Created in COMSOL Multiphysics 5.5

Magnetic Lens

Introduction

Scanning electron microscopes take images of samples by scanning with a high-energy beam of electrons. The subsequent electron interactions produce signals such as secondary and back-scattered electrons that contain information about the sample surface topography. Electromagnetic lenses are used to focus this electron beam down to a spot about 10 nm wide on the sample surface.

Note: This application requires the AC/DC Module and the Particle Tracing Module.

Model Definition

Particles (electrons) are released from near the bottom boundary of the simulation space and pass through a collimator. This collimator can typically be adjusted to remove stray electrons. A simple DC coil produces an axial magnetic field. This rotationally symmetric, inhomogeneous magnetic field results in non-axial electrons experiencing a radial force causing them to spiral about the axis. As they begin to spiral, they have a larger velocity component perpendicular to the mainly axial magnetic field, therefore the radius of their spiral/helical path decreases. Thus, a parallel beam of electrons entering the lens converges to a point.

If the region in which the magnetic field acts upon the electrons is sufficiently small, this coil acts as a 'thin' convex lens and the thin lens expression holds.

MODEL EQUATIONS

A simple model is set up to test the magnetic force within the Charged Particle Tracing interface. The equations solved are the equation of motion of a charged particle in a magnetic field (Lorentz force):

$$\frac{\mathrm{d}}{\mathrm{d}t}(m\mathbf{v}) = q(\mathbf{v}\times\mathbf{B})$$

where q (SI unit: C) is the particle charge, $\mathbf{v}$ (SI unit: m/s) is the particle velocity, and $\mathbf{B}$ (SI unit: T) is the magnetic flux density. The total work done on a particle by a magnetic field is zero.

Results and Discussion

The magnetic flux density is plotted in Figure 1. The strength of the lens depends upon the coil configuration and current. The lenses within electron microscopes are generally very strong, in some cases focusing the electron beam within the lens itself.

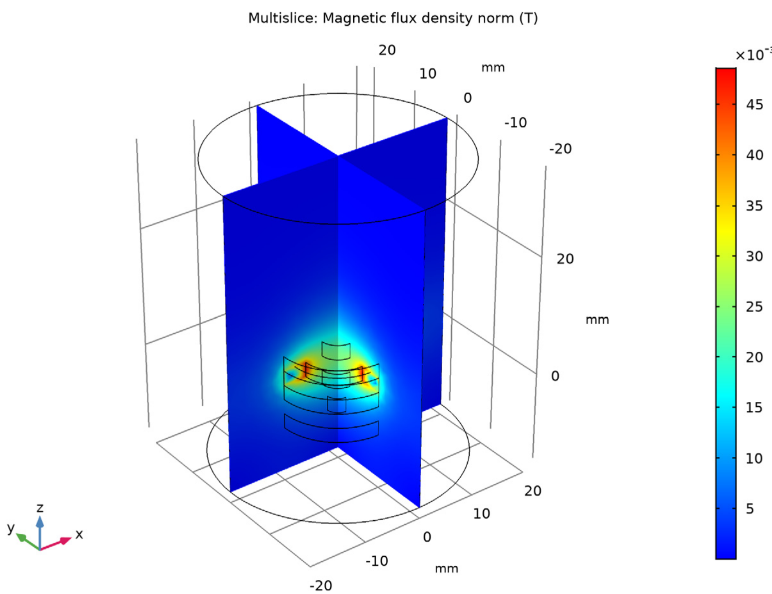

Figure 1: Plot of the magnetic flux density in the magnetic lens.

Figure 2 plots the electron trajectories as they travel through the coil. The electrons are focused at a point along the z-axis. The focal length is given by:

$$f = K\frac{V}{i^2}$$

where K is a constant based on the coil geometry and number of turns, V is the accelerating voltage and i is the coil current. The focal length increases with electron energy (that is, V) because their high velocity means they spend less time experiencing a force due the magnetic field. However, as the current increases so does the magnetic field strength, therefore the electrons spiral in tighter paths bringing the focal length closer.

