Fabry-Perot Cavity

Introduction

A Fabry-Perot cavity is a slab of material of higher refractive index than its surroundings, as shown in Figure 1. Such a structure can act as a resonator at certain frequencies. Although such solutions can be found analytically, this model demonstrates how to find the resonant frequencies and the Q-factor using a COMSOL Multiphysics simulation.

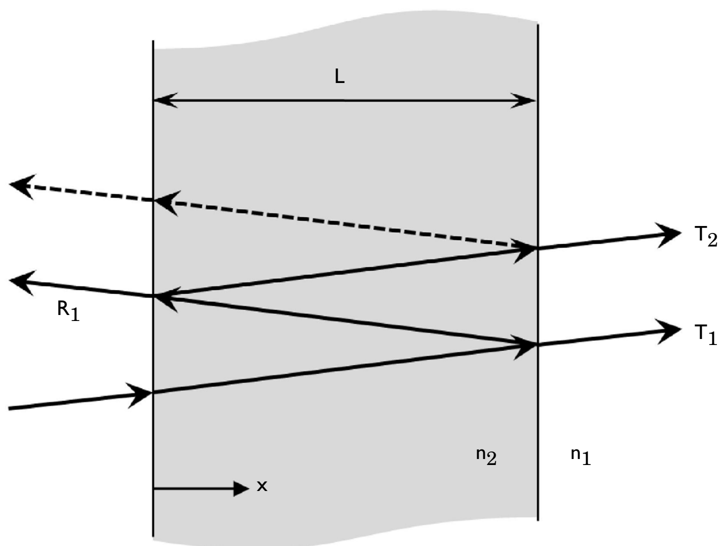


Figure 1: A Fabry-Perot cavity. An electromagnetic wave, traveling at normal incidence, is partially reflected and transmitted at each interface between differing dielectrics. When the length, L, is an integer fraction of the wavelength, this acts as a resonator.

Model Definition

The geometry is a slab of a material with refractive index higher than the surrounding medium. It is assumed that the mode of interest is polarized with the electric field out of the plane, and that the wave vector of the mode of interest is parallel to the x-axis.

Because the mode of interest propagates in the x direction, the model's y-dimension is arbitrary. The model space is composed of three types of domains:

- a central domain of unit width and refractive index $n = 4$
- domains of $n = 1$ on both sides of the central domain
- two outer perfectly-matched-layer (PML) domains

The PMLs absorb without reflection any incoming evanescent or propagating wave. The boundary condition on the top and bottom edges is perfect magnetic conductor (PMC), which implies that the solution is mirror symmetric about those planes. A scattering boundary condition (SBC) applies at the left and right sides. This boundary condition is only perfectly transparent to an incoming plane wave, and partially reflects any other component. Using a PML backed by an SBC reduces any artificial reflections due to the boundary conditions.

EIGENFREQUENCY MODEL

First, solve the model as an eigenvalue problem, which requires that you specify the number of eigenfrequencies to solve for and the frequency range around which to search. The PML and the SBC make this problem nonlinear by introducing a damping term that depends upon the frequency. This, in turn, requires that you specify an eigenvalue transform point, which only needs to be within an order of magnitude or so of the expected resonant frequency.

FREQUENCY-DOMAIN MODEL

The approach described above has several drawbacks. First, the results must be manually examined to identify the spurious, nonphysical, modes. Second, it requires solving a nonlinear eigenvalue problem using a memory-intensive direct solver. For a 2D model, this is not a computational hurdle, but for structurally complex 3D cases, where far more mesh elements are required, it can be a concern. The convergence rate and solution time of the eigenvalue solver also depend on the choice of starting guess at the resonant frequency, the number of modes requested as output, and the spacing between these modes.

An alternative approach to determining the resonant frequency and Q-factor is to recast this as a frequency-domain model and to excite the structure over a range of frequencies. The excitation should be as isotropic as possible, so that it can excite all possible modes. The present example uses a line current condition applied to a point. The model is run in the frequency domain over a range covering the expected resonances.

