Created in COMSOL Multiphysics 5.6

Magnetic Prospecting of Iron Ore Deposits

Introduction

Magnetic prospecting is one of the methods of geological exploration applicable to certain types of iron ore deposits, in particular those made up of magnetite and hematite. Estimates of center-of-mass coordinates and the spatial extent of iron-rich layers help decrease the cost of exploration. Passive magnetic prospecting relies on accurate mapping of local geomagnetic anomalies — deviations of the natural magnetostatic fields from their predicted values based on the magnetic dipole model of the Earth's field. This application investigates magnetic anomaly estimation for both surface and aerial prospecting.

Crustal magnetic anomalies may result from either induced or remnant magnetization of iron-rich rocks. Among all terrestrial iron minerals, magnetite has the largest magnetic permeability (up to $\mu_r = 650$ in small-grain magnetite) and the strongest naturally occurring thermo-remnant magnetization (up to $M_r = 5000$ A/m) (Ref. 1). Another mineral with relatively strong magnetization is hematite (Ref. 1). Magnetite and hematite are the major minerals of high-grade iron ore. This application uses modest values of magnetic permeability ($\mu_r = 3.5$) and remnant magnetization ($M_r = 60$ A/m) that are typical for terrestrial magnetite with a grain size distribution of 20–200 μm (Ref. 1). The concentration of magnetite in the iron-rich ore is assumed to be 25%.

Model Definition

Magnetostatic problems without significant electrical currents can be solved in terms of a scalar magnetic potential. Variations of the magnetic permeability or remnant magnetization cause small deviations of magnetic fields from the norm (local geomagnetic anomalies), which you can model accurately using the reduced potential formulation of the Magnetic Fields, No Currents interface. The background magnetic field is assumed to be uniform within the simulation domain. Intensity and orientation of the natural magnetic field is estimated using the data from National Geophysical Data Center of the U.S. Government (Ref. 2).

This application neglects the induced and remnant magnetization of the rocks outside the iron ore body. The iron ore bed is approximated by a uniform flat ellipsoid with a maximum vertical thickness of 100 m, a maximum North-South width of 400 m, and an East-West extent of 2000 m. Center-of-mass coordinates of the ore bed (2500, 1500) meters from the lower-left corner (200 meters above sea level) are positioned underneath the Eastern Pit of the Eagle Mountain Mine, a former Kaiser Steel Co. mining operation in Riverside County, California (Ref. 3). The specific location and shape of magnetic ore in this application are not based on any actual geological prospecting but are chosen to simulate a basic size and shape of such an ore. Figure 1 shows the model geometry.

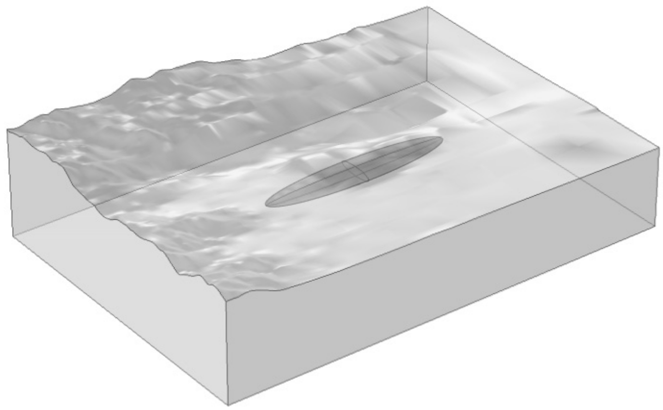


Figure 1: The model geometry.

TOPOGRAPHY

The topographical map used for this application consists of a rectangular grid containing 157-by-111 points spaced 1 arc second (1/3600°) apart in both horizontal directions. The curvature of the geoid is neglected in drawing this geometry from elevation data. The lower-left (south-west) corner of the map is located at a latitude 33.85° (North) and longitude 115.5° (West). The size of the simulation domain is 4836 m in an East-West (x-axis), 3410 m in a North-South direction (y-axis), and 1934.4 m in the up-down direction (z-axis). A satellite image of this area can be seen in (Ref. 4).