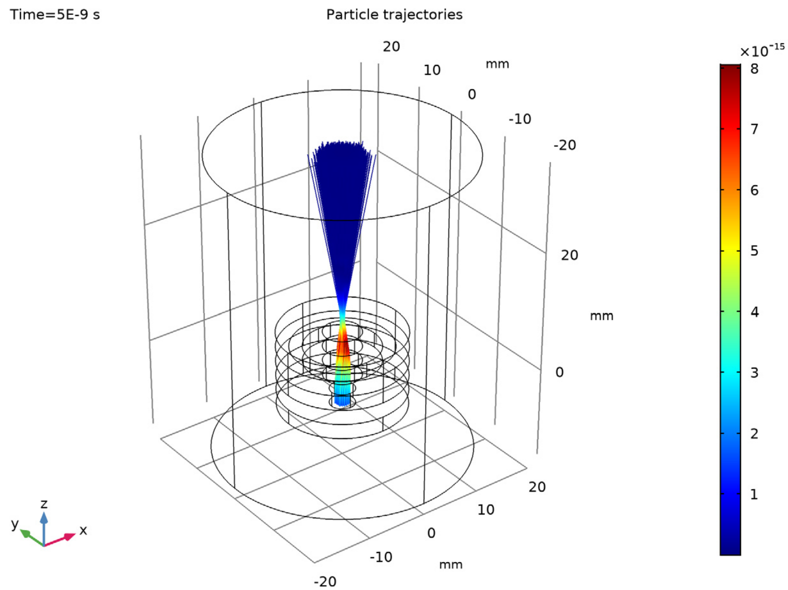

Figure 2: Plot of the electron trajectories traveling through the magnetic lens.

When charged particle beams are released, additional global variables are used to define beam properties such as the emittance and the Twiss parameters. These global variables can be used to characterize the shape of a beam and the transverse phase space distribution of the beam particles. In Figure 3 the hyperemittance is plotted along the average beam trajectory as a color expression and as a tube radius expression. The nominal trajectory reaches maximum thickness shortly after entering the lens and appears to be pinched off at the location where the beam is focused.

The ability to change the focal length of a lens is useful as it allows the focusing onto a surface in addition to adjusting the magnification. The effect of the focusing can be seen in Figure 4 which shows a Poincaré map of the particle position at three different snapshots in time. The sharpness of the cross-over can be improved using multiple lenses.

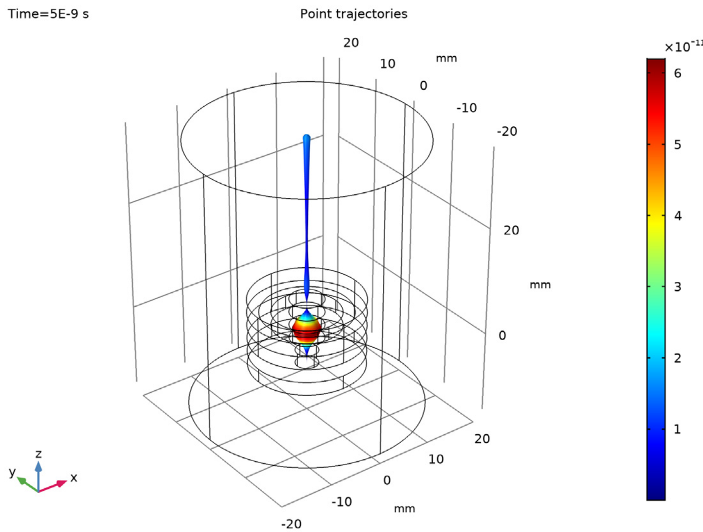

Figure 3: The nominal beam trajectory is plotted, with a color and thickness proportional to the 1-rms hyperemittance of the beam...

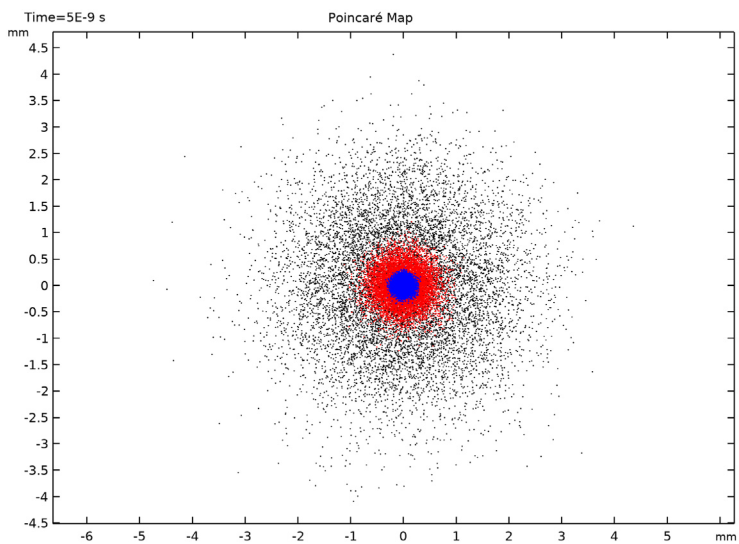

Figure 4: Poincaré maps of the particle location in the xy-plane initially (red), at the focal point of the lens (blue), and at the last time step (black).

