

Two-Phase Flow Modeling of a Dense Suspension

Introduction

Liquid-solid mixtures (suspensions) are important in a variety of industrial fields, such as oil and gas refinement, paper manufacturing, food processing, slurry transport, and wastewater treatment. Several different modeling approaches have been developed, ranging from discrete, particle-based methods to macroscopic, semi-empirical two-phase descriptions. Particle-based methods are suitable when there is a limited number of solid particles. When, on the other hand, there are many particles, it is better to use a macroscopic, or averaged, model that tracks the volume fractions of the phases.

The following example illustrates how you can set up a macroscopic two-phase flow model in COMSOL Multiphysics using the Phase Transport Mixture Model, Laminar Flow multiphysics interface. The model is based on the "diffusive flux" model described in Ref. 1, Ref. 2, and Ref. 3, suitable for liquid-solid mixtures with high concentrations of solid particles. It accounts for not only buoyancy effects but also shear-induced migration; that is, the tendency of particles to migrate toward regions of lower shear rates.

The model simulates the flow of a dense suspension consisting of light, solid particles in a liquid placed between two concentric cylinders. The inner cylinder rotates while the outer one is fixed.

Model Definition

A suspension is a mixture of solid particles and a liquid. The dynamics of a suspension can be modeled by a momentum transport equation for the mixture, a continuity equation, and a transport equation for the solid phase volume fraction. The Phase Transport Mixture Model, Laminar Flow multiphysics interface automatically sets up these equations. It uses the following equation to model the momentum transport:

$$\rho \frac{\partial \mathbf{j}}{\partial t} + \rho(\mathbf{j} \cdot \nabla)\mathbf{j} = -\nabla p - \nabla \cdot \mu[\nabla \mathbf{j} + \nabla \mathbf{j}^{T}] + \rho \mathbf{g}$$
$$-\nabla \cdot [\rho_{s} \phi_{s} (1 - \rho_{s} \phi_{s}) \mathbf{u}_{slip} \mathbf{u}_{slip}^{T}]$$

where **j** is the mass-averaged mixture velocity (m/s), *p* denotes the pressure (Pa), **g** refers to the acceleration of gravity (m/s²), ε is the reduced density difference, and **u**_{slip} gives the relative velocity between the solid and the liquid phases (m/s). Further, $\rho = (1 - \phi_s)\rho_f + \phi_s\rho_s$ is the mixture density, where ρ_f and ρ_s are the pure-phase densities

 (kg/m^3) of liquid and solids, respectively, and where ϕ_s is the solid-phase volume fraction (m^3/m^3) .

Finally, μ represents the mixture viscosity (Ns/m^2) according to the Krieger-type expression

$$\mu = \mu_f \left(1 - \frac{\phi_s}{\phi_{max}} \right)^{-2.5\phi}$$
(1)

where μ_f is the dynamic viscosity of the pure fluid and ϕ_{max} is the maximum packing concentration.

The mixture model uses the following form of the continuity equation

$$\frac{\partial}{\partial t} \boldsymbol{\rho} + \nabla \cdot (\boldsymbol{\rho} \mathbf{j}) = \mathbf{0}$$
⁽²⁾

The transport equation for the solid-phase volume fraction is

$$\frac{\partial}{\partial t}(\rho_{\rm s}\phi_{\rm s}) + \nabla \cdot (\rho_{\rm s}\phi_{\rm s}\mathbf{u}_{\rm s}) = 0$$
(3)

The solid-phase velocity, \mathbf{u}_s , is given by $\mathbf{u}_s = \mathbf{j} + (1 - c_s) \mathbf{u}_{slip}$, where $c_s = \phi_s \rho_s / \rho$ is the dimensionless particle mass fraction and \mathbf{j} is the mass-averaged mixture velocity (m/s). Assuming ρ_s is constant, this means that Equation 3 is equivalent to

$$\frac{\partial \phi_{\rm s}}{\partial t} + \nabla \cdot (\phi_{\rm s} \mathbf{j} + \phi_{\rm s} (1 - c_s) \mathbf{u}_{\rm slip}) = 0$$
(4)