A digital elevation model (DEM) for this example can be derived from the National Elevation Dataset (NED) of the U.S. Geological Survey data center (Ref. 5). The free program MicroDEM (Ref. 6) converts the USGS elevation data into an ASCII file containing the elevation matrix, which you can import into COMSOL Multiphysics as an Interpolation function feature. From the interpolation function you can then create a geometry by using a Parametric Surface feature. Optionally, the resulting geometry can be saved in the internal binary CAD file format (.mphbin). Importing the saved CAD file into the COMSOL Desktop is the starting point for the step-by-step instructions for this application. Figure 2 contains a plot of the resulting elevation map as it appears in COMSOL Multiphysics.

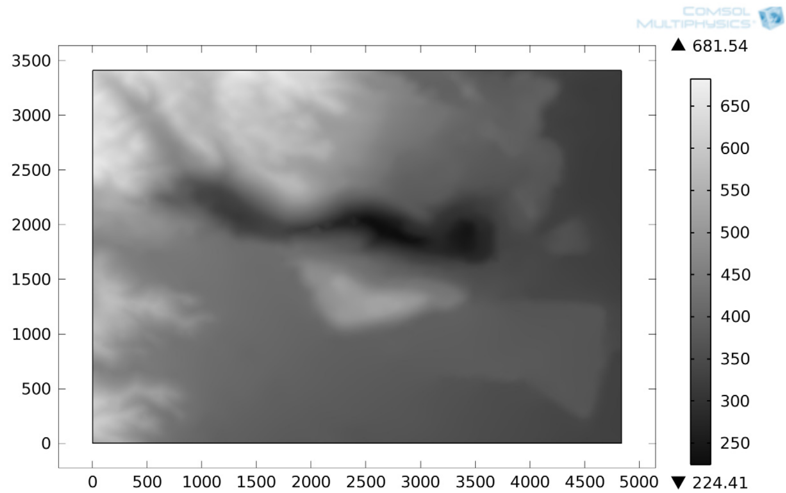


Figure 2: Elevation map of the simulation domain (boundary plot of z-coordinate on Boundary 6).

DOMAIN EQUATIONS

In a current-free region, where

$$\nabla \times \mathbf{H} = 0$$

it is possible to define a scalar magnetic potential, V_{m}, such that

$$\mathbf{H} = -\nabla V_{\mathrm{m}}$$

This is analogous to the definition of the electric potential for static electric fields. Using the constitutive relation between the magnetic flux density and magnetic field,

$$\mathbf{B} = \mu\mathbf{H} + \mathbf{B}_{\mathrm{r}}$$

together with the equation

$$\nabla \cdot \mathbf{B} = 0$$

one can derive an equation for V_{m}:

$$\nabla \cdot (-\mu\nabla V_{\mathrm{m}} + \mathbf{B}_{\mathrm{r}}) = 0$$

The reduced potential formulation used in this model splits the total magnetic potential into external and reduced parts, $V_{\text{tot}} = V_{\text{ext}} + V_{\text{red}}$, where the reduced potential V_{red} is the dependent variable:

$$\nabla \cdot (-\mu\nabla(V_{\text{ext}} + V_{\text{red}}) + \mathbf{B}_{\text{r}}) = 0$$

The external magnetic potential is more easily defined as an external magnetic field, so the equation that is used in practice is rather

$$\nabla \cdot (-\mu\nabla(V_{\text{red}}) + \mathbf{B}_{\text{r}} + \mu\mathbf{H}_{\text{ext}}) = 0$$

To simulate the background geomagnetic field, components of the external magnetic field are expressed through total intensity, magnetic declination, and inclination angles as follows:

$$\begin{aligned}
\text{extHOx} &= \text{HO} \cdot \cos(\text{Incl}) \cdot \sin(\text{Decl}) \\
\text{extHOy} &= \text{HO} \cdot \cos(\text{Incl}) \cdot \cos(\text{Decl}) \\
\text{extHOz} &= -\text{HO} \cdot \sin(\text{Incl})
\end{aligned}$$

Based on the data provided by NOAA’s data center (Ref. 3), inclination and declination angles for this location (N 33.85°, W 115.5°) are approximately $\text{Incl} = 59.357°$ and $\text{Decl} = 12.275°$, respectively. The magnitude of natural magnetic flux density ($B_0 = \mu_0 H_0$) is estimated as 48.163 µT.