Results and Discussion

EIGENFREQUENCY MODEL

The results of the eigenvalue analysis are plotted in Figure 2, and the Q-factor is reported in Table 1. The settings for the eigenfrequency solver have been adjusted so only physical modes are returned. However, sometimes also nonphysical modes can be returned. Those modes represent numerical modes — that is, solutions to the numerical eigenvalue problem that have no physically meaningful interpretation. These nonphysical eigenmodes can be identified in two ways:

- A visual examination of the field solutions can reveal that some modes exist purely in the PML regions. This is, however, a manual task, and it is not always obvious that a mode is indeed physical.
- Alternatively, it is possible to examine the Q-factor for each mode. A nonphysical mode has a Q-factor less than 1/2.

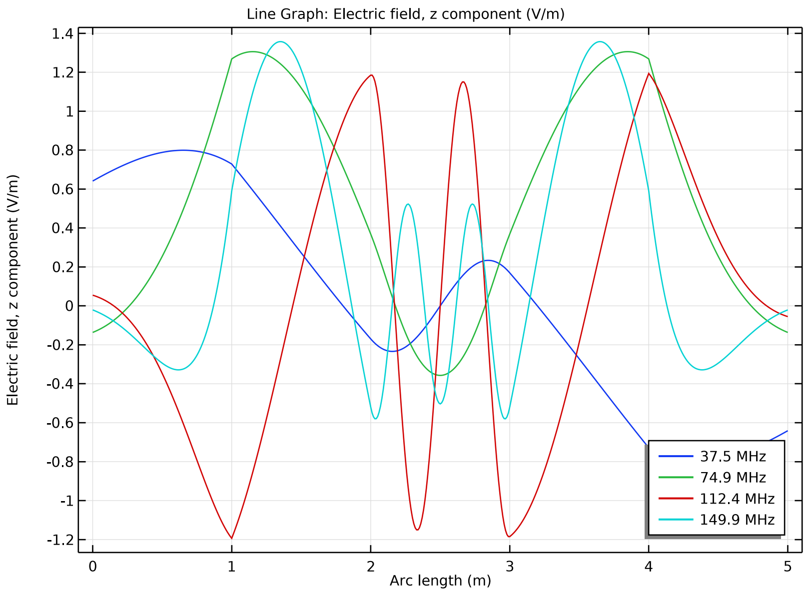


Figure 2: The electric field across the entire modeling domain for various solutions to the eigenvalue problem.

TABLE 1: COMPUTED RESONANT FREQUENCY AND Q-FACTORS.

Resonant frequency (MHz)	Q-factor	Note
37.5	3.08	
74.9	6.15	
112.4	9.22	
149.9	12.3	

FREQUENCY-DOMAIN MODEL

Figure 3 plots the results of the frequency-domain analysis. The total energy density is monitored at a point inside the cavity region. The peaks in this graph correspond to the resonant frequencies, f_0, and the Q-factor can be computed as $Q = f_0/\Delta f$, where Δf is the full width at half maximum. This method is an alternative approach to finding the resonant frequencies and Q-factors that requires less memory, an important concern for 3D models. This approach entirely avoids the problem of finding and eliminating spurious modes. The only limitations are that some care must be taken to ensure that the desired modes are indeed excited and that evaluation of the Q-factor requires manual postprocessing of the data.

