

# Computing Capacitance

A capacitor, in its simplest form, is a two terminal electrical device that stores electric energy when a voltage is applied across the terminals. The stored electric energy is proportional to the applied voltage squared and is determined by the capacitance of the device. This example introduces a model of a simple capacitor. The electric field and device capacitance are solved for under electrostatic conditions.

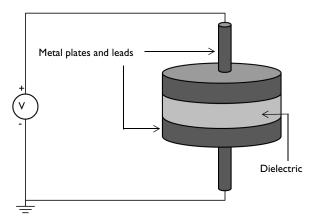


Figure 1: A simple capacitor composed of a disk of dielectric with metal plates on either side, and lead wires.

# Model Definition

The capacitor being modeled is shown in Figure 1. Two metal disks, with leads, are separated by a disk of dielectric material. Since there can be significant fringing fields around the capacitor plates, an air volume is included in the model. The size of this air volume truncates the modeling space. In actuality, the fringing electric fields extend to infinity, but drop off in proportion to the inverse cube of the distance. They rapidly become small enough to be considered numerically insignificant. Here, it is assumed that the air volume is large enough to accurately capture the fringing fields, this could be checked by increasing the air volume size an comparing the results.

Under the assumption of electrostatic conditions, the entire surface of each electrode must be at the same potential, otherwise current would flow through these conductors. The air and dielectric are assumed to be perfect insulators. The quantity of interest is the electric

potential profile in the air and in the dielectric — it is not necessary to solve for the potential in the electrode, because it is constant. There are two approaches that can be followed:

- apply the **Terminal** domain feature on the electrodes, or
- remove the electrode domains from the selection of the physics interface and apply appropriate boundary conditions (such as Ground or boundary Terminal) at the interface with the air or dielectric.

This model serves as an example of the second approach.

## Results and Discussion

The electric field is plotted in Figure 2. The field is relatively uniform between the plates, but some strong variations are seen at the edges of the plates. The fringing fields are observed to extend a small distance away from the capacitor. The capacitance of the device is evaluated to be approximately 43 pF. In Figure 3, a slice plot of the electric potential shows that there is an equipotential surface exactly midway between the capacitor plates.

Surface: Electric potential (V) Slice: Electric field norm (V/m) Arrow Volume: Electric field

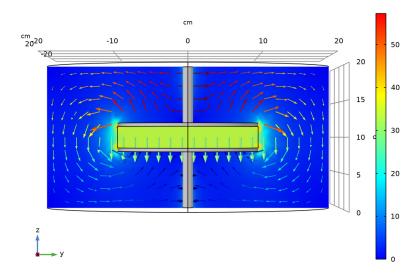


Figure 2: The electric field strength in the dielectric and air domain surrounding the capacitor.

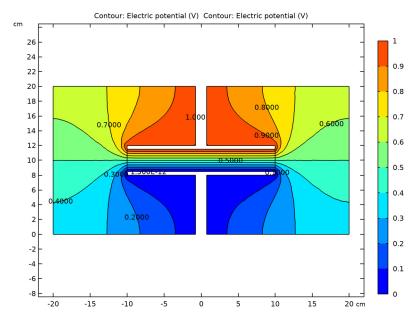


Figure 3: Contours of the voltage field at a slice through the center of the capacitor.

**Application Library path:** ACDC\_Module/Capacitive\_Devices/capacitor\_dc

# Modeling Instructions

From the File menu, choose New.

#### NEW

In the New window, click Model Wizard.

# MODEL WIZARD

- I In the Model Wizard window, click **3D**.
- 2 In the Select Physics tree, select AC/DC>Electric Fields and Currents>Electrostatics (es).
- 3 Click Add.
- 4 Click Study.

- 5 In the Select Study tree, select General Studies>Stationary.
- 6 Click M Done.

#### GEOMETRY I

- I In the Model Builder window, under Component I (compl) click Geometry I.
- 2 In the Settings window for Geometry, locate the Units section.
- **3** From the **Length unit** list, choose **cm**.

First, create a cylinder for the model domain.

Cylinder I (cyll)

- I In the Geometry toolbar, click Cylinder.
- 2 In the Settings window for Cylinder, locate the Size and Shape section.
- 3 In the Radius text field, type 20.
- 4 In the Height text field, type 20.
- 5 Click | Build Selected.

Choose wireframe rendering to get a better view of the interior parts.

6 Click the Wireframe Rendering button in the Graphics toolbar.

Then, add a cylinder for the disc of dielectric with the two metal plates.

