

Transfer Impedance of a Perforate

Perforates are plates with a distribution of small perforations or holes. They are used in muffler systems, sound absorbing panels, and in many other places as liners, where it is important to control attenuation precisely. As the perforations become smaller and smaller, the viscous and thermal losses become more important. The attenuation behavior, which is also frequency dependent, can be controlled by selecting the perforate size and distribution in a plate. Nonlinear loss mechanisms occur at higher sound levels or in the presence of a flow (through/bias or over/grazing the perforate). Only the linear effects due to viscosity and thermal conduction are studied in this tutorial model. These effects are modeled in detail using the Thermoviscous Acoustics, Frequency Domain interface.

Perforates have been theoretically studied for many years. Typically, analytical or semianalytical models only apply for simple geometries. A numerical approach is necessary for systems where the holes have various cross sections, if the perforations are tapered, or if the distribution of holes is uneven or of mixed sizes.

In this model, a simple perforate with circular holes is studied. The transfer impedance, surface normal impedance, and attenuation coefficient of the system is determined. The transfer impedance determined in this detailed model can, for example, be used in a larger system simulation using the interior impedance condition that exists in the *Pressure* Acoustics, Frequency Domain interface.

Model Definition

A sketch of the perforate system simulated is depicted in Figure 1. The model uses the symmetries that exist in the geometry along the x- and y-directions.

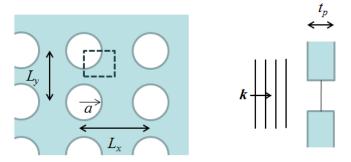


Figure 1: Schematic of the system, front and side views. The modeled region is marked by a box.

In Figure 1 the plate thickness is t_p , the hole radius is named a, the hole-hole x and y distances are named $L_{\rm x}$ and $L_{\rm v}$, respectively. In this model, the default values are $t_{\rm p}$ = 1.0 mm, a = 0.5 mm, $L_{\rm x}$ = 1.4 mm, and $L_{\rm v}$ = 2.0 mm. A plane wave is incident from one side, modeled using the Background Acoustic Fields feature in the Thermoviscous Acoustics interface.

The transfer impedance Z_{trans} , surface normal impedance Z_{n} , and absorption coefficient α are defined as

$$Z_{\text{trans}} = \frac{\Delta p}{v_{\text{bole}}} \qquad Z_{\text{n}} = \frac{p}{\mathbf{v} \cdot \mathbf{n}} \qquad \alpha = 1 - |R|^2$$
 (1)

where Δp is the pressure drop across the plate, v is the mean velocity in the perforation hole, \bf{n} is the surface normal of the perforate, and R is the reflection coefficient. These expressions are defined as variables in the model (imported from a file), under the **Definitions** node.

The transfer impedance of a perforate (perforated plate) is a well-studied problem in acoustics. Several models exist: some are pure analytical and some are semi-analytical. In this tutorial model, the results obtained in the simulation are compared to a semi-analytical expression for the transfer impedance $Z_{\rm trans}$ presented in Ref. 3 and Ref. 4. The expression combines an expression for the linear losses including viscosity (given in Ref. 1) and an expression for nonlinear effects at high levels (given in Ref. 2). The model also includes hole-hole interaction effects (semi-empirical). In this tutorial, the COMSOL model only consider linear acoustics. We include thermal and viscous losses as well as holehole interaction fully. In Equation 2, retaining the nonlinear term is out of interest and shows how it can be added to an analytical expression in COMSOL Multiphysics. The magnitude of that term is very small with the chosen levels. The semi-analytical model is given by the expression

$$\begin{split} \frac{Z_{\text{trans}}}{\rho_{0}c_{0}} &= \left(t_{\text{p}} + \frac{16a\Psi(\sqrt{\sigma})}{3\pi}\right) \frac{i\omega}{c_{0}\sigma\left[1 - \frac{2}{k_{s}} \frac{J_{1}(k_{s}a)}{J_{0}(k_{s}a)}\right]} + \frac{1.2(1 - \sigma^{2})}{2c_{0}(\sigma C_{\text{d}})^{2}} U_{\text{rms}} \\ k_{s}^{2} &= -i\rho_{0}\omega/\mu \end{split} \tag{2}$$

where ρ_0 is the fluid density, c_0 the speed of sound, μ is the dynamic viscosity, $J_{\rm n}$ is the Bessel function of the first kind, $k_{\rm s}$ is the shear (viscous) wave number, $U_{\rm rms}$ the rms acoustic velocity in the hole, σ the porosity, and C_d is the discharge coefficient (a typical value is 0.76). Ψ is the so-called Fok function is defined by

$$\Psi(\sqrt{\sigma}) = 1 - 1.41(\sqrt{\sigma}) + 0.34(\sqrt{\sigma})^3 + 0.07(\sqrt{\sigma})^5 - 0.02(\sqrt{\sigma})^6 + 0.06(\sqrt{\sigma})^7 - 0.016(\sqrt{\sigma})^8$$
(3)