Remnant magnetic flux density is assumed to be aligned with the local contemporary direction of geomagnetic field:

$$\begin{aligned}
B_{r,x} &= B_{r,\text{ore}} \cdot \cos(\text{Incl}) \cdot \sin(\text{Decl}) \\
B_{r,y} &= B_{r,\text{ore}} \cdot \cos(\text{Incl}) \cdot \cos(\text{Decl}) \\
B_{r,z} &= -B_{r,\text{ore}} \cdot \sin(\text{Incl})
\end{aligned}$$

In general, thermo-remnant magnetization does not have to be aligned with the Earth’s magnetic field.

The values of magnetic permeability and remnant flux used for the iron ore are based on the typical parameters of magnetite ore and a simple homogenization model taking into account dilution of magnetization in the ore bed:

$$\begin{aligned}
\mu_{\text{r, ore}} - 1 &= (\mu_{\text{r, magn}} - 1) \cdot c_{\text{magn}} \\
\mathbf{B}_{\text{r, ore}} &= \mathbf{B}_{\text{r, magn}} \cdot c_{\text{magn}}
\end{aligned}$$

Concentration of magnetite c_{magn} in the ore is assumed to be 25%.

BOUNDARY CONDITIONS

Along the exterior boundaries of the surrounding box, the perturbation of the magnetic field (H_{red}) is assumed to be tangential to the boundaries. The natural boundary condition from the equation is

$$\mathbf{n} \cdot \mathbf{B}_{red} = \mathbf{n} \cdot \mu \cdot \nabla(V_m) = 0$$

Thus, the reduced magnetic field is made tangential to the boundary by a Neumann condition on the reduced potential. Interior boundaries are modeled as continuity boundary conditions and require no user input.

The Magnetic Fields, No Currents interface automatically adds a point constraint, $V_m = 0$, using a weak term on the equation-system level if there are no boundary conditions that constrain the value of V_m (such as Magnetic potential or Zero potential). This ensures that the scalar potential V_m is uniquely defined in problems with pure Neumann boundary conditions.

Results and Discussion

Figure 3 shows deviations of the magnetic field intensity on the ground (boundary plot), and at an altitude of 1300 meters. The maximum magnetic field perturbation on the ground is 2 A/m and at 1300 meters altitude it has fallen to 0.1 A/m. In both locations the maximum is located approximately above the center of mass of the magnetite ore body. These results show that aerial magnetic prospecting may reveal the location and provide estimates of the horizontal extent of magnetite-rich deposits but also that ground-based magnetic prospecting apparently is a much more sensitive exploration tool than aerial prospecting.

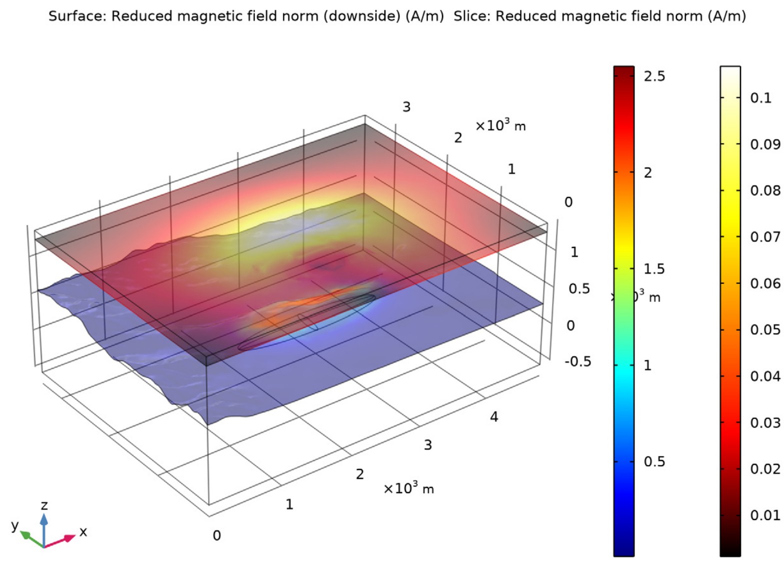


Figure 3: The slice color plot shows perturbations of magnetic intensity (norm of reduced magnetic field) at an altitude 1300 m. The boundary plot shows the same quantity on the ground.