Cylinder 2 (cyl2)

- I In the Geometry toolbar, click ( Cylinder.
- 2 In the Settings window for Cylinder, locate the Size and Shape section.
- 3 In the Radius text field, type 10.
- 4 In the Height text field, type 4.
- **5** Locate the **Position** section. In the **z** text field, type **8**.
- 6 Click to expand the Layers section. In the table, enter the following settings:

Layer name	Thickness (cm)
Layer 1	5[mm]

- 7 Clear the Layers on side check box.
- 8 Select the Layers on bottom check box.
- **9** Select the Layers on top check box.
- 10 Click Build Selected.

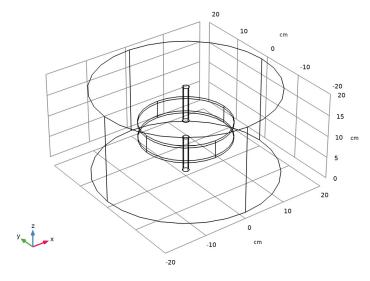
Finish the geometry by adding two cylinders for the leads.

Cylinder 3 (cyl3)

- I In the **Geometry** toolbar, click **Cylinder**.
- 2 In the Settings window for Cylinder, locate the Size and Shape section.
- 3 In the Radius text field, type 0.75.
- 4 In the Height text field, type 8.

Cylinder 4 (cyl4)

- I Right-click Cylinder 3 (cyl3) and choose Duplicate.
- 2 In the Settings window for Cylinder, locate the Position section.
- 3 In the z text field, type 12.
- 4 Click **Build All Objects**.



The result should look like the image above.

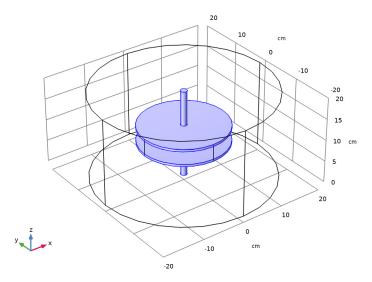
#### DEFINITIONS

The model is composed of a disc of dielectric material with metal plates on either side and two lead wires. Create a set of selections to use when setting up the physics. First, create a selection for the metal domains.

Metal

- I In the **Definitions** toolbar, click **\( \bigcap\_{\bigcap} \) Explicit**.
- 2 In the Settings window for Explicit, type Metal in the Label text field.

**3** Select Domains 2 and 4–6 only.

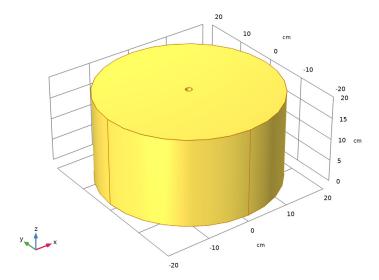


Add a **Complement** selection for the insulators.

# Insulators

- I In the **Definitions** toolbar, click **\( \frac{1}{2} \) Complement**.
- 2 In the Settings window for Complement, type Insulators in the Label text field.
- 3 Locate the Input Entities section. Under Selections to invert, click + Add.
- 4 In the Add dialog box, select Metal in the Selections to invert list.

# 5 Click OK.

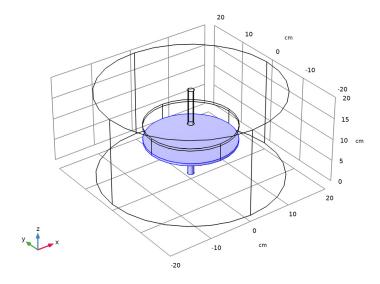


Add a selection for the ground boundaries, and the terminal boundaries.

# Ground

- I In the **Definitions** toolbar, click **\( \big|\_{\bigsymbol{a}} \) Explicit**.
- 2 In the Settings window for Explicit, type Ground in the Label text field.
- 3 Select Domains 2 and 5 only.

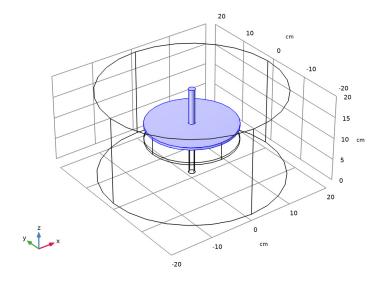
**4** Locate the **Output Entities** section. From the **Output entities** list, choose **Adjacent boundaries**.