The transfer impedance expression in Equation 2 does not include thermal effects. These can be added by modifying the linear part of the transfer impedance expression. The thermal effects are small for thin perforated plate (in the small $k \cdot t_{\mathbf{p}}$ limit) where shear and viscous losses dominate, see Ref. 5.

Results and Discussion

The total acoustic pressure in the system is depicted in Figure 2 evaluated at 2000 Hz. The acoustic particle velocity is depicted in Figure 3 at two different frequencies (400 and 2000 Hz). From the figure, the viscous boundary layer is easily seen as well as its frequency dependence (decreases with increasing frequency). Moreover, the end correction extent is also visualized here, as the part of the fluid moving axially, that extends out of the orifice. In Figure 4, the acoustic temperature variations can be seen. Here, as for the viscous boundary layer, the thermal boundary layer can be visualized.

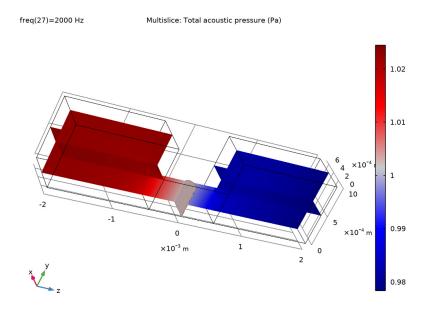


Figure 2: Pressure distribution in the system.

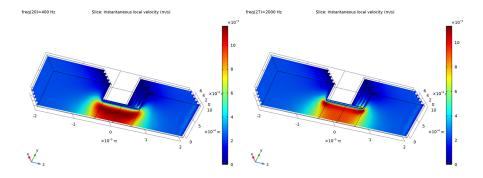


Figure 3: Velocity distribution at 400 Hz and 2000 Hz.

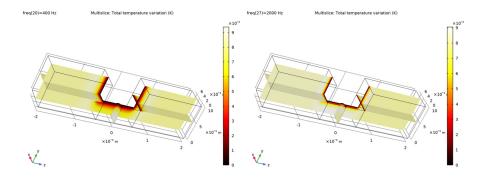


Figure 4: Temperature distribution at 400 Hz and 2000 Hz.

The simulated value of the transfer impedance is depicted in Figure 5 and compared to the semi-analytical model given by Equation 2. The results are seen to coincide well. On the logarithmic y-axis scale, there is a constant difference between the curves. This corresponds to a factor multiplied on the linear scale. This could be due to the end correction and the hole-hole interaction modeled with the Fok function. In the present case, the holes are not equidistant in all directions. This can also make the simulation results deviate from the analytical assumptions.

The surface normal impedance experienced by a plane wave is depicted in Equation 6 and the absorption coefficient is depicted in Equation 7.

Finally, the instantaneous acoustic particle velocity is depicted using mirror data sets in the last Figure 8.

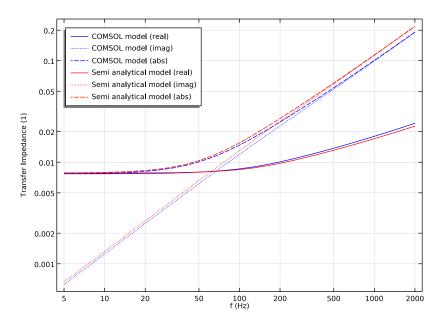


Figure 5: Transfer impedance as function of frequency. Comparing the COMSOL Multiphysics model results and the semianalytical expression.

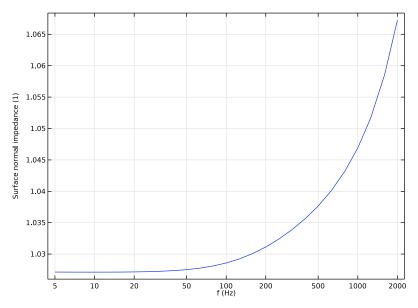


Figure 6: Surface normal impedance of the perforated plate.