Rao and others (Ref. 2) formulate the continuity equation and the particle transport in a slightly different way. Instead of the slip velocity, \mathbf{u}_{slip} , they define a particle flux, \mathbf{J}_s (kg $/(\text{m}^2 \cdot \text{s})$), and write the solid phase transport according to

$$\frac{\partial \phi_{\rm s}}{\partial t} + \nabla \cdot (\phi_{\rm s} \mathbf{j}) = -\frac{\nabla \cdot \mathbf{J}_{\rm s}}{\rho_{\rm s}}$$
(5)

By comparing Equation 5 with Equation 4, it is clear that they are equivalent if

$$\mathbf{u}_{\text{slip}} = \frac{\mathbf{J}_{\text{s}}}{\phi_{\text{s}}\rho_{\text{s}}(1-c_{\text{s}})}$$

In this example you use the model for the particle flux, \mathbf{J}_{s} , as suggested by Subia and others (Ref. 3) and Rao and others (Ref. 2), but the open and editable format of COMSOL Multiphysics makes it possible to specify the expression arbitrarily.

Following Rao and others, the particle flux is

$$\frac{\mathbf{J}_{\rm s}}{\rho_{\rm s}} = - [\phi \mathbf{D}_{\phi} \nabla(\dot{\gamma} \phi) + \phi^2 \dot{\gamma} \mathbf{D}_{\eta} \nabla(\ln \mu)] + f_{\rm h} \mathbf{u}_{\rm st} \phi$$

Here, \bm{u}_{st} is the settling velocity (m/s) of a single particle surrounded by fluid and D_{ϕ} and D_{η} are empirically fitted parameters (m²) given by

$$D_{\phi} = 0.41a^2$$
$$D_{\eta} = 0.62a^2$$

where a is the particle radius (m).

The shear rate tensor, $\dot{\gamma}$ (1/s), is given by

$$\dot{\gamma} = \nabla \mathbf{j} + \nabla \mathbf{j}^{T}$$

and its magnitude by

$$\dot{\gamma} = \sqrt{\frac{1}{2}}(\dot{\gamma};\dot{\gamma})$$

which for a 2-dimensional problem is

$$\dot{\gamma} = \sqrt{\frac{1}{2}(4j_{x,x}^2 + 2(j_{x,y} + j_{y,x})^2 + 4j_{y,y}^2)}$$

The settling velocity, \mathbf{u}_{st} , for a single spherical particle surrounded by pure fluid is given by

$$\mathbf{u}_{\rm st} = \frac{2}{9} \frac{a^2(\rho_{\rm s} - \rho_{\rm f})}{\eta_0} \mathbf{g}$$

For several particles in a fluid, the settling velocity is lower. To account for the surrounding particles, the settling velocity for a single particle is multiplied by the hindering function, $f_{\rm h}$, defined as

$$f_{\rm h} = \frac{\mu_{\rm f}(1 - \phi_{\rm av})}{\mu}$$

where ϕ_{av} is the average solid phase volume fraction in the suspension, μ_f is the dynamic viscosity of the pure fluid (Ns/m²), and μ is the mixture viscosity (Equation 1).

NAME	VALUE	DESCRIPTION
$\rho_{\rm s}$	1180 kg/m ³	Density of particles
ρ_{f}	1250 kg/m ³	Density of pure fluid
a	678 μm	Particle radius
$\mu_{\mathbf{f}}$	0.589 Pa·s	Viscosity of pure fluid

The following table gives the physical properties of the solid and the liquid phases:

BOUNDARY CONDITIONS

The suspension is placed in a Couette device, that is, between two concentric cylinders. The inner cylinder rotates while the outer one is fixed. The radii of the two cylinders are 0.64 cm and 2.54 cm, respectively. The inner cylinder rotates at a steady rate of 55 rpm. With the cylinder centered at (0,0), this corresponds to a velocity of

$$(j_x, j_y) = \frac{110\pi}{60}(y, -x)$$

The fluid and particle motion is small along the direction of the cylinder axis. You can therefore use a 2-dimensional model. Figure 1 shows the corresponding geometry.



Figure 1: Geometry of the Couette device. The inner cylinder rotates, the outer one is fixed.