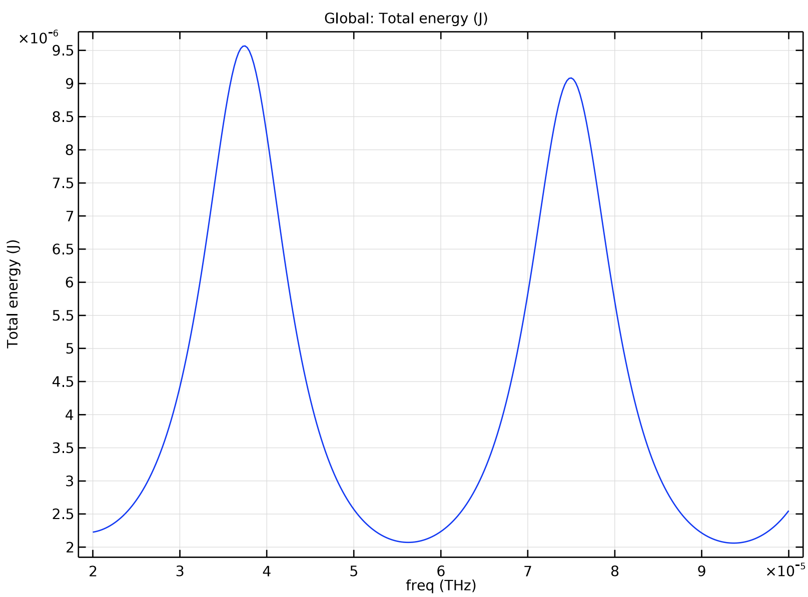


Figure 3: Plot of energy density within the cavity over a range of frequencies. This plot can be used to find the resonant frequencies and Q-factors.

Application Library path: `Wave_Optics_Module/Verification_Examples/fabry_perot`

Modeling Instructions

From the **File** menu, choose **New**.

NEW

In the **New** window, click **Model Wizard**.

MODEL WIZARD

1 In the **Model Wizard** window, click **2D**.

2 In the **Select Physics** tree, select **Optics>Wave Optics>Electromagnetic Waves, Frequency Domain (ewfd)**.

3 Click **Add**.

4 Click **Study**.

5 In the **Select Study** tree, select **General Studies>Eigenfrequency**.

6 Click **Done**.

GLOBAL DEFINITIONS

Parameters 1

1 In the **Model Builder** window, under **Global Definitions** click **Parameters 1**.

2 In the **Settings** window for **Parameters**, locate the **Parameters** section.

3 In the table, enter the following settings:

Name	Expression	Value	Description
`f_min`	`20[MHz]`	2E7 Hz	`Minimum frequency in sweep`
`f_max`	`100[MHz]`	1E8 Hz	`Maximum frequency in sweep`

GEOMETRY 1

Rectangle 1 (r1)

1 In the **Geometry** toolbar, click **Rectangle**.

2 In the **Settings** window for **Rectangle**, locate the **Size and Shape** section.

3 In the **Height** text field, type `0.2`.

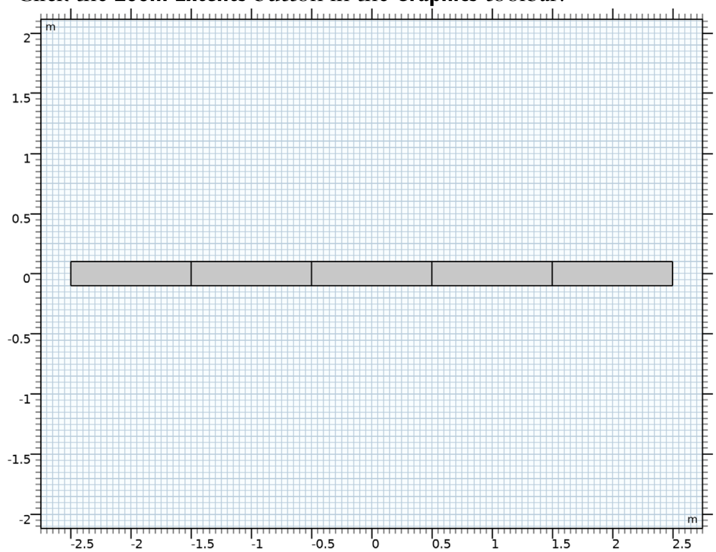
4 Locate the **Position** section. From the **Base** list, choose **Center**.

5 In the **x** text field, type `-2`.

6 Click **Build Selected**.

Array 1 (arr1)

1 In the **Geometry** toolbar, click **Transforms** and choose **Array**.
2 Select the object **r1** only.
3 In the **Settings** window for **Array**, locate the **Size** section.
4 From the **Array type** list, choose **Linear**.
5 In the **Size** text field, type 5.
6 Locate the **Displacement** section. In the **x** text field, type 1.
7 Click **Build All Objects**.
8 Click the **Zoom Extents** button in the **Graphics** toolbar.