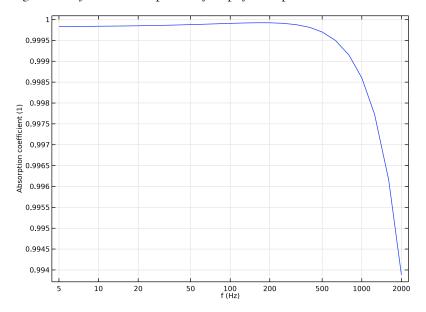


Figure 7: Absorption coefficient of the perforated plate.

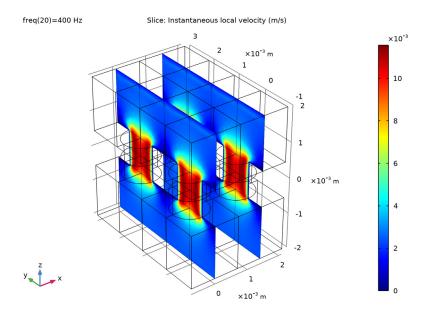


Figure 8: Instantaneous velocity plotted using the symmetry data sets.

Notes About the COMSOL Implementation

SOLVER

This model does not use the default direct solver, but an iterative approach with a so-called direct preconditioner. This is one of the predefined iterative solver suggestions generated by COMSOL Multiphysics (it is then simply to enable it). This solver strategy is described in the Acoustics Module User's Guide in the Modeling with Thermoviscous Acoustics section. The approach is ideal for medium sized pure thermoviscous acoustics models; the iterative solver is both faster and more memory efficient than the default direct solver.

The present model utilized the symmetries of the geometry to reduce the model size. Some perforated plate conflagrations may not have the same possibilities and thus require solving a larger problem. If the resulting problem becomes too large for the first iterative solver suggestion (used in this tutorial), the second iterative solver suggestion may be used. It is based on the domain decomposition (DD) method. This solver is very memory efficient but slower. An example can be found on the COMSOL homepage at:

www.comsol.com/model/transfer-impedance-of-a-perforate-12585

MESH

When setting up the mesh, the parameter dvisc has been used as a measure of the viscous boundary layer thickness at the maximal study frequency. The viscous boundary layer thickness is given by

$$\delta_{\text{visc}} = \sqrt{\frac{2\mu}{\omega \rho_0}}$$

which at 100 Hz is equal to 0.22 mm for air. Thus the expression 220[um]* sqrt(100[Hz]/fmax) used in the parameter list for dvisc.

REFLECTION COEFFICIENT

In the expression for the reflection coefficient R given under **Definitions > Variables 1**, the down() operator has been used in order to access the values of the background and scattered field variables on the side of the boundary where the Background Acoustic Fields feature is defined. The orientation up/down is relative to the surface normal which can be visualized with an arrow surface plot of the normal vector components nx, ny, and nz.

References

- 1. I. B. Crandall, "Theory of Vibrating Systems and Sound", D. Van Nostrand Company, New York (1926).
- 2. T. H. Melling, "The acoustic impedance of perforates at medium and high sound pressure levels", J. Sound Vibration 29, 1-65 (1973).
- 3. T. Schultz, F. Liu, L. Cattafesta, M. Sheplak, and M. Jones, "A Comparison Study of Normal-Incidence Acoustic Impedance Measurements of a Perforated Liner", NASA Technical Reports Server LF99-801 (2009).
- 4. T. Schultz, F. Liu, L. Cattafesta, M. Sheplak, and M. Jones, "A Comparison Study of Normal-Incidence Acoustic Impedance Measurements of a Perforated Liner", 15th AIAA/CEAS Aeroacoustics conference, AIAA 2009-3301.
- 5. A. W. Nolle, "Small-Signal Impedance of Short Tubes", J. Acoust. Soc. Am. 25, 32-39 (1952).

Application Library path: Acoustics Module/Tutorials, Thermoviscous Acoustics/transfer impedance perforate From the File menu, choose New.

NEW

In the New window, click Model Wizard.

MODEL WIZARD

- I In the Model Wizard window, click **3D**.
- 2 In the Select Physics tree, select Acoustics>Thermoviscous Acoustics> Thermoviscous Acoustics, Frequency Domain (ta).
- 3 Click Add.
- 4 Click 🗪 Study.
- 5 In the Select Study tree, select General Studies>Frequency Domain.
- 6 Click **Done**.

GLOBAL DEFINITIONS

Start by loading the global parameters that define the geometry and mesh parameters.