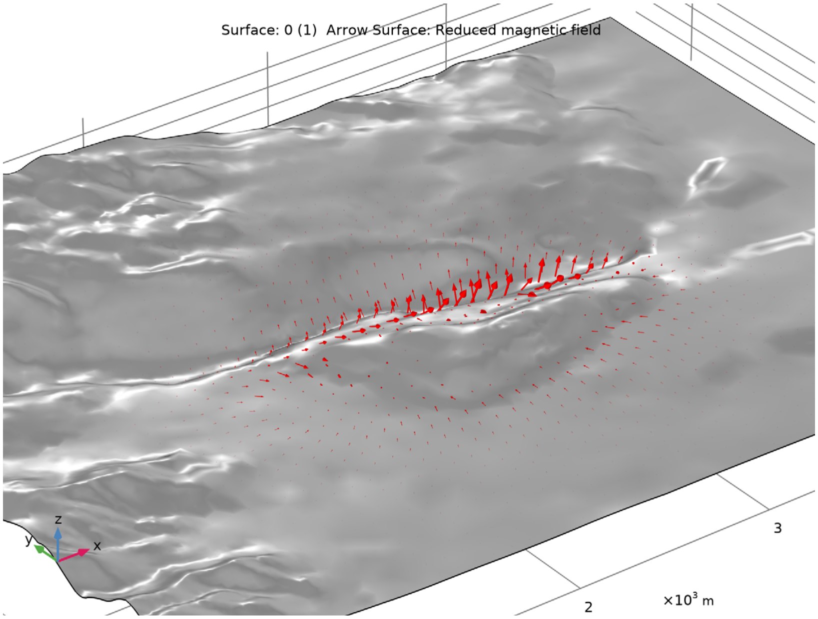


Figure 4: Arrows show the direction and relative magnitude of the reduced magnetic field on the Earth's surface.

Figure 5 shows a slice plot of the dimensionless ratio of remanent and induced magnetizations, $B_r/(\mu_0 M)$, inside the iron ore body, at an elevation of 200 m above sea level. This plot indicates that contributions of remanent and induced magnetizations to the magnetic anomaly can be comparable for realistic magnetic parameters of magnetite (Ref. 1).

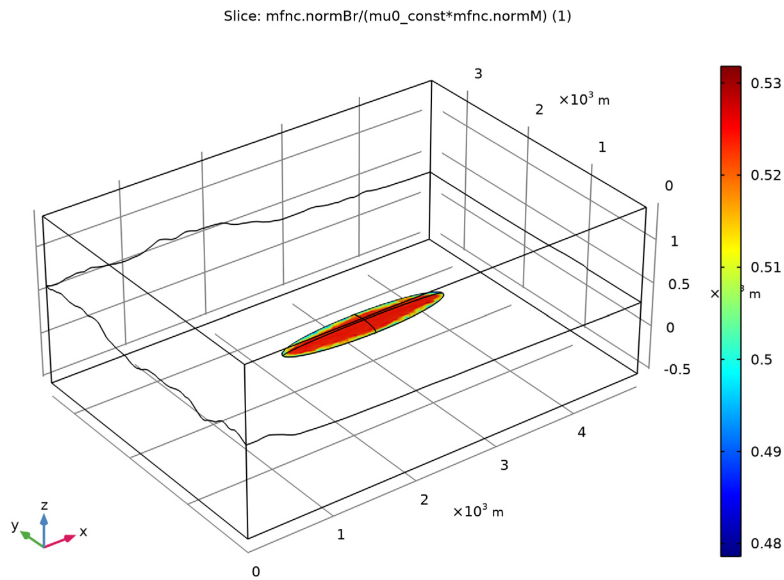


Figure 5: Slice plot of the dimensionless ratio of remanent to induced magnetizations inside the iron ore body at an elevation of 200 m above sea level. The two contributions to the magnetic anomaly are of comparable magnitude.

References

1. G. Kletetschka, P.J. Wasilewski, and P.T. Taylor, "Hematite vs. magnetite as the signature for planetary magnetic anomalies", *Physics of the Earth and Planetary Interiors*, vol. 119, pp. 259–267, 2000.