ELECTROMAGNETIC WAVES, FREQUENCY DOMAIN (EWFD)

Now set up the physics. The model is based on differences in refractive indices and you solve for the E-field's out-of-plane component.

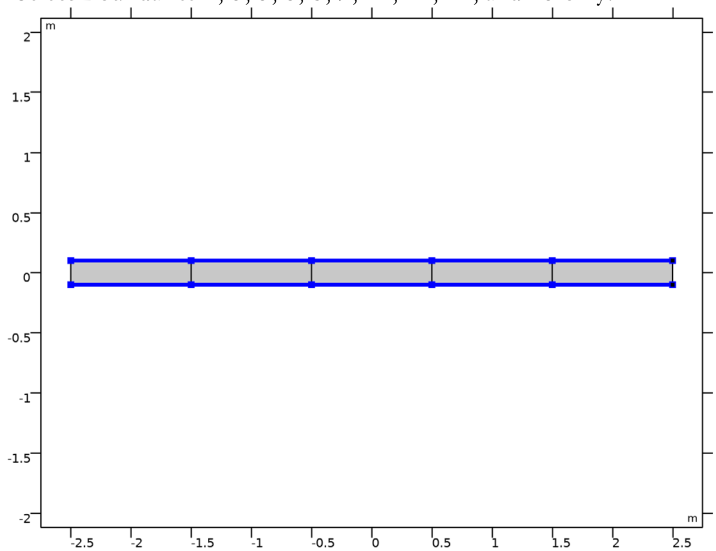
1 In the **Model Builder** window, under **Component 1 (comp1)** click **Electromagnetic Waves, Frequency Domain (ewfd)**.
2 In the **Settings** window for **Electromagnetic Waves, Frequency Domain**, locate the **Components** section.
3 From the **Electric field components solved for** list, choose **Out-of-plane vector**.

Perfect Magnetic Conductor 1

Assign a PMC condition on the top and bottom edges.

1 In the **Physics** toolbar, click **Boundaries** and choose **Perfect Magnetic Conductor**.

2 Select Boundaries 2, 3, 5, 6, 8, 9, 11, 12, 14, and 15 only.

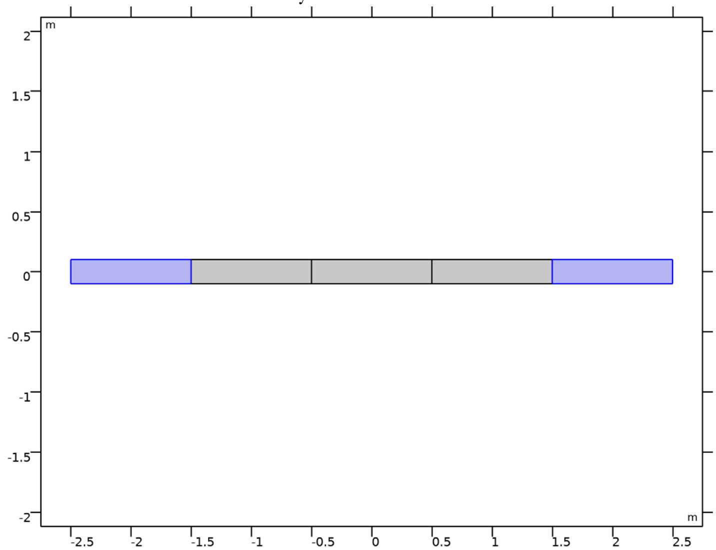


DEFINITIONS

Perfectly Matched Layer 1 (pml1)