Parameters 1

- I In the Model Builder window, under Global Definitions click Parameters I.
- 2 In the Settings window for Parameters, locate the Parameters section.
- 3 Click Load from File.
- 4 Browse to the model's Application Libraries folder and double-click the file transfer impedance perforate parameters.txt.

GEOMETRY I

The next task is to set up the geometry of the perforate. The following instructions walk you through the steps of creating a fully parameterized geometry of one quarter of a hole as depicted in Figure 1. The instructions are quite lengthy.

If you want to skip these steps, the geometry sequence can be imported by right-clicking the **Geometry** node and selecting **Insert Sequence**. Browse to your COMSOL installation folder under Multiphysics>applications>Acoustics_Module>Tutorials and select transfer impedance perforate.mph. After this, just continue setting up the model from the **Definitions** section below.

Cvlinder I (cvl I)

I In the Geometry toolbar, click (Cylinder.

- 2 In the Settings window for Cylinder, locate the Size and Shape section.
- 3 In the Radius text field, type a.
- 4 In the Height text field, type tp.
- 5 Locate the **Position** section. In the z text field, type -tp/2.
- 6 Click to expand the Layers section. Clear the Layers on side check box.
- 7 Select the Layers on bottom check box.
- **8** In the table, enter the following settings:

Layer name	Thickness (m)
Layer 1	tp/2

Block I (blk I)

- I In the Geometry toolbar, click Block.
- 2 In the Settings window for Block, locate the Size and Shape section.
- 3 In the Width text field, type Lx/2.
- 4 In the **Depth** text field, type Ly/2.
- 5 In the **Height** text field, type Lz+Lpml.
- 6 Locate the **Position** section. In the **z** text field, type tp/2.
- 7 Click to expand the Layers section. Find the Layer position subsection. Clear the Bottom check box.
- **8** Select the **Top** check box.
- **9** In the table, enter the following settings:

Layer name	Thickness (m)
Layer 1	Lpml

Block 2 (blk2)

- I In the Geometry toolbar, click Block.
- 2 In the Settings window for Block, locate the Size and Shape section.
- 3 In the Width text field, type Lx/2.
- 4 In the **Depth** text field, type Ly/2.
- 5 In the Height text field, type Lz+Lpml.
- 6 Locate the **Position** section. In the **z** text field, type (Lz+Lpm1+tp/2).

7 Locate the Layers section. In the table, enter the following settings:

Layer name	Thickness (m)
Layer 1	Lpml

Work Plane I (wbl)

- I In the Geometry toolbar, click Swork Plane.
- 2 In the Settings window for Work Plane, locate the Plane Definition section.
- 3 From the Plane list, choose yz-plane.

Work Plane 2 (wb2)

- I In the Geometry toolbar, click Work Plane.
- 2 In the Settings window for Work Plane, locate the Plane Definition section.
- 3 From the Plane list, choose zx-plane.

Partition Objects I (par I)

- I In the Geometry toolbar, click Booleans and Partitions and choose Partition Objects.
- **2** Select the object **cyll** only.
- 3 In the Settings window for Partition Objects, locate the Partition Objects section.
- 4 From the Partition with list, choose Work plane.
- 5 From the Work plane list, choose Work Plane I (wpl).
- 6 Click | Build Selected.

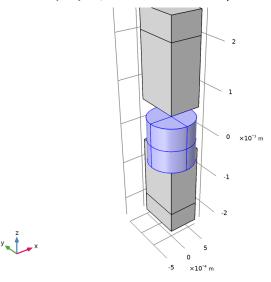
Partition Objects 2 (par2)

- I In the Geometry toolbar, click Booleans and Partitions and choose Partition Objects.
- 2 Select the object parl only.
- 3 In the Settings window for Partition Objects, locate the Partition Objects section.
- 4 From the Partition with list, choose Work plane.
- 5 Click Pauld Selected.

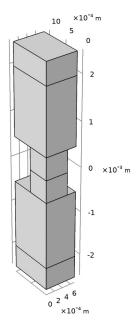
Delete Entities I (dell)

- I In the Model Builder window, right-click Geometry I and choose Delete Entities.
- 2 In the Settings window for Delete Entities, locate the Entities or Objects to Delete section.
- 3 From the Geometric entity level list, choose Domain.

4 On the object par2, select Domains 1–6 only.



- 5 Click **Build All Objects**.
- 6 Click the Zoom Extents button in the Graphics toolbar.