2. NOAA, National Centers for Environmental Information, https://www.ngdc.noaa.gov/geomag-web/

3. E.R. Force, "Eagle Mountain Mine – Geology of the former Kaiser Steel operation in Riverside county, California", *U.S. Geological Survey Open-File Report* 01-237.

4. Google Maps, https://maps.google.com/maps?f=q&hl=en&geocode=&q=33.865,-115.475&ie=UTF8&t=h&z=16&iwloc=addr

5. Data available from U.S. Geological Survey, National Elevation Data set, using The National Map Advanced Viewer, https://viewer.nationalmap.gov/advanced-viewer/

6. MICRODEM Home Page, http://www.usna.edu/Users/oceano/pguth/website/microdem/microdem.htm

Application Library path: `ACDC_Module/Magnetostatics/magnetic_prospecting`

Modeling Instructions

From the **File** menu, choose **New**.

NEW

In the **New** window, click **Model Wizard**.

MODEL WIZARD

1 In the **Model Wizard** window, click **3D**.

2 In the **Select Physics** tree, select **AC/DC>Magnetic Fields, No Currents>Magnetic Fields, No Currents (mfnc)**.

3 Click **Add**.

4 Click **Study**.

5 In the **Select Study** tree, select **General Studies>Stationary**.

6 Click **Done**.

GLOBAL DEFINITIONS

Parameters 1

1 In the **Model Builder** window, under **Global Definitions** click **Parameters 1**.

2 In the **Settings** window for **Parameters**, locate the **Parameters** section.

3 In the table, enter the following settings:

Name	Expression	Value	Description
mur_magn	3.5	3.5	Relative permittivity of magnetite
c_magn	0.25	0.25	Magnetite concentration in ore
Mr_magn	60[A/m]	60 A/m	Remanent magnetization of magnetite

Name	Expression	Value	Description
H0	48163[nT]/ mu0_const	38.327 A/m	Geomagnetic field
Incl	59.357[deg]	1.036 rad	Local inclination
Decl	12.275[deg]	0.21424 rad	Local declination

Variables 1

1 In the **Home** toolbar, click **Variables** and choose **Global Variables**.

2 In the **Settings** window for **Variables**, locate the **Variables** section.

3 In the table, enter the following settings:

Name	Expression	Unit	Description
Br_magn	Mr_magn*mu0_const	T	Remanent flux density of magnetite
Br	Br_magn*c_magn	T	Remanent flux density of ore
Gx	cos(Incl)*sin(Decl)		Geomagnetic field direction, x-component
Gy	cos(Incl)*cos(Decl)		Geomagnetic field direction, y-component
Gz	-sin(Incl)		Geomagnetic field direction, z-component
mur_ore	1+(mur_magn-1)*c_magn		Model for the relative permittivity of ore

GEOMETRY 1

The geometry obtained from the heightmap is available in an MPHBIN-file (COMSOL Multiphysics' native CAD file format) that can be imported in the geometry node.

Import 1 (imp1)

1 In the **Home** toolbar, click **Import**.

2 In the **Settings** window for **Import**, locate the **Import** section.

3 Click **Browse**.

4 Browse to the model's Application Libraries folder and double-click the file `magnetic_prospecting.mphbin`.

5 Click **Import**.

Create an ellipsoid that models the iron ore deposit.

Ellipsoid 1 (elp1)

1 In the **Geometry** toolbar, click **More Primitives** and choose **Ellipsoid**.

2 In the **Settings** window for **Ellipsoid**, locate the **Size and Shape** section.

3 In the **a-semiaxis** text field, type 1000.

4 In the **b-semiaxis** text field, type 200.

5 In the **c-semiaxis** text field, type 50.

6 Locate the **Position** section. In the **x** text field, type 2500.

7 In the **y** text field, type 1500.

8 In the **z** text field, type 200.

9 Click **Build All Objects**.

Activate the transparency to verify that the deposit is in the correct position.

10 Click the **Transparency** button in the **Graphics** toolbar.

MATERIALS

Use one material for the background (air and nonmagnetic rocks) and another for the iron ore. For the ore, you specify the relative permeability as a material property but add the remanent magnetization in the physics settings.

Background Material

1 In the **Model Builder** window, under **Component 1 (comp1)** right-click **Materials** and choose **Blank Material**.