Reference

1. M.J. Pritchard, *Manipulation of Ultracold Atoms Using Magnetic and Optical Fields*, PhD thesis, Durham University, September 2006, http://massey.dur.ac.uk/resources/mjpritchard/thesis_pritchard.pdf.

Application Library path: `ACDC_Module/Particle_Tracing/magnetic_lens`

Modeling Instructions

From the **File** menu, choose **New**.

NEW

In the **New** window, click **Model Wizard**.

MODEL WIZARD

1. In the **Model Wizard** window, click **3D**.
2. In the **Select Physics** tree, select **AC/DC>Electromagnetic Fields>Magnetic Fields (mf)**.
3. Click **Add**.
4. Click **Study**.
5. In the **Select Study** tree, select **General Studies>Stationary**.
6. Click **Done**.

GLOBAL DEFINITIONS

Parameters 1

1. In the **Model Builder** window, under **Global Definitions** click **Parameters 1**.
2. In the **Settings** window for **Parameters**, locate the **Parameters** section.
3. In the table, enter the following settings:

Name	Expression	Value	Description
Ic	0.32[A]	0.32 A	Coil current
Nc	1000	1000	Number of turns in coil

GEOMETRY 1

The coil geometry is constructed using cylinders, and it is available as a separate file in the Application Library. Insert the prepared geometry sequence from the file. You can read the instructions for creating the geometry in the appendix.

1 In the **Geometry** toolbar, click **Insert Sequence**.

2 Browse to the model's Application Libraries folder and double-click the file `magnetic_lens_geom_sequence.mph`.

MATERIALS

Material 1 (mat1)

1 In the **Model Builder** window, under **Component 1 (comp1)** right-click **Materials** and choose **Blank Material**.

2 In the **Settings** window for **Material**, locate the **Material Contents** section.

3 In the table, enter the following settings:

Property	**Variable**	**Value**	**Unit**	**Property group**
Relative permeability	mur_iso ; murii = mur_iso, murij = 0	1	1	Basic
Electrical conductivity	sigma_iso ; sigmaii = sigma_iso, sigmaij = 0	6e7	S/m	Basic
Relative permittivity	epsilonr_iso ; epsilonrii = epsilonr_iso, epsilonrij = 0	1	1	Basic

Material 2 (mat2)

1 Right-click **Materials** and choose **Blank Material**.

2 Select Domain 1 only.

3 In the **Settings** window for **Material**, locate the **Material Contents** section.

4 In the table, enter the following settings:

Property	**Variable**	**Value**	**Unit**	**Property group**
Relative permeability	mur_iso ; murii = mur_iso, murij = 0	1	1	Basic
Electrical conductivity	sigma_iso ; sigmaii = sigma_iso, sigmaij = 0	0	S/m	Basic
Relative permittivity	epsilonr_iso ; epsilonrii = epsilonr_iso, epsilonrij = 0	1	1	Basic

MAGNETIC FIELDS (MF)

Coil 1

1 In the **Model Builder** window, under **Component 1 (comp1)** right-click **Magnetic Fields (mf)** and choose the domain setting **Coil**.

2 Select Domain 4 only.

3 In the **Settings** window for **Coil**, locate the **Coil** section.

4 From the **Conductor model** list, choose **Homogenized multi-turn**.

5 From the **Coil type** list, choose **Circular**.

6 Locate the **Homogenized Multi-Turn Conductor** section. In the N text field, type `Nc`.

7 Locate the **Coil** section. In the I_{coil} text field, type `Ic`.

Specify the reference edges to be used in the calculation of the current path for the circular coil. To obtain the best results, the selected edges should have a radius close to the average coil radius. In this case, select the edges created for this purpose in previous steps.