There is no particle flux through the boundaries, and the suspension velocity satisfies no slip conditions at all walls.

INITIAL CONDITIONS

There are two different initial particle distributions. In the first example, the particles are evenly distributed within the device. In the second example, the particles are initially gathered at the top of the device.

Results

CASE I — INITIALLY EVENLY DISTRIBUTED PARTICLES

A suspension with particles lighter than the fluid is placed in a concentric Couette device. Initially, the particles are evenly distributed with a constant volume fraction of 0.35. The shear rate in the device varies radially across the gap and thus it is expected that particles migrate (shear-induced migration) from regions of high shear to regions of low shear (toward the outer wall). Because the particles are lighter than the fluid, they also rise.



Figure 2: The particle concentration ϕ_s at different times. The particles move to regions with lower shear rate and rise because of buoyancy.

Figure 2 shows the particle concentration ϕ_s in the device at t = 0 s, t = 30 s, t = 100 s and t = 1000 s. The migration of the particles toward the outer wall is apparent. As a result of the shear induced migration and gravity, the solid phase volume fraction approaches the value for maximum packing close to the upper right outer wall. The suspension viscosity thus becomes high in this region. The results compare well with those presented in Ref. 2.

CASE 2 — **PARTICLES INITIALLY GATHERED AT THE TOP OF THE DEVICE** In this case the particles are initially gathered at the top of the device. The particle volume fraction is initially zero in the lower part, while it is 0.59 at the top.



Figure 3: Particle concentrations for t = 0 s, 10 s, 20 s, and 100 s with particles initially at the top. Note that the same color range values are used in each of the plots.

Figure 3 shows the numerically predicted particle concentration at times 0 s, 10 s, 20 s, and 100 s. Initially, the particle motion is dominated by inertia and the effect of the shear-induced migration is not visible. At later times, shear-induced migration causes the particles to move toward the outer boundary. In this case also, the results agree well with the results in Ref. 2.

References

1. R.J. Phillips, R.C. Armstrong, R.A. Brown, A.L. Graham, and J.R. Abbot, "A Constitutive Equation for Concentrated Suspensions that Accounts for Shear-induced Particle Migration," *Phys. Fluids A*, vol. 4, pp. 30–40, 1992.

2. R. Rao, L. Mondy, A. Sun, and S. Altobelli, "A Numerical and Experimental Study of Batch Sedimentation and Viscous Resuspension," *Int. J. Num. Methods in Fluids*, vol. 39, pp. 465–483, 2002.

3. S.R. Subia, M.S. Ingber, L.A. Mondy, S.A. Altobelli, and A.L. Graham, "Modelling of Concentrated Suspensions Using a Continuum Constitutive Equation," *J. Fluid Mech.*, vol. 373, pp. 193–219, 1998.

Notes About the COMSOL Implementation

To set up the model with COMSOL Multiphysics, open the Mixture Model, Laminar Flow interface. The shear rate is discretized as an additional equation to improve accuracy because the particle flux contains derivatives of this quantity, which in turn depend on the derivatives of the velocity.

Application Library path: CFD_Module/Verification_Examples/

dense_suspension

Modeling Instructions

From the File menu, choose New.

NEW

In the New window, click 🔗 Model Wizard.

MODEL WIZARD

- I In the Model Wizard window, click 🤬 2D.
- 2 In the Select Physics tree, select Fluid Flow>Multiphase Flow> Phase Transport Mixture Model>Laminar Flow.
- 3 Click Add.
- 4 In the Added physics interfaces tree, select Phase Transport (phtr).
- 5 In the Volume fractions table, enter the following settings:

phic phid

6 In the Select Physics tree, select Mathematics>PDE Interfaces>General Form PDE (g).

- 7 Click Add.
- 8 In the **Dependent variables** table, enter the following settings:

gamma

9 Click i Define Dependent Variable Unit.

IO In the **Dependent variable quantity** table, enter the following settings:

Dependent variable quantity	Unit	
Custom unit	1/s	

II Click 🛋 Define Source Term Unit.