2 In the **Settings** window for **Material**, locate the **Material Contents** section.

3 In the table, enter the following settings:

Property	Variable	Value	Unit	Property group
Relative permeability	mur_iso ; murii = mur_iso, murij = 0	1	1	Basic

4 Right-click **Material 1 (mat1)** and choose **Rename**.

5 In the **Rename Material** dialog box, type `Background Material` in the **New label** text field.

6 Click **OK**.

Ore

1 In the **Model Builder** window, right-click **Materials** and choose **Blank Material**.

2 Select Domain 3 only.

3 In the **Settings** window for **Material**, locate the **Material Contents** section.

4 In the table, enter the following settings:

Property	Variable	Value	Unit	Property group
Relative permeability	mur_iso ; murii = mur_iso, murij = 0	`mur_ore`	1	Basic

5 Right-click **Material 2 (mat2)** and choose **Rename**.

6 In the **Rename Material** dialog box, type `Ore` in the **New label** text field.

7 Click **OK**.

MAGNETIC FIELDS, NO CURRENTS (MFNC)

Magnetic Flux Conservation 2

1 In the **Model Builder** window, under **Component 1 (comp1)** right-click **Magnetic Fields, No Currents (mfnc)** and choose **Magnetic Flux Conservation**.

2 Select Domain 3 only.

3 In the **Settings** window for **Magnetic Flux Conservation**, locate the **Constitutive Relation B-H** section.

4 From the **Magnetization model** list, choose **Remanent flux density**.

5 From the $\|\mathbf{B}_\mathrm{r}\|$ list, choose **User defined**. In the associated text field, type `Br`.

6 Specify the **e** vector as

`Gx`	x
`Gy`	y
`Gz`	z

Specify the local geomagnetic field as the background field for the problem.

7 In the **Model Builder** window, click **Magnetic Fields, No Currents (mfnc)**.

8 In the **Settings** window for **Magnetic Fields, No Currents**, locate the **Background Magnetic Field** section.

9 From the **Solve for** list, choose **Reduced field**.

10 Specify the $\mathbf{H}_b$ vector as

H0*Gx	x
H0*Gy	y
H0*Gz	z

External Magnetic Flux Density 1

1 In the **Physics** toolbar, click **Boundaries** and choose **External Magnetic Flux Density**.

Apply **External Magnetic Flux Density** as the boundary condition instead of **Magnetic Insulation**. The former operates only on the relative field, while the latter sets a condition for the total field.

2 Select Boundaries 1–5, 7–9, 18, and 19 only.

STUDY 1

Disable the automatic generation of the default plots. You will manually create the plots after the solution.

1 In the **Model Builder** window, click **Study 1**.

2 In the **Settings** window for **Study**, locate the **Study Settings** section.

3 Clear the **Generate default plots** check box.

4 In the **Home** toolbar, click **Compute**.

RESULTS

In the **Model Builder** window, expand the **Results** node.

Create a second solution with a selection active only on the boundary corresponding to the Earth's surface. You will use this solution when plotting surface data.

Study 1/Solution 1 (2) (sol1)

1 In the **Model Builder** window, expand the **Results>Datasets** node.

2 Right-click **Results>Datasets** and choose **Solution**.

Selection

1 In the **Results** toolbar, click **Attributes** and choose **Selection**.

2 In the **Settings** window for **Selection**, locate the **Geometric Entity Selection** section.

3 From the **Geometric entity level** list, choose **Boundary**.

4 Select Boundary 6 only.

3D Plot Group 1

In the **Results** toolbar, click **3D Plot Group**.

Surface 1

1 Right-click **3D Plot Group 1** and choose **Surface**.

2 In the **Settings** window for **Surface**, locate the **Data** section.

3 From the **Dataset** list, choose **Study 1/Solution 1 (2) (sol1)**.

4 Click **Replace Expression** in the upper-right corner of the **Expression** section. From the menu, choose **Component 1 (comp1)>Magnetic Fields, No Currents>Magnetic>mfnc.normredH - Reduced magnetic field norm - A/m**.

5 Locate the **Expression** section. In the **Expression** text field, type `down(mfnc.normredH)`.

6 Select the **Description** check box.

7 In the associated text field, type `Reduced magnetic field norm (downside)`.

The `down()` operator indicates in this case that the value on the ground side should be used, i.e. not the default mean of the values on each side.