# Terminal

- I In the **Definitions** toolbar, click **\( \bigcap\_{\text{a}} \) Explicit**.
- 2 In the Settings window for Explicit, type Terminal in the Label text field.
- **3** Select Domains 4 and 6 only.

4 Locate the Output Entities section. From the Output entities list, choose Adjacent boundaries.

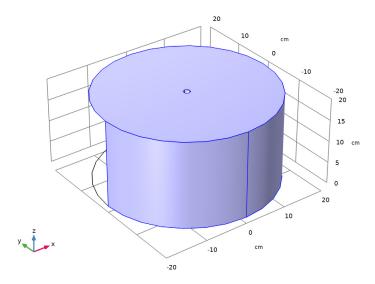


To get a better view, hide some of the boundaries. Begin by selecting the **Electrostatics** interface, then add a Hide node.

Hide for Physics 1

- I In the Model Builder window, right-click View I and choose Hide for Physics.
- 2 In the Settings window for Hide for Physics, locate the Geometric Entity Selection section.
- 3 From the Geometric entity level list, choose Boundary.

4 Select Boundaries 1, 4, and 23 only.



#### ELECTROSTATICS (ES)

- I In the Model Builder window, under Component I (compl) click Electrostatics (es).
- 2 In the Settings window for Electrostatics, locate the Domain Selection section.
- 3 From the Selection list, choose Insulators.

#### Ground I

- I In the Physics toolbar, click **Boundaries** and choose **Ground**.
- 2 In the Settings window for Ground, locate the Boundary Selection section.
- 3 From the Selection list, choose Ground.

#### Terminal I

- I In the Physics toolbar, click **Boundaries** and choose **Terminal**.
- 2 In the Settings window for Terminal, locate the Boundary Selection section.
- **3** From the **Selection** list, choose **Terminal**.
- 4 Locate the Terminal section. From the Terminal type list, choose Voltage.

Next, assign material properties to the model. Begin by specifying Air for all domains.

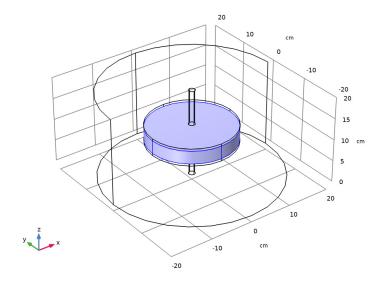
## ADD MATERIAL

I In the Home toolbar, click 👯 Add Material to open the Add Material window.

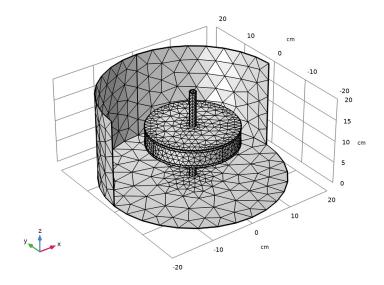
- 2 Go to the Add Material window.
- 3 In the tree, select Built-in>Air.
- 4 Click Add to Component in the window toolbar. Override the dielectric disc with glass (quartz).
- 5 In the tree, select Built-in>Glass (quartz).
- 6 Click Add to Component in the window toolbar.
- 7 In the Home toolbar, click **Add Material** to close the Add Material window.

# MATERIALS

Glass (quartz) (mat2) Select Domain 3 only.



# MESH I In the Model Builder window, under Component I (compl) right-click Mesh I and choose **Build All.**



#### STUDY I

- I In the Model Builder window, click Study I.
- 2 In the Settings window for Study, locate the Study Settings section.
- 3 Clear the Generate default plots check box.
- 4 In the Home toolbar, click **Compute**.

Add a solution and define a Selection for the metal parts. Domains that are excluded from the selection will be hidden in the corresponding plots.

5 In the Results toolbar, click More Datasets and choose Solution.

#### RESULTS

#### Selection

- I In the Results toolbar, click \( \frac{1}{2} \) Attributes and choose Selection.
- 2 In the Settings window for Selection, locate the Geometric Entity Selection section.
- 3 From the Geometric entity level list, choose Domain.
- 4 From the Selection list, choose Metal.

3D Plot Group 1

In the Results toolbar, click **a 3D Plot Group**.

The metal parts can be visualized by choosing **Uniform** for the coloring type. In this case the purpose of the plot is not to show a quantity, but to show a shape. The variable that the plot is based on is of no importance. Create a surface plot for this purpose.

