0.08.
- **5** Locate the **Plot Settings** section. From the **View** list, choose **View 2**.

Surface

- I In the Model Builder window, expand the Velocity: Y Component node, then click Surface.
- 2 In the Settings window for Surface, locate the Expression section.
- **3** In the **Expression** text field, type mbd.u_tY.
- 4 Locate the Coloring and Style section. From the Color table list, choose AuroraAustralis.
- 5 In the Velocity: Y Component toolbar, click Plot.

In order to visualize the motion of the spider gears with respect to the ring gear, you can use the option of defining a reference frame available in the Multibody Dynamics interface and plot the postprocessing variables for displacement and velocity with respect to the reference frame.

MULTIBODY DYNAMICS (MBD)

- I In the Model Builder window, under Component I (compl) click Multibody Dynamics (mbd).
- 2 In the Settings window for Multibody Dynamics, click to expand the Results section.
- 3 From the Body defining reference frame list, choose Bevel Gear: Ring Gear.

You can update the solution to plot the postprocessing variables available for the new reference frame.

STUDY I

In the Study toolbar, click C Update Solution.

RESULTS

Study I/Solution I (3) (soll)

In the Model Builder window, under Results>Datasets right-click Study 1/ Solution I (2) (soll) and choose Duplicate.

Selection

- I In the Model Builder window, expand the Study I/Solution I (3) (soll) node, then click Selection.
- 2 In the Settings window for Selection, locate the Geometric Entity Selection section.
- 3 In the list, choose 2 and 3.
- 4 Click Remove from Selection.
- **5** Select Domains 1, 4, and 5 only.

Velocity: Ring Gear Reference

- I In the Model Builder window, right-click Displacement (mbd) and choose Duplicate.
- 2 In the Model Builder window, click Displacement (mbd) 1.
- 3 In the Settings window for 3D Plot Group, type Velocity: Ring Gear Reference in the Label text field.
- 4 Locate the Data section. From the Dataset list, choose Study I/Solution I (3) (soll).
- 5 From the Time (s) list, choose 0.08.
- **6** Locate the **Plot Settings** section. Clear the **Plot dataset edges** check box.

Surface

- I In the Model Builder window, click Surface.
- 2 In the Settings window for Surface, locate the Expression section.
- 3 In the Expression text field, type mbd.vel ref.

Deformation

- I In the Model Builder window, expand the Surface node, then click Deformation.
- 2 In the Settings window for Deformation, locate the Expression section.
- **3** In the **X** component text field, type u_ref.
- 4 In the Y component text field, type v ref.
- 5 In the **Z** component text field, type w ref.

Velocity: Ring Gear Reference

I In the Model Builder window, click Velocity: Ring Gear Reference.

- 2 In the Settings window for 3D Plot Group, locate the Plot Settings section.
- 3 From the View list, choose New view.
- 4 Click the Zoom Extents button in the Graphics toolbar.
- 5 In the Velocity: Ring Gear Reference toolbar, click Plot.

View 3D 3

- I In the Model Builder window, expand the Results>Views node, then click View 3D 3.
- 2 In the Settings window for View 3D, locate the View section.
- 3 Select the Lock camera check box.

Follow the instructions below to plot the angular speed of both the wheels and the spider gears shown in Figure 5 and Figure 6 respectively.

Angular Velocity: Wheels

- I In the Home toolbar, click **Add Plot Group** and choose ID Plot Group.
- 2 In the Settings window for ID Plot Group, type Angular Velocity: Wheels in the Label text field.

Global I

- I Right-click Angular Velocity: Wheels and choose Global.
- 2 In the Settings window for Global, click Replace Expression in the upper-right corner of the y-Axis Data section. From the menu, choose Component I (compl)> Multibody Dynamics>Hinge joints>Hinge Joint 3>mbd.hgj3.th_t -Relative angular velocity - rad/s.
- 3 Click Add Expression in the upper-right corner of the y-Axis Data section. From the menu, choose Component I (compl)>Multibody Dynamics>Hinge joints>Hinge joint 4> mbd.hgj4.th_t - Relative angular velocity - rad/s.
- 4 Click to expand the Coloring and Style section. In the Width text field, type 2.
- 5 Click to expand the Legends section. From the Legends list, choose Manual.
- **6** In the table, enter the following settings:

Legends Inner wheel Outer wheel

Angular Velocity: Wheels

- I In the Model Builder window, click Angular Velocity: Wheels.
- 2 In the Settings window for ID Plot Group, click to expand the Title section.

- 3 From the Title type list, choose None.
- 4 Locate the Plot Settings section. Select the y-axis label check box.
- 5 In the associated text field, type Angular velocity (rad/s).
- 7 Click the $| \leftarrow |$ Zoom Extents button in the Graphics toolbar.

Angular Velocity: Spider Gear

- I Right-click Angular Velocity: Wheels and choose Duplicate.
- 2 In the Settings window for ID Plot Group, type Angular Velocity: Spider Gear in the Label text field.
- 3 Locate the Title section. From the Title type list, choose Manual.
- 4 In the Title text area, type Spider gear.

Global I

- I In the Model Builder window, expand the Angular Velocity: Spider Gear node, then click Global I.
- 2 In the Settings window for Global, click Replace Expression in the upper-right corner of the y-Axis Data section. From the menu, choose Component I (compl)> Multibody Dynamics>Hinge joints>Hinge Joint I>mbd.hgjl.th_t -Relative angular velocity - rad/s.
- 3 Locate the Legends section. Clear the Show legends check box.
- 4 In the Angular Velocity: Spider Gear toolbar, click Plot.
- 5 Click the | Zoom Extents button in the Graphics toolbar.

Animation I

- I In the Results toolbar, click Animation and choose Player.
- 2 In the Settings window for Animation, locate the Scene section.
- 3 From the Subject list, choose Velocity: Magnitude.
- 4 Locate the Frames section. In the Number of frames text field, type 100.

Animation 2

- I Right-click Animation I and choose Duplicate.
- 2 In the Settings window for Animation, locate the Scene section.
- 3 From the Subject list, choose Velocity: Y Component.

Animation 3

I Right-click Animation 2 and choose Duplicate.

- 2 In the Settings window for Animation, locate the Scene section.
- 3 From the Subject list, choose Velocity: Ring Gear Reference.