8 In the **3D Plot Group 1** toolbar, click **Plot**.

3D Plot Group 1

The plot of the relative magnetic field at the altitute of 1300 m can be obtained with a simple slice plot.

Slice 1

1 In the **Model Builder** window, right-click **3D Plot Group 1** and choose **Slice**.

2 In the **Settings** window for **Slice**, locate the **Plane Data** section.

3 From the **Plane** list, choose **xy-planes**.

4 From the **Entry method** list, choose **Coordinates**.

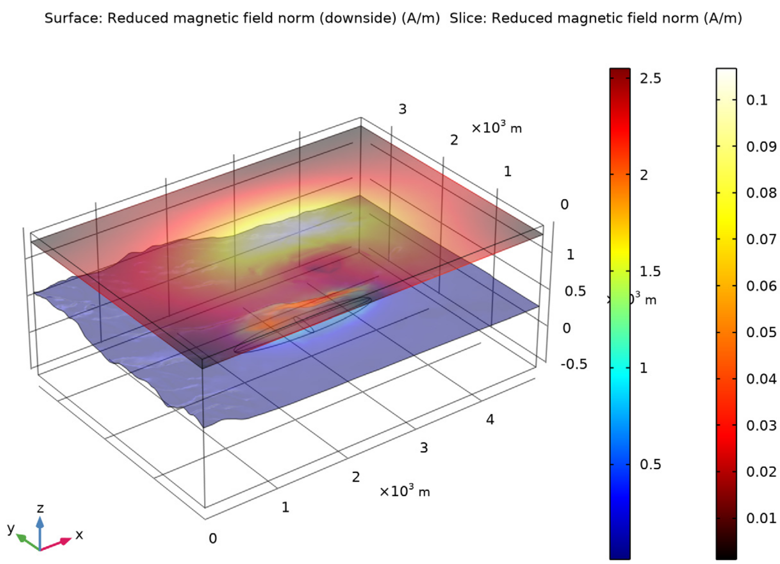
5 In the **z-coordinates** text field, type `1300`.

6 Click **Replace Expression** in the upper-right corner of the **Expression** section. From the menu, choose **Component 1 (comp1)>Magnetic Fields, No Currents>Magnetic>mfnc.normredH - Reduced magnetic field norm - A/m**.

7 Locate the **Coloring and Style** section. From the **Color table** list, choose **Thermal**.

8 In the **3D Plot Group 1** toolbar, click **Plot**.

The plot shows the alteration of the geomagnetic field due to the presence of a magnetized material. Note how the relative magnetic field intensity at ground level is about 20 times larger than at 1300 m.



9 Click the **Transparency** button in the **Graphics** toolbar to return to the default state.

Next, add another plot group to visualize the vector field.

3D Plot Group 2

In the **Home** toolbar, click **Add Plot Group** and choose **3D Plot Group**.

Add a uniform gray surface representing the ground.

Surface 1

Right-click **3D Plot Group 2** and choose **Surface**.

3D Plot Group 2

1 In the **Settings** window for **3D Plot Group**, locate the **Data** section.

2 From the **Dataset** list, choose **Study 1/Solution 1 (2) (sol1)**.

Surface 1

1 In the **Model Builder** window, click **Surface 1**.

2 In the **Settings** window for **Surface**, locate the **Expression** section.

3 In the **Expression** text field, type 0.

4 Locate the **Coloring and Style** section. From the **Coloring** list, choose **Uniform**.

5 From the **Color** list, choose **Gray**.

3D Plot Group 2

Now plot the relative magnetic field at ground level.