## Surface I

- I Right-click 3D Plot Group I and choose Surface.
- 2 In the Settings window for Surface, locate the Data section.
- 3 From the Dataset list, choose Study I/Solution I (2) (soll).
- 4 Locate the Coloring and Style section. From the Coloring list, choose Uniform.
- 5 From the Color list, choose Gray.

3D Plot Group 1

Add a slice plot for the norm of the electric fields and an arrow plot for the electric fields.

#### Slice 1

- I In the Model Builder window, right-click 3D Plot Group I and choose Slice.
- 2 In the Settings window for Slice, click Replace Expression in the upper-right corner of the Expression section. From the menu, choose Component I (compl)>Electrostatics> Electric>es.normE Electric field norm V/m.
- 3 Locate the Plane Data section. In the Planes text field, type 1.
- 4 Locate the Coloring and Style section. From the Color table list, choose RainbowLight.

Arrow Volume 1

- I Right-click 3D Plot Group I and choose Arrow Volume.
- 2 In the Settings window for Arrow Volume, locate the Arrow Positioning section.
- 3 Find the x grid points subsection. In the Points text field, type 1.
- 4 Find the y grid points subsection. In the Points text field, type 24.
- 5 Find the z grid points subsection. In the Points text field, type 11.
- 6 Locate the Coloring and Style section. From the Arrow length list, choose Logarithmic.

Color Expression 1

- I Right-click Arrow Volume I and choose Color Expression.
- 2 In the Settings window for Color Expression, locate the Coloring and Style section.
- 3 Clear the Color legend check box.
- 4 Click the YZ Go to YZ View button in the Graphics toolbar.

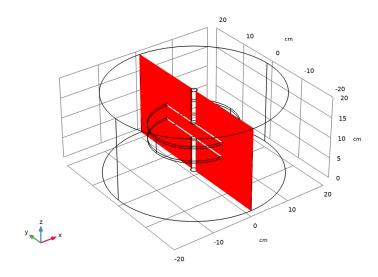
**5** Click the **Zoom Extents** button in the **Graphics** toolbar.

The plot should look like that in Figure 2.

Create a Cut Plane that may serve as a basis for 2D plots.

## Cut Plane I

- I In the Results toolbar, click Cut Plane.
- 2 In the Settings window for Cut Plane, click Plot.
- 3 Click the Go to Default View button in the Graphics toolbar.



On this dataset, base a **2D Plot Group** with contour plots for the electric potential.

# 2D Plot Group 2

In the **Results** toolbar, click 2D Plot Group.

#### Contour I

- I Right-click 2D Plot Group 2 and choose Contour.
- 2 In the Settings window for Contour, locate the Levels section.
- 3 From the Entry method list, choose Levels.
- 4 Click Range.
- 5 In the Range dialog box, type 0.1 in the Start text field.
- 6 In the Step text field, type 0.1.
- 7 In the **Stop** text field, type 0.9.

- 8 Click Replace.
- 9 In the Settings window for Contour, locate the Coloring and Style section.
- **10** From the Contour type list, choose Filled.
- II From the Color table list, choose RainbowLight.

#### Contour 2

- I In the Model Builder window, right-click 2D Plot Group 2 and choose Contour.
- 2 In the Settings window for Contour, locate the Levels section.
- 3 From the Entry method list, choose Levels.
- 4 Click Range.
- 5 In the Range dialog box, type 0 in the Start text field.
- 6 In the Step text field, type 0.1.
- 7 In the **Stop** text field, type 1.
- 8 Click Replace.
- 9 In the Settings window for Contour, locate the Coloring and Style section.
- 10 Select the Level labels check box.
- II From the Coloring list, choose Uniform.
- 12 From the Color list, choose Black.
- 13 Clear the Color legend check box.
- 14 In the 2D Plot Group 2 toolbar, click Plot.
- **15** Click the **Zoom Extents** button in the **Graphics** toolbar.

Compare the resulting plot with Figure 3.

The **Terminal** feature automatically computes the capacitance. The computed value is available in a global variable and can be accessed using a Global Evaluation.

## Global Evaluation 1

- I In the Results toolbar, click (8.5) Global Evaluation.
- 2 In the Settings window for Global Evaluation, click Replace Expression in the upper-right corner of the Expressions section. From the menu, choose Component I (compl)> Electrostatics>Terminals>es.CII - Maxwell capacitance - F.
- 3 Click **= Evaluate**.

# TABLE

I Go to the Table window.

The computed capacitance should be around 43 pF.