DEFINITIONS

Load the variables that define the transfer impedance, normal surface impedance, and absorption coefficient. The variables also define the semianalytical transfer impedance model defined in Equation 2.

Variables 1

- I In the Model Builder window, expand the Component I (compl)>Definitions node.
- 2 Right-click **Definitions** and choose **Variables**.
- 3 In the Settings window for Variables, locate the Variables section.
- 4 Click **Load from File**.
- **5** Browse to the model's Application Libraries folder and double-click the file transfer_impedance_perforate_variables.txt.

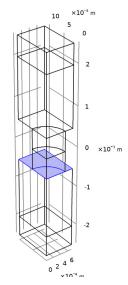
Now, first set up integration operators on the two sides of the plate (in and out) as well as in the center of the tube (mid). Create selections for the symmetry planes and the wall. Finally, add the perfectly matched layers (PMLs) that truncate the computational domain.

Integration I (intopl)

- I In the **Definitions** toolbar, click Nonlocal Couplings and choose Integration.
- 2 Click the Zoom Extents button in the Graphics toolbar.
- 3 Click the Wireframe Rendering button in the Graphics toolbar.
- 4 In the Settings window for Integration, locate the Source Selection section.
- 5 From the Geometric entity level list, choose Boundary.

6 Select Boundaries 9 and 21 only.

It might be easier to select the boundaries by using the Selection List window. To open this window, in the Home toolbar click Windows and choose Selection List. (If you are running the cross-platform desktop, you find **Windows** in the main menu.)





7 In the Operator name text field, type intop in.

Integration 2 (intob2)

- I In the Definitions toolbar, click / Nonlocal Couplings and choose Integration.
- 2 In the Settings window for Integration, locate the Source Selection section.
- 3 From the Geometric entity level list, choose Boundary.
- 4 Select Boundaries 15 and 23 only.
- 5 In the Operator name text field, type intop out.

Integration 3 (intob3)

- I In the Definitions toolbar, click Nonlocal Couplings and choose Integration.
- 2 In the Settings window for Integration, locate the Source Selection section.
- 3 From the Geometric entity level list, choose Boundary.
- 4 Select Boundary 12 only.
- 5 In the Operator name text field, type intop mid.

Symmetry

- I In the **Definitions** toolbar, click **\(\frac{1}{2} \) Explicit**.
- 2 In the Settings window for Explicit, type Symmetry in the Label text field.
- 3 Locate the Input Entities section. From the Geometric entity level list, choose Boundary.
- 4 Select the **Group by continuous tangent** check box, to simplify selection and click on one face on each symmetry planes. Alternatively select the following faces.
- **5** Select Boundaries 1, 2, 4, 5, 7, 8, 10, 11, 13, 14, 16, 17, and 24–31 only.

Wall

- I In the **Definitions** toolbar, click **\(\frac{1}{2} \) Explicit**.
- 2 In the Settings window for Explicit, type Wall in the Label text field.
- 3 Locate the Input Entities section. From the Geometric entity level list, choose Boundary.
- **4** Select Boundaries 20–23 only.

Perfectly Matched Layer I (pml1)

- I In the Definitions toolbar, click M Perfectly Matched Layer.
- 2 Select Domains 1 and 6 only.

MATERIALS

Add air as the material and set the bulk viscosity to 0.

ADD MATERIAL

- I In the Home toolbar, click 🤼 Add Material to open the Add Material window.
- 2 Go to the Add Material window.
- 3 In the tree, select Built-in>Air.
- 4 Click Add to Component I (compl).
- 5 In the Home toolbar, click **‡ Add Material** to close the **Add Material** window. Proceed to set up the physics.

THERMOVISCOUS ACOUSTICS, FREQUENCY DOMAIN (TA)

Symmetry I

- I In the Model Builder window, under Component I (compl) right-click Thermoviscous Acoustics, Frequency Domain (ta) and choose Symmetry.
- 2 In the Settings window for Symmetry, locate the Boundary Selection section.
- 3 From the Selection list, choose Symmetry.

Finally, add an incident plane wave using the Background Acoustic Fields feature with the Plane Wave option. Since the domain is backed by a PML, this configuration will allow reflections from the surface to leave the computational domain.