Coil Geometry 1

1 In the **Model Builder** window, expand the **Coil 1** node, then click **Coil Geometry 1**.

2 In the **Settings** window for **Coil Geometry**, locate the **Edge Selection** section.

3 Click **Clear Selection**.

4 Select Edges 22, 23, 57, and 82 only.

MESH 1

Scale 1

1 In the **Model Builder** window, under **Component 1 (comp1)** right-click **Mesh 1** and choose **Scale**.
2 In the **Settings** window for **Scale**, locate the **Geometric Entity Selection** section.
3 From the **Geometric entity level** list, choose **Domain**.
4 Select Domains 2–5 only.
5 Locate the **Scale** section. In the **Element size scale** text field, type `0.5`.

Free Triangular 1

1 In the **Model Builder** window, right-click **Mesh 1** and choose **More Operations> Free Triangular**.
2 Select Boundary 30 only.

Size 1

1 Right-click **Free Triangular 1** and choose **Size**.

   Use a fine mesh on the surface where particles will be released.
2 In the **Settings** window for **Size**, locate the **Element Size** section.
3 From the **Predefined** list, choose **Extremely fine**.

Free Tetrahedral 1

1 In the **Model Builder** window, right-click **Mesh 1** and choose **Free Tetrahedral**.
2 Click **Build All**.

STUDY 1

In the **Home** toolbar, click **Compute**.

RESULTS

Multislice 1

1 In the **Model Builder** window, expand the **Results>Magnetic Flux Density Norm (mf)** node, then click **Multislice 1**.
2 In the **Settings** window for **Multislice**, locate the **Multiplane Data** section.
3 Find the **Z-planes** subsection. In the **Planes** text field, type `0`.
4 Click the **Zoom Extents** button in the **Graphics** toolbar.
5 In the **Magnetic Flux Density Norm (mf)** toolbar, click **Plot**. Compare the resulting image to Figure 1.

ADD PHYSICS

1 In the **Home** toolbar, click **Add Physics** to open the **Add Physics** window.

2 Go to the **Add Physics** window.

3 In the tree, select **AC/DC>Particle Tracing>Charged Particle Tracing (cpt)**.

4 Find the **Physics interfaces in study** subsection. In the table, clear the **Solve** check box for **Study 1**.

5 Click **Add to Component 1** in the window toolbar.

6 In the **Model Builder** window, click **Component 1 (comp1)**.

7 In the **Home** toolbar, click **Add Physics** to close the **Add Physics** window.

ADD STUDY

1 In the **Home** toolbar, click **Add Study** to open the **Add Study** window.

2 Go to the **Add Study** window.

3 Find the **Studies** subsection. In the **Select Study** tree, select **General Studies> Time Dependent**.

4 Find the **Physics interfaces in study** subsection. In the table, clear the **Solve** check box for the **Magnetic Fields (mf)** interface.

5 Click **Add Study** in the window toolbar.

6 In the **Home** toolbar, click **Add Study** to close the **Add Study** window.

GEOMETRY 1

1 In the **Model Builder** window, collapse the **Component 1 (comp1)>Geometry 1** node.

2 In the **Settings** window for **Charged Particle Tracing**, locate the **Domain Selection** section.

3 Click **Clear Selection**.

4 Select Domain 1 only.

CHARGED PARTICLE TRACING (CPT)

Particle Properties 1

1 In the **Model Builder** window, under **Component 1 (comp1)>Charged Particle Tracing (cpt)** click **Particle Properties 1**.

2 In the **Settings** window for **Particle Properties**, locate the **Particle Species** section.

3 From the **Particle species** list, choose **Electron**.

You need to provide the forces acting on the particles; in this case, the magnetic (Lorentz) force.

Magnetic Force 1

1 In the **Physics** toolbar, click **Domains** and choose **Magnetic Force**.
2 Select Domain 1 only.
3 In the **Settings** window for **Magnetic Force**, locate the **Magnetic Force** section.
4 From the $\mathbf{B}$ list, choose **Magnetic flux density (mf)**.

Particle Beam 1

1 In the **Physics** toolbar, click **Boundaries** and choose **Particle Beam**.
2 Select Boundary 30 only.
3 In the **Settings** window for **Particle Beam**, locate the **Initial Position** section.
4 In the N text field, type `10000`.
5 Locate the **Initial Transverse Velocity** section. In the $\varepsilon_{\mathrm{rms}}$ text field, type `0.1[um]`.
6 Locate the **Initial Longitudinal Velocity** section. In the E text field, type `0.5[keV]`.