Arrow Surface 1

1 In the **Model Builder** window, right-click **3D Plot Group 2** and choose **Arrow Surface**.

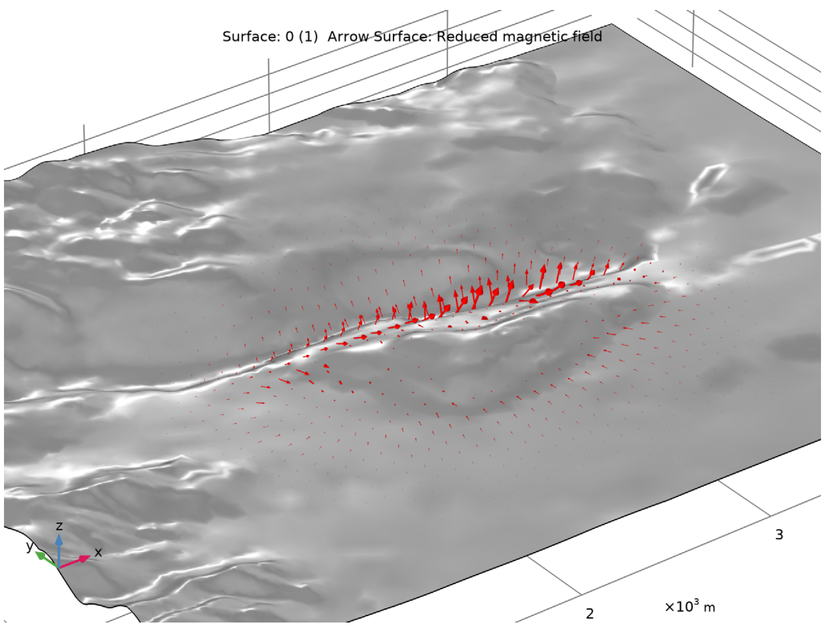
2 In the **Settings** window for **Arrow Surface**, click **Replace Expression** in the upper-right corner of the **Expression** section. From the menu, choose **Component 1 (comp1)> Magnetic Fields, No Currents>Magnetic>mfnc.redHx,...,mfnc.redHz - Reduced magnetic field**.

3 Locate the **Arrow Positioning** section. In the **Number of arrows** text field, type 2000.

4 In the **3D Plot Group 2** toolbar, click **Plot**.

5 Click the **Zoom In** button in the **Graphics** toolbar.

The plot shows the vector field for the magnetic field perturbation caused by the iron ore deposit.



Finally, plot the ratio between the remanent flux density and the induced magnetization.

3D Plot Group 3

In the **Home** toolbar, click **Add Plot Group** and choose **3D Plot Group**.

Slice 1

1 Right-click **3D Plot Group 3** and choose **Slice**.

2 In the **Settings** window for **Slice**, locate the **Plane Data** section.

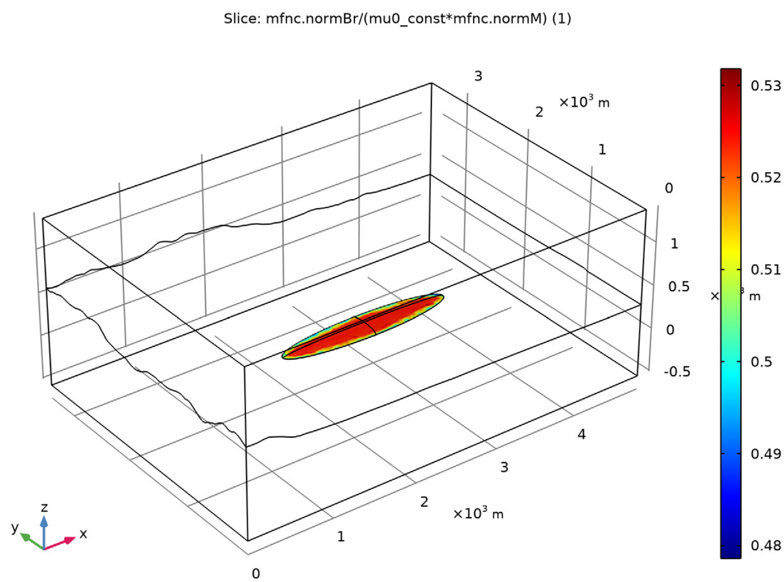
3 From the **Plane** list, choose **xy-planes**.

4 From the **Entry method** list, choose **Coordinates**.

5 In the **z-coordinates** text field, type `200`.

6 Locate the **Expression** section. In the **Expression** text field, type `mfnc.normBr/(mu0_const*mfnc.normM)`.

7 In the **3D Plot Group 3** toolbar, click **Plot**.



The plot shows that the two contributions are comparable in magnitude.

8 Click the **Go to Default View** button in the **Graphics** toolbar.