1 In the **Definitions** toolbar, click **Perfectly Matched Layer**.

2 Select Domains 1 and 5 only.

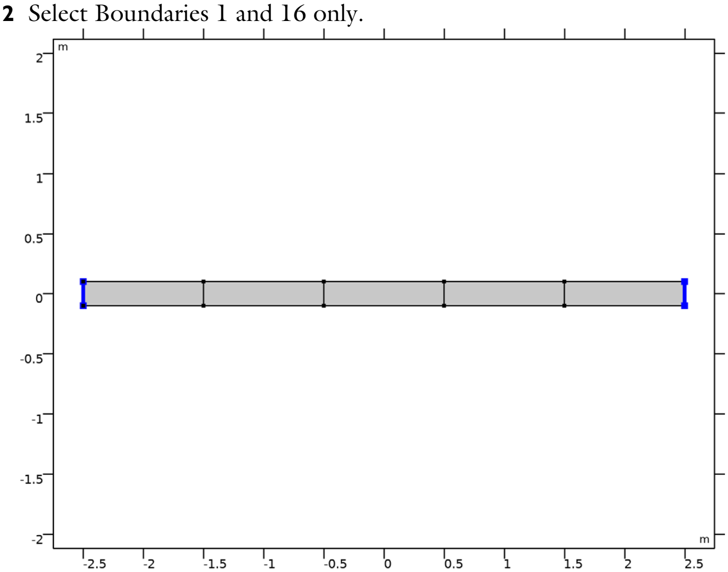


ELECTROMAGNETIC WAVES, FREQUENCY DOMAIN (EWFD)

Scattering Boundary Condition 1

1 In the **Physics** toolbar, click **Boundaries** and choose **Scattering Boundary Condition**.

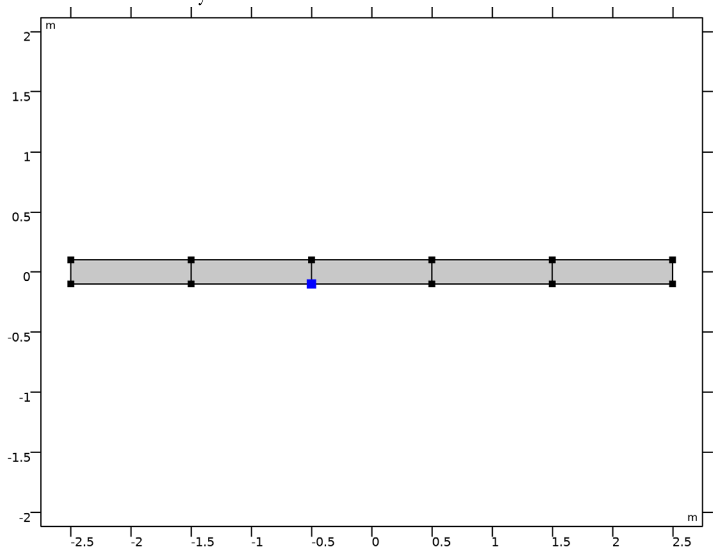
2 Select Boundaries 1 and 16 only.



Line Current (Out-of-Plane) 1

1 In the **Physics** toolbar, click **Points** and choose **Line Current (Out-of-Plane)**.

2 Select Point 5 only.



3 In the **Settings** window for **Line Current (Out-of-Plane)**, locate the **Line Current (Out-of-Plane)** section.

4 In the I_0 text field, type 1.

MATERIALS

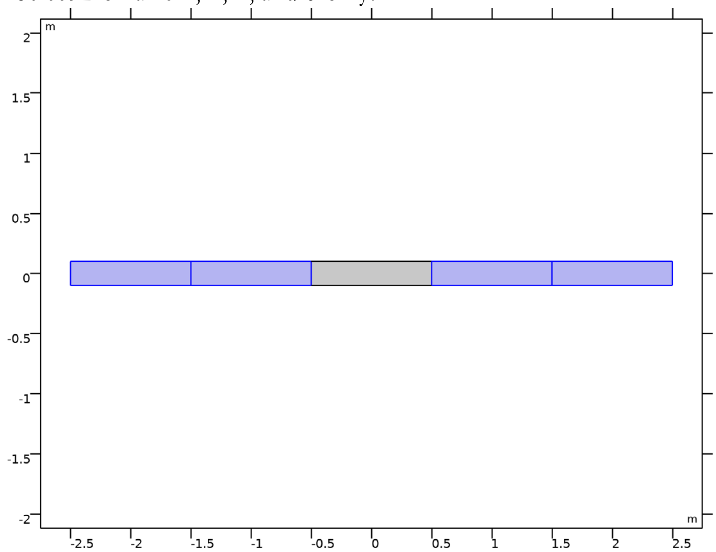
Now, specify the material properties. First, define the medium surrounding the slab.