STUDY 2

Step 1: Time Dependent

1 In the **Model Builder** window, under **Study 2** click **Step 1: Time Dependent**.
2 In the **Settings** window for **Time Dependent**, click to expand the **Values of Dependent Variables** section.
3 Find the **Values of variables not solved for** subsection. From the **Settings** list, choose **User controlled**.
4 From the **Method** list, choose **Solution**.
5 From the **Study** list, choose **Study 1, Stationary**.
6 Locate the **Study Settings** section. Click **Range**.
7 In the **Range** dialog box, choose **Number of values** from the **Entry method** list.
8 In the **Stop** text field, type `5e-9`.
9 In the **Number of values** text field, type `50`.
10 Click **Replace**.
11 In the **Home** toolbar, click **Compute**.

RESULTS

Particle Trajectories (cpt)

In the **Model Builder** window, expand the **Particle Trajectories (cpt)** node.

Particle Trajectories 1

1 In the **Model Builder** window, expand the **Results>Particle Trajectories (cpt)> Particle Trajectories 1** node, then click **Particle Trajectories 1**.
2 In the **Settings** window for **Particle Trajectories**, locate the **Coloring and Style** section.
3 Find the **Line style** subsection. From the **Type** list, choose **Line**.
4 Find the **Point style** subsection. From the **Type** list, choose **None**.

Color Expression 1

1 In the **Model Builder** window, click **Color Expression 1**.
2 In the **Settings** window for **Color Expression**, locate the **Expression** section.
3 In the **Expression** text field, type `sqrt(cpt.Ftx^2+cpt.Fty^2+cpt.Ftz^2)`.
4 In the **Particle Trajectories (cpt)** toolbar, click **Plot**.
5 Click the **Zoom Extents** button in the **Graphics** toolbar. Compare the resulting image to Figure 2.

Now observe the beam hyperemittance along the nominal beam trajectory.

Average Beam Position (cpt)

1 In the **Model Builder** window, under **Results** click **Average Beam Position (cpt)**.
2 In the **Settings** window for **3D Plot Group**, type `Average Beam Position and Hyperemittance` in the **Label** text field.

Point Trajectories 1

1 In the **Model Builder** window, expand the **Results> Average Beam Position and Hyperemittance** node, then click **Point Trajectories 1**.
2 In the **Settings** window for **Point Trajectories**, locate the **Coloring and Style** section.
3 Find the **Line style** subsection. From the **Type** list, choose **Tube**.
4 Click **Replace Expression**.
5 Right-click and choose **Component 1>Charged Particle Tracing>Beam properties> cpt.e1hrms - 1-RMS beam hyperemittance - m²** from the menu.
6 Select the **Radius scale factor** check box.
7 In the associated text field, type `4E10`.
8 From the **Interpolation** list, choose **Uniform**.
9 In the **Average Beam Position and Hyperemittance** toolbar, click **Plot**. Compare the resulting image to Figure 3.

Now construct a **Poincaré Map** to visualize the radial distribution of particles initially, at the focal point, and at the exit of the modeling domain.

Cut Plane 1

1. In the **Results** toolbar, click **Cut Plane**.
2. In the **Settings** window for **Cut Plane**, locate the **Plane Data** section.
3. From the **Plane** list, choose **XY-planes**.
4. In the **Z-coordinate** text field, type `-6`.
5. Locate the **Data** section. From the **Dataset** list, choose **Particle 1**.

Cut Plane 2

1. Right-click **Cut Plane 1** and choose **Duplicate**.
2. In the **Settings** window for **Cut Plane**, locate the **Plane Data** section.
3. In the **z-coordinate** text field, type `7`.

Cut Plane 3

1. Right-click **Cut Plane 2** and choose **Duplicate**.
2. In the **Settings** window for **Cut Plane**, locate the **Plane Data** section.
3. In the **z-coordinate** text field, type `34`.

2D Plot Group 4

1. In the **Results** toolbar, click **2D Plot Group**.
2. In the **Settings** window for **2D Plot Group**, type `Poincaré Maps` in the **Label** text field.
3. Click to expand the **Title** section. From the **Title type** list, choose **Manual**.
4. In the **Title** text area, type `Poincaré Map`.