12 In the Source term quantity table, enter the following settings:

Source term quantity	Unit	
Custom unit	1/s	

I3 Click \bigcirc Study.

I4 In the Select Study tree, select General Studies>Time Dependent.

15 Click 🗹 Done.

GLOBAL DEFINITIONS

Parameters 1

- I In the Model Builder window, under Global Definitions click Parameters I.
- 2 In the Settings window for Parameters, locate the Parameters section.
- **3** Click **b** Load from File.
- 4 Browse to the model's Application Libraries folder and double-click the file dense_suspension_parameters.txt.

GEOMETRY I

Circle 1 (c1)

- I In the **Geometry** toolbar, click \bigcirc **Circle**.
- 2 In the Settings window for Circle, locate the Size and Shape section.
- 3 In the Radius text field, type 0.0064.
- 4 Click 틤 Build Selected.

Circle 2 (c2)

- I In the **Geometry** toolbar, click Circle.
- 2 In the Settings window for Circle, locate the Size and Shape section.
- 3 In the Radius text field, type 0.0254.
- 4 Click 틤 Build Selected.

Difference I (dif1)

- I In the Geometry toolbar, click Pooleans and Partitions and choose Difference.
- 2 Select the object c2 only.
- 3 In the Settings window for Difference, locate the Difference section.
- **4** Find the **Objects to subtract** subsection. Select the **Delta Activate Selection** toggle button.
- **5** Select the object **cl** only.
- 6 Click 🔚 Build Selected.

DEFINITIONS

Variables I

- I In the Home toolbar, click a= Variables and choose Local Variables.
- 2 In the Settings window for Variables, locate the Variables section.
- 3 Click 📂 Load from File.
- 4 Browse to the model's Application Libraries folder and double-click the file dense_suspension_variables.txt.

LAMINAR FLOW (SPF)

- I In the Model Builder window, under Component I (compl) click Laminar Flow (spf).
- 2 In the Settings window for Laminar Flow, locate the Physical Model section.
- **3** Select the **Include gravity** check box.

Pressure Point Constraint I

- I In the Physics toolbar, click 💭 Points and choose Pressure Point Constraint.
- 2 Select Point 1 only.

Wall 2

- I In the Physics toolbar, click Boundaries and choose Wall.
- 2 Select Boundaries 3, 4, 6, and 7 only.
- 3 In the Settings window for Wall, click to expand the Wall Movement section.

- **4** From the **Translational velocity** list, choose **Manual**.
- **5** Specify the **u**_{tr} vector as

c_vel*y	x
-c vel*x	у

PHASE TRANSPORT (PHTR)

Initial Values 1

- I In the Model Builder window, under Component I (compl)>Phase Transport (phtr) click Initial Values I.
- 2 In the Settings window for Initial Values, locate the Initial Values section.
- **3** In the $s_{0,\text{phid}}$ text field, type phi0.

GENERAL FORM PDE (G)

- I In the Model Builder window, under Component I (compl) click General Form PDE (g).
- 2 In the Settings window for General Form PDE, click to expand the Discretization section.
- **3** From the **Element order** list, choose **Linear**.

General Form PDE 1

- I In the Model Builder window, under Component I (comp1)>General Form PDE (g) click General Form PDE I.
- 2 In the Settings window for General Form PDE, locate the Conservative Flux section.
- **3** Specify the Γ vector as

0	x
0	у

- 4 Locate the Source Term section. In the f text field, type gamma-sqrt(0.5*(4*ux^2+2* (uy+vx)^2+4*vy^2)+eps).
- **5** Locate the **Damping or Mass Coefficient** section. In the d_a text field, type **0**.

Flux/Source 1

- I In the Physics toolbar, click Boundaries and choose Flux/Source.
- 2 In the Settings window for Flux/Source, locate the Boundary Selection section.
- 3 From the Selection list, choose All boundaries.