Background Acoustic Fields 1

- I In the Physics toolbar, click **Domains** and choose Background Acoustic Fields.
- 2 Select Domain 2 only.
- 3 In the Settings window for Background Acoustic Fields, locate the **Background Acoustic Fields** section.
- 4 From the Acoustic field type list, choose Plane wave.
- **5** In the $|p_b|$ text field, type 1.
- **6** Specify the \mathbf{e}_k vector as

0	x
0	у
1	z

MESH I

Now create a mesh that will resolve the acoustic boundary layers. The thickness of the viscous layer at 2000 Hz is defined as a parameter dvisc, use the parameter when setting up a boundary layer mesh. The mesh should also be structured in the PML regions with at least 8 layers (used for the polynomial stretching type of the PML).

Free Tetrahedral I

In the Mesh toolbar, click A Free Tetrahedral.

Size

- I In the Model Builder window, click Size.
- 2 In the Settings window for Size, locate the Element Size section.
- **3** Click the **Custom** button.
- 4 Locate the Element Size Parameters section. In the Maximum element size text field, type a/3.
- 5 In the Minimum element size text field, type dvisc/2.
- 6 In the Resolution of narrow regions text field, type 4.

Free Tetrahedral I

I In the Model Builder window, click Free Tetrahedral I.

- 2 In the Settings window for Free Tetrahedral, locate the Domain Selection section.
- 3 From the Geometric entity level list, choose Domain.
- **4** Select Domains 2–5 only.

Size 1

- I Right-click Free Tetrahedral I and choose Size.
- 2 In the Settings window for Size, locate the Geometric Entity Selection section.
- 3 From the Geometric entity level list, choose Edge.
- 4 Select Edges 23 and 27 only.
- **5** Locate the **Element Size** section. Click the **Custom** button.
- 6 Locate the Element Size Parameters section. Select the Maximum element size check box.
- 7 In the associated text field, type dvisc.

Swebt I

In the Mesh toolbar, click A Swept.

Distribution I

- I Right-click Swept I and choose Distribution.
- 2 In the Settings window for Distribution, locate the Distribution section.
- 3 In the Number of elements text field, type 10.
- 4 Click III Build All.

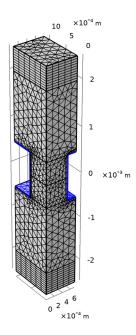
Boundary Layers 1

- I In the Mesh toolbar, click Boundary Layers.
- 2 In the Settings window for Boundary Layers, click to expand the Transition section.
- 3 Clear the Smooth transition to interior mesh check box.

Boundary Layer Properties

- I In the Model Builder window, click Boundary Layer Properties.
- 2 In the Settings window for Boundary Layer Properties, locate the Geometric Entity Selection section.
- 3 From the Selection list, choose Wall.
- 4 Locate the Boundary Layer Properties section. In the Number of boundary layers text field, type 2.
- 5 From the Thickness of first layer list, choose Manual.
- 6 In the Thickness text field, type 0.4*dvisc.

7 Click **Build All**.



STUDY I

Step 1: Frequency Domain

Proceed to select the frequencies for which to solve the model. Use the built in ISO preferred frequencies. In order to see these, first activate the advanced study option view.

- I In the Model Builder window, under Study I click Step I: Frequency Domain.
- 2 In the Settings window for Frequency Domain, locate the Study Settings section.
- 3 Click Range.
- 4 In the Range dialog box, choose ISO preferred frequencies from the Entry method list.
- 5 In the Start frequency text field, type 5.
- 6 In the Stop frequency text field, type 2000.
- 7 From the Interval list, choose 1/3 octave.

8 Click Replace.

This gives a list with the ISO preferred frequencies between 5 and 2000 Hz with 1/3 octave spacing.

In the next steps, set up an iterative solver to solve this thermoviscous acoustics problem. Because of the problem size (not too large) an iterative solver with a direct preconditioner is the best choice over the default direct solver. A discussion of different solver strategies is given in the modeling section under the Thermoviscous Acoustics chapter in the Acoustics Module User's Guide.

Start by generating the default solver, expand the nodes, and then enable the iterative solver suggestion that uses the direct preconditioner.

Solution I (soll)

- I In the Study toolbar, click Show Default Solver.
- 2 In the Model Builder window, expand the Solution I (soll) node.
- 3 In the Model Builder window, expand the Study I>Solver Configurations> Solution I (soll)>Stationary Solver I node.
- 4 Right-click Suggested Iterative Solver (GMRES with Direct Precon.) (ta) and choose Enable.

Step 1: Frequency Domain

In the **Study** toolbar, click **Compute**.

RESULTS

Acoustic Pressure (ta)

Add a selection to the dataset to remove plotting in the PML region. Then, rotate the figure to produce the pressure, velocity, and temperature plots depicted in Figure 2, Figure 3, and Figure 4.