Material 1 (mat1)

1 In the **Model Builder** window, under **Component 1 (comp1)** right-click **Materials** and choose **Blank Material**.

2 In the **Settings** window for **Material**, type n=1 in the **Label** text field.

3 Select Domains 1, 2, 4, and 5 only.



4 Locate the **Material Contents** section. In the table, enter the following settings:

Property	**Variable**	**Value**	**Unit**	**Property group**
Refractive index, real part	n_iso ; nii = n_iso, nij = 0	1	1	Refractive index

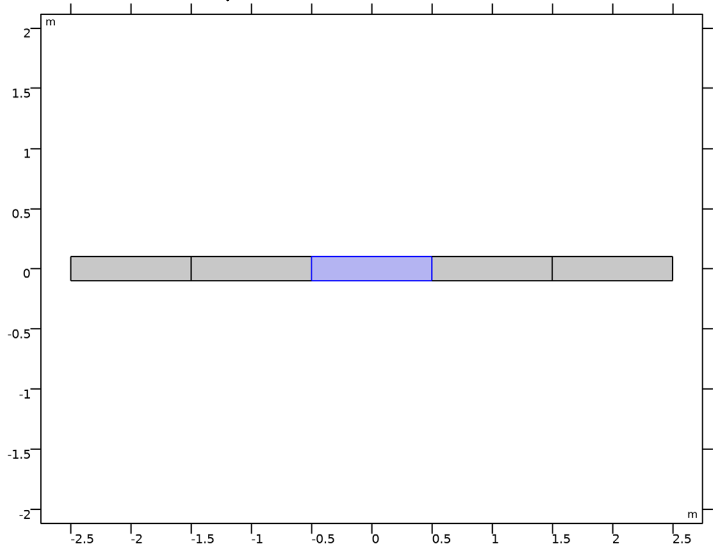
The refractive index of the slab is 4.

Material 2 (mat2)

1 Right-click **Materials** and choose **Blank Material**.

2 In the **Settings** window for **Material**, type n=4 in the **Label** text field.

3 Select Domain 3 only.



4 Locate the **Material Contents** section. In the table, enter the following settings:

Property	Variable	Value	Unit	Property group
Refractive index, real part	n_iso ; nii = n_iso, nij = 0	4	1	Refractive index

MESH 1

Free Triangular 1

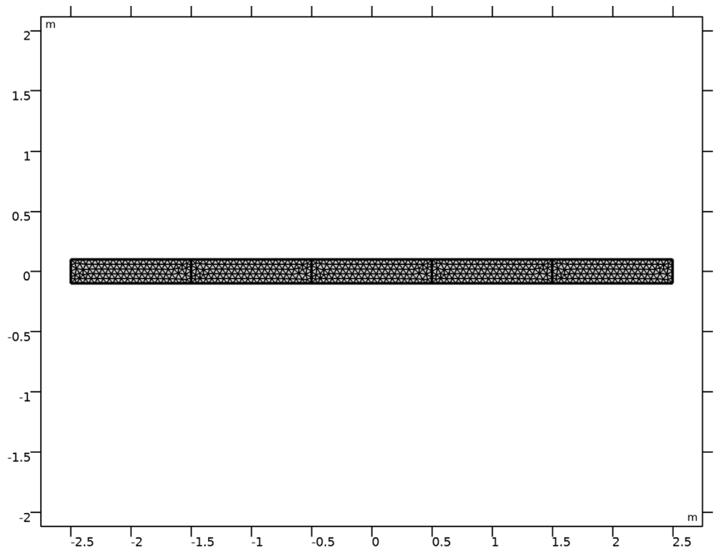
In the **Model Builder** window, under **Component 1 (comp1)** right-click **Mesh 1** and choose **Free Triangular**.