Poincaré Map 1

1. In the **Poincaré Maps** toolbar, click **More Plots** and choose **Poincaré Map**.
2. In the **Settings** window for **Poincaré Map**, locate the **Data** section.
3. From the **Cut plane** list, choose **Cut Plane 3**.
4. Locate the **Coloring and Style** section. From the **Color** list, choose **Black**.
5. Click the **Zoom Extents** button in the **Graphics** toolbar.

Poincaré Map 2

1. Right-click **Poincaré Map 1** and choose **Duplicate**.
2. In the **Settings** window for **Poincaré Map**, locate the **Data** section.
3. From the **Cut plane** list, choose **Cut Plane 1**.

4 Locate the **Coloring and Style** section. From the **Color** list, choose **Red**.

Poincaré Map 3

1 Right-click **Poincaré Map 2** and choose **Duplicate**.

2 In the **Settings** window for **Poincaré Map**, locate the **Data** section.

3 From the **Cut plane** list, choose **Cut Plane 2**.

4 Locate the **Coloring and Style** section. From the **Color** list, choose **Blue**.

5 In the **Poincaré Maps** toolbar, click **Plot**.

6 Click the **Zoom Extents** button in the **Graphics** toolbar. Compare the resulting image to Figure 4.

Appendix A - Geometry Instructions

ADD COMPONENT

In the **Home** toolbar, click **Add Component** and choose **3D**.

GEOMETRY 1

1 In the **Settings** window for **Geometry**, locate the **Units** section.

2 From the **Length unit** list, choose **mm**.

Cylinder 1 (cyl1)

1 In the **Geometry** toolbar, click **Cylinder**.

2 In the **Settings** window for **Cylinder**, locate the **Size and Shape** section.

3 In the **Radius** text field, type `10`.

4 In the **Height** text field, type `2.5`.

Cylinder 2 (cyl2)

1 In the **Geometry** toolbar, click **Cylinder**.

2 In the **Settings** window for **Cylinder**, locate the **Size and Shape** section.

3 In the **Radius** text field, type `6`.

4 In the **Height** text field, type `2.5`.

5 Click **Build Selected**.

Cylinder 3 (cyl3)

1 In the **Model Builder** window, under **Component 1 (comp1)>Geometry 1** right-click **Cylinder 1 (cyl1)** and choose **Duplicate**.

2 In the **Settings** window for **Cylinder**, locate the **Position** section.

3 In the **z** text field, type `-7.5`.

Cylinder 4 (cyl4)

1 In the **Geometry** toolbar, click **Cylinder**.

2 In the **Settings** window for **Cylinder**, locate the **Size and Shape** section.

3 In the **Radius** text field, type `2`.

4 In the **Height** text field, type `2.5`.

5 Locate the **Position** section. In the **z** text field, type `-7.5`.

Cylinder 5 (cyl5)

1 Right-click **Cylinder 1 (cyl1)** and choose **Duplicate**.

2 In the **Settings** window for **Cylinder**, locate the **Position** section.

3 In the **z** text field, type `-2.5`.

Cylinder 6 (cyl6)

1 In the **Geometry** toolbar, click **Cylinder**.

2 In the **Settings** window for **Cylinder**, locate the **Size and Shape** section.

3 In the **Radius** text field, type `3`.

4 In the **Height** text field, type `2.5`.

5 Locate the **Position** section. In the **z** text field, type `-2.5`.

Cylinder 7 (cyl7)

1 Right-click **Cylinder 1 (cyl1)** and choose **Duplicate**.

2 In the **Settings** window for **Cylinder**, locate the **Position** section.

3 In the **z** text field, type `2.5`.

Cylinder 8 (cyl8)

1 In the **Geometry** toolbar, click **Cylinder**.

2 In the **Settings** window for **Cylinder**, locate the **Size and Shape** section.

3 In the **Radius** text field, type `3`.

4 In the **Height** text field, type `2.5`.

5 Locate the **Position** section. In the **z** text field, type `2.5`.

Cylinder 9 (cyl9)

1 In the **Geometry** toolbar, click **Cylinder**.
2 In the **Settings** window for **Cylinder**, locate the **Size and Shape** section.
3 In the **Radius** text field, type 20.
4 In the **Height** text field, type 50.
5 Locate the **Position** section. In the **z** text field, type -15.

Difference 1 (dif1)

1 In the **Geometry** toolbar, click **Booleans and Partitions** and choose **Difference**.
2 Select the objects **cyl1**, **cyl3**, **cyl5**, and