Note that to analyze the damping inside the PML domains, it can be a good idea not to add the selection and plot the sound pressure level ta.Lp.

Study I/Solution I (soll)

In the Model Builder window, expand the Results>Datasets node, then click Study 1/ Solution I (soll).

Selection

- I In the Results toolbar, click has a Attributes and choose Selection.
- 2 In the Settings window for Selection, locate the Geometric Entity Selection section.

- 3 From the Geometric entity level list, choose Domain.
- **4** Select Domains 2–5 only.

Multislice

- I In the Model Builder window, expand the Results>Acoustic Pressure (ta) node, then click
- 2 In the Settings window for Multislice, locate the Coloring and Style section.
- 3 Clear the Symmetrize color range check box.

Acoustic Velocity (ta)

- I In the Model Builder window, click Acoustic Velocity (ta).
- 2 In the Settings window for 3D Plot Group, locate the Data section.
- 3 From the Parameter value (freq (Hz)) list, choose 400.
- 4 In the Acoustic Velocity (ta) toolbar, click **Plot**.

Multislice

- I In the Model Builder window, expand the Results>Temperature Variation (ta) node, then click Multislice.
- 2 In the Settings window for Multislice, locate the Coloring and Style section.
- 3 Clear the Symmetrize color range check box.

Temperature Variation (ta)

- I In the Model Builder window, click Temperature Variation (ta).
- 2 In the Settings window for 3D Plot Group, locate the Data section.
- 3 From the Parameter value (freq (Hz)) list, choose 400.
- 4 In the Temperature Variation (ta) toolbar, click Plot.

Now, create the transfer impedance plot depicted in Figure 5.

Transfer Impedance

- I In the Home toolbar, click **Add Plot Group** and choose ID Plot Group.
- 2 In the Settings window for ID Plot Group, type Transfer Impedance in the Label text field.
- 3 Click to expand the **Title** section. From the **Title type** list, choose **None**.
- **4** Locate the **Plot Settings** section. Select the **x-axis label** check box.
- 5 In the associated text field, type f (Hz).
- 6 Select the y-axis label check box.

- 7 In the associated text field, type Transfer Impedance (1).
- 8 Locate the Axis section. Select the x-axis log scale check box.
- **9** Select the **y-axis log scale** check box.
- 10 Locate the Legend section. From the Position list, choose Upper left.

Global I

- I Right-click Transfer Impedance and choose Global.
- 2 In the Settings window for Global, locate the y-Axis Data section.
- **3** In the table, enter the following settings:

Expression	Unit	Description
real(Ztrans)	1	COMSOL model (real)
imag(Ztrans)	1	COMSOL model (imag)
abs(Ztrans)	1	COMSOL model (abs)

- 4 Click to expand the Coloring and Style section. Find the Line style subsection. From the Line list, choose Cycle.
- 5 From the Color list, choose Blue.

Point Graph 1

- I In the Model Builder window, right-click Transfer Impedance and choose Point Graph.
- **2** Select Point 5 only.
- 3 In the Settings window for Point Graph, locate the y-Axis Data section.
- 4 In the Expression text field, type real(Ztrans ana).
- 5 Click to expand the Coloring and Style section. From the Color list, choose Red.
- 6 Click to expand the Legends section. Select the Show legends check box.
- 7 From the Legends list, choose Manual.
- **8** In the table, enter the following settings:

Leger	ıds			
Semi	analytical	model	(real)	

Point Graph 2

- I Right-click Transfer Impedance and choose Point Graph.
- **2** Select Point 5 only.
- 3 In the Settings window for Point Graph, locate the y-Axis Data section.
- 4 In the Expression text field, type imag(Ztrans ana).

- 5 Locate the Coloring and Style section. Find the Line style subsection. From the Line list, choose Dotted
- 6 From the Color list, choose Red.
- 7 Locate the **Legends** section. Select the **Show legends** check box.
- 8 From the Legends list, choose Manual.
- **9** In the table, enter the following settings:

Legends Semi analytical model (imag)

Point Graph 3

- I Right-click Transfer Impedance and choose Point Graph.
- **2** Select Point 5 only.
- 3 In the Settings window for Point Graph, locate the y-Axis Data section.
- **4** In the **Expression** text field, type abs(Ztrans_ana).
- 5 Locate the Coloring and Style section. Find the Line style subsection. From the Line list, choose Dashed.
- 6 From the Color list, choose Red.
- 7 Locate the Legends section. Select the Show legends check box.
- 8 From the Legends list, choose Manual.
- **9** In the table, enter the following settings:

Legends Semi analytical model (abs)

10 In the **Transfer Impedance** toolbar, click **10 Plot**.

Create the surface impedance plot depicted in Figure 6.