MULTIPHYSICS

Mixture Model I (mfmm1)

- I In the Model Builder window, under Component I (compl)>Multiphysics click Mixture Model I (mfmml).
- 2 In the Settings window for Mixture Model, locate the Physical Model section.
- **3** From the **Slip model** list, choose **User defined**. From the **Mixture viscosity model** list, choose **Krieger type**.
- **4** In the ϕ_{max} text field, type phi_max.
- 5 Locate the Continuous Phase Properties section. From the ρ_c list, choose User defined. In the associated text field, type rho_f.
- 6 From the μ_c list, choose User defined. In the associated text field, type eta_f.
- 7 Locate the Dipsersed Phase 2 Properties section. From the ρ_{phid} list, choose User defined. In the associated text field, type rho_s.
- **8** Specify the **u**_{slip,phid} vector as

J1/(phid*rho_s*(1-cd)) x J2/(phid*rho_s*(1-cd)) y

MESH I

Free Triangular 1

In the Mesh toolbar, click 🦳 Free Triangular.

Size

- I In the Model Builder window, click Size.
- 2 In the Settings window for Size, locate the Element Size section.
- 3 From the Calibrate for list, choose Fluid dynamics.
- 4 From the Predefined list, choose Finer.
- **5** Click the **Custom** button.
- **6** Locate the **Element Size Parameters** section. In the **Maximum element size** text field, type 0.0012.
- 7 Click 📗 Build All.

STUDY I

Step 1: Time Dependent

- I In the Model Builder window, under Study I click Step I: Time Dependent.
- 2 In the Settings window for Time Dependent, locate the Study Settings section.
- 3 In the **Output times** text field, type 0 30 100 1000.

Solution 1 (soll)

- I In the Study toolbar, click The Show Default Solver.
- 2 In the Model Builder window, expand the Solution I (soll) node.
- 3 In the Model Builder window, click Time-Dependent Solver I.
- **4** In the **Settings** window for **Time-Dependent Solver**, click to expand the **Time Stepping** section.
- **5** In the **Study** toolbar, click **= Compute**.

RESULTS

Velocity (spf)

To visualize the volume fraction of the **Dispersed Phase** as a surface plot along with the mixture velocity field as an arrow surface plot, follow the steps given below.

Surface

- I In the Model Builder window, expand the Volume Fraction (phtr) node, then click Surface.
- 2 In the Settings window for Surface, locate the Expression section.
- 3 In the Expression text field, type phid.
- 4 Click to expand the Range section. Select the Manual color range check box.

Fix the color range so that the solutions for different time values use the same color legend.

- **5** In the **Minimum** text field, type **0.21**.
- 6 In the Maximum text field, type 0.45.

Arrow Surface 1

- I In the Model Builder window, right-click Volume Fraction (phtr) and choose Arrow Surface.
- 2 In the Settings window for Arrow Surface, locate the Coloring and Style section.
- 3 From the Color list, choose White.
- 4 In the Volume Fraction (phtr) toolbar, click 💽 Plot.

Volume Fraction (phtr)

- I In the Model Builder window, click Volume Fraction (phtr).
- 2 In the Settings window for 2D Plot Group, locate the Data section.
- 3 From the Time (s) list, choose 0.
- 4 In the Volume Fraction (phtr) toolbar, click 🗿 Plot.

Repeat the two previous instructions for the times 30 s, 100 s and 1000 s to generate Figure 2.

This completes Case 1. Now model the case with the particles initially gathered at the top.

DEFINITIONS

Define the particle concentration distribution using the **Step** function.

Step I (step I)

- I In the Home toolbar, click f(x) Functions and choose Local>Step.
- 2 In the Settings window for Step, click to expand the Smoothing section.
- 3 In the Size of transition zone text field, type 2*2.

Variables I

- I In the Model Builder window, click Variables I.
- 2 In the Settings window for Variables, locate the Variables section.
- **3** In the table, enter the following settings:

Name	Expression	Unit	Description
phi0_2	step1(y[1/mm]-8)*0.59		Initial concentration

PHASE TRANSPORT (PHTR)

In the Model Builder window, under Component I (compl) click Phase Transport (phtr).

Initial Values 2

- I In the Physics toolbar, click **Domains** and choose Initial Values.
- 2 In the Settings window for Initial Values, locate the Domain Selection section.
- 3 From the Selection list, choose All domains.
- **4** Locate the **Initial Values** section. In the $s_{0,\text{phid}}$ text field, type phi0_2.