Size

1 In the **Settings** window for **Size**, locate the **Element Size** section.

2 From the **Predefined** list, choose **Extremely fine**.

3 Click **Build All**.



STUDY 1

Step 1: Eigenfrequency

1 In the **Model Builder** window, under **Study 1** click **Step 1: Eigenfrequency**.

2 In the **Settings** window for **Eigenfrequency**, locate the **Study Settings** section.

3 Select the **Desired number of eigenfrequencies** check box.

4 In the associated text field, type 8.

5 Find the **Search region** subsection. From the **Unit** list, choose **MHz**.

6 In the **Search for eigenfrequencies around** text field, type 10.

7 From the **Eigenfrequency search method around shift** list, choose **Larger real part**. This will search for eigenfrequencies with a larger real part than the **Search for eigenfrequencies around** value (10 MHz). Thus, the eigenfrequency solver should not return low-frequency spurious modes.

8 In the **Model Builder** window, click **Study 1**.

9 In the **Settings** window for **Study**, locate the **Study Settings** section.

10 Clear the **Generate default plots** check box.

11 In the **Home** toolbar, click **Compute**.

RESULTS

Global Evaluation 1

1 In the **Results** toolbar, click **Global Evaluation**.

2 In the **Settings** window for **Global Evaluation**, click **Replace Expression** in the upper-right corner of the **Expressions** section. From the menu, choose **Component 1> Electromagnetic Waves, Frequency Domain>Global>ewfd.Qfactor - Quality factor**.

3 Click **Evaluate**.

TABLE

1 Go to the **Table** window.

Review the evaluated eigenfrequencies and Q-factors; Q-factors less than 0.5 correspond to unphysical eigenmodes.

RESULTS

1D Plot Group 1

1 In the **Results** toolbar, click **1D Plot Group**.

2 In the **Settings** window for **1D Plot Group**, locate the **Legend** section.

3 From the **Position** list, choose **Lower right**.

4 Locate the **Data** section. From the **Eigenfrequency selection** list, choose **Manual**.

5 In the **Eigenfrequency indices (1-8)** text field, type `1 2 3 4`, to plot the four modes with the lowest eigenfrequencies.

Line Graph 1

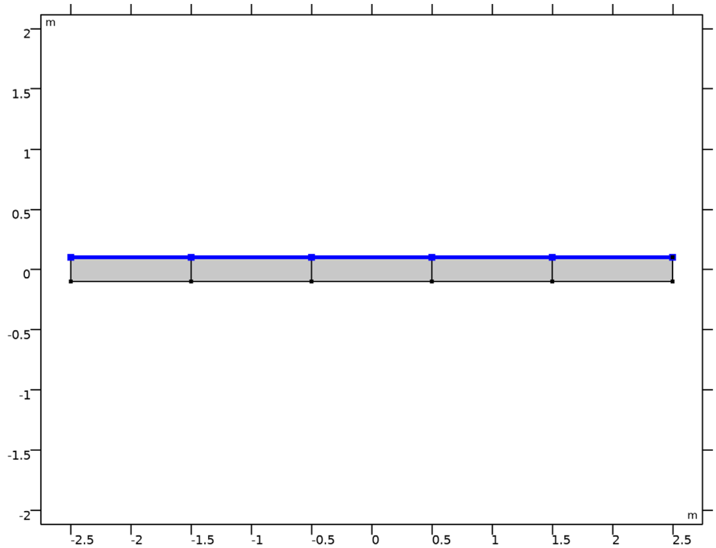
1 Right-click **1D Plot Group 1** and choose **Line Graph**.

2 In the **Settings** window for **Line Graph**, locate the **Selection** section.

3 Click **Paste Selection**.

4 In the **Paste Selection** dialog box, type `3, 6, 9, 12, 15` in the **Selection** text field.

5 Click **OK**.



6 In the **Settings** window for **Line Graph**, locate the **y-Axis Data** section.

7 In the **Expression** text field, type `Ez`.

8 Click to expand the **Legends** section. Select the **Show legends** check box.

9 From the **Legends** list, choose **Manual**.

10 In the table, enter the following settings:

Legends
`37.5 MHz`
`74.9 MHz`
`112.4 MHz`
`149.9 MHz`

11 In the **1D Plot Group 1** toolbar, click **Plot**. Compare the plot with that shown in Figure 2.

ROOT

Add a new study as an alternative approach to examine the Q-factor for each mode.

ADD STUDY

1 In the **Home** toolbar, click **Add Study** to open the **Add Study** window.

2 Go to the **Add Study** window.

3 Find the **Studies** subsection. In the **Select Study** tree, select **General Studies> Frequency Domain**.

4 Click **Add Study** in the window toolbar.

5 In the **Home** toolbar, click **Add Study** to close the **Add Study** window.

STUDY 2

Step 1: Frequency Domain

1 In the **Settings** window for **Frequency Domain**, locate the **Study Settings** section.

2 In the **Frequencies** text field, type `range(f_min,0.25[MHz],f_max)`.

3 In the **Home** toolbar, click **Compute**.

RESULTS

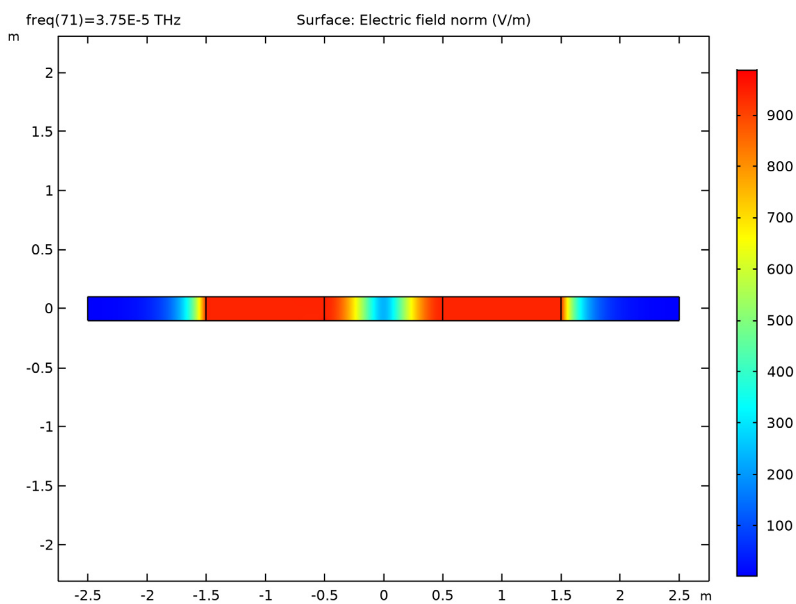
Electric Field (ewfd)

1 In the **Settings** window for **2D Plot Group**, locate the **Data** section.

2 From the **Parameter value (freq (THz))** list, choose **3.75E-5**.

3 In the **Electric Field (ewfd)** toolbar, click **Plot**.

4 Click the **Zoom Extents** button in the **Graphics** toolbar.



This is the E-field norm at 37.5 MHz.

1D Plot Group 3

Finally, reproduce the plot in Figure 3.

1 In the **Home** toolbar, click **Add Plot Group** and choose **1D Plot Group**.

2 In the **Settings** window for **1D Plot Group**, locate the **Data** section.

3 From the **Dataset** list, choose **Study 2/Solution 2 (sol2)**.

Global 1

1 Right-click **1D Plot Group 3** and choose **Global**.

2 In the **Settings** window for **Global**, locate the **y-Axis Data** section.

3 In the table, enter the following settings:

Expression	Unit	Description
`ewfd.intWe+ewfd.intWm`	`J`	`Total energy`

4 Click to expand the **Legends** section. Clear the **Show legends** check box.

5 In the **1D Plot Group 3** toolbar, click **Plot**.

Using the definition $Q = f_0/\Delta f$ you can use this plot to evaluate the Q-factor at each resonance.

Electric Field (ewfd)

Click the **Zoom Extents** button in the **Graphics** toolbar.