Surface Normal Impedance

- I In the Home toolbar, click **Add Plot Group** and choose ID Plot Group.
- 2 In the Settings window for ID Plot Group, type Surface Normal Impedance in the **Label** text field.
- 3 Locate the Title section. From the Title type list, choose None.
- 4 Locate the Plot Settings section. Select the x-axis label check box.
- 5 In the associated text field, type f (Hz).
- 6 Select the y-axis label check box.

- 7 In the associated text field, type Surface normal impedance (1).
- 8 Locate the Axis section. Select the x-axis log scale check box.
- **9** Locate the **Legend** section. Clear the **Show legends** check box.

Global I

- I Right-click Surface Normal Impedance and choose Global.
- 2 In the Settings window for Global, locate the y-Axis Data section.
- **3** In the table, enter the following settings:

Expression	Unit	Description
abs(Zn)	1	

4 In the Surface Normal Impedance toolbar, click Plot.

Next, create the absorption coefficient plot depicted in Figure 7.

Absorption coefficient

- I In the Home toolbar, click **Add Plot Group** and choose ID Plot Group.
- 2 In the Settings window for ID Plot Group, type Absorption coefficient in the Label text field.
- **3** Locate the **Title** section. From the **Title type** list, choose **None**.
- **4** Locate the **Plot Settings** section. Select the **x-axis label** check box.
- 5 In the associated text field, type f (Hz).
- 6 Select the y-axis label check box.
- 7 In the associated text field, type Absorption coefficient (1).
- **8** Locate the **Axis** section. Select the **x-axis log scale** check box.
- **9** Locate the **Legend** section. Clear the **Show legends** check box.

Global I

- I Right-click Absorption coefficient and choose Global.
- 2 In the Settings window for Global, locate the y-Axis Data section.
- **3** In the table, enter the following settings:

Expression	Unit	Description
alpha	1	

4 In the Absorption coefficient toolbar, click Plot.

Finally, create a series of mirror datasets to plot the solution in a larger domain (using the symmetries of the model). This will reproduce the plot in Figure 8.

Mirror 3D I

In the **Results** toolbar, click **More Datasets** and choose **Mirror 3D**.

- I In the Results toolbar, click More Datasets and choose Mirror 3D.
- 2 In the Settings window for Mirror 3D, locate the Data section.
- 3 From the Dataset list, choose Mirror 3D 1.
- 4 Locate the Plane Data section. From the Plane list, choose xz-planes.

Mirror 3D 3

- I In the Results toolbar, click More Datasets and choose Mirror 3D.
- 2 In the Settings window for Mirror 3D, locate the Data section.
- 3 From the Dataset list, choose Mirror 3D 2.
- 4 Locate the Plane Data section. From the Plane type list, choose General.
- 5 From the Plane entry method list, choose Point and normal vector.
- **6** Find the **Point** subsection. In the x text field, type Lx/2.
- 7 Find the Normal vector subsection. In the z text field, type 0.
- 8 In the x text field, type 1.

Mirror 3D 4

- I In the Results toolbar, click More Datasets and choose Mirror 3D.
- 2 In the Settings window for Mirror 3D, locate the Data section.
- 3 From the Dataset list, choose Mirror 3D 3.
- 4 Locate the Plane Data section. From the Plane type list, choose General.
- 5 From the Plane entry method list, choose Point and normal vector.
- 6 Find the Point subsection. In the y text field, type Ly/2.
- 7 Find the Normal vector subsection. In the z text field, type 0.
- 8 In the y text field, type 1.

Mirror Plot: Velocity

- I In the Results toolbar, click **3D Plot Group**.
- 2 In the Settings window for 3D Plot Group, type Mirror Plot: Velocity in the Label text field.

- 3 Locate the Data section. From the Dataset list, choose Mirror 3D 4.
- 4 From the Parameter value (freq (Hz)) list, choose 400.

Slice 1

- I Right-click Mirror Plot: Velocity and choose Slice.
- 2 In the Settings window for Slice, locate the Expression section.
- 3 In the Expression text field, type ta.v_inst.
- 4 Locate the Plane Data section. In the Planes text field, type 2.
- 5 In the Mirror Plot: Velocity toolbar, click Plot.