In order to avoid dividing by zero in the region where the particle volume fraction is initially zero, modify the expressions for the slip velocity.

MULTIPHYSICS

Mixture Model I (mfmm1)

- I In the Model Builder window, under Component I (compl)>Multiphysics click Mixture Model I (mfmml).
- **2** In the **Settings** window for **Mixture Model**, locate the **Dipsersed Phase 2 Properties** section.
- **3** Specify the $\mathbf{u}_{\text{slip,phid}}$ vector as

J1/(phid*rho_s*(1-cd)+eps) x J2/(phid*rho_s*(1-cd)+eps) y

COMPONENT I (COMPI)

Create a finer mesh compared to the one used in the previous case.

MESH 2

In the Mesh toolbar, click Add Mesh.

Free Triangular 1 In the **Mesh** toolbar, click **See Triangular**.

Size

- I In the Settings window for Size, locate the Element Size section.
- 2 From the **Predefined** list, choose **Extra fine**.
- 3 Click the **Custom** button.
- 4 Locate the **Element Size Parameters** section. In the **Maximum element size** text field, type 0.0006.
- 5 Click 📗 Build All.

Next, add a Time Dependent study to compute the particle distribution.

ADD STUDY

- I In the Home toolbar, click \sim Add Study to open the Add Study window.
- 2 Go to the Add Study window.
- 3 Find the Studies subsection. In the Select Study tree, select General Studies> Time Dependent.
- 4 Click Add Study in the window toolbar.
- 5 In the Home toolbar, click 2 Add Study to close the Add Study window.

STUDY 2

Step 1: Time Dependent

I In the Settings window for Time Dependent, locate the Study Settings section.

2 In the **Output times** text field, type range(0, 10, 100).

3 Click to expand the Mesh Selection section. In the table, enter the following settings:

Geometry	Mesh
Geometry I	Mesh 2

4 In the **Home** toolbar, click **= Compute**.

RESULTS

To visualize the volume fraction of the dispersed phase as a surface plot along with the arrow plot of the mixture velocity (Figure 3), follow the steps given below.

Volume Fraction (phtr) 1

- I In the Model Builder window, under Results click Volume Fraction (phtr) I.
- 2 In the Settings window for 2D Plot Group, locate the Data section.
- 3 From the Time (s) list, choose 0.

Surface

- I In the Model Builder window, expand the Volume Fraction (phtr) I node, then click Surface.
- 2 In the Settings window for Surface, locate the Expression section.
- **3** In the **Expression** text field, type phid.
- 4 Locate the Range section. Select the Manual color range check box.
- 5 In the Minimum text field, type 0.
- 6 In the Maximum text field, type 0.62.

Arrow Surface 1

- I In the Model Builder window, right-click Volume Fraction (phtr) I and choose Arrow Surface.
- 2 In the Settings window for Arrow Surface, locate the Coloring and Style section.
- 3 From the Color list, choose White.
- 4 In the Volume Fraction (phtr) I toolbar, click 🗿 Plot.

Volume Fraction (phtr) 1

- I In the Model Builder window, click Volume Fraction (phtr) I.
- 2 In the Settings window for 2D Plot Group, locate the Data section.
- 3 From the Time (s) list, choose 10.
- 4 In the Volume Fraction (phtr) I toolbar, click 💽 Plot.
- 5 From the Time (s) list, choose 20.
- 6 In the Volume Fraction (phtr) I toolbar, click 🗿 Plot.
- 7 From the Time (s) list, choose 100.
- 8 In the Volume Fraction (phtr) I toolbar, click **O** Plot.

To make **Study I** behave as when it was first created, the feature **Initial Values 2** added for **Study 2** must be disabled.

STUDY I

Step 1: Time Dependent

- I In the Model Builder window, under Study I click Step I: Time Dependent.
- 2 In the Settings window for Time Dependent, locate the Physics and Variables Selection section.
- **3** Select the Modify model configuration for study step check box.
- 4 In the Physics and variables selection tree, select Component I (compl)> Phase Transport (phtr)>Initial Values 2.
- 5 Click 📿 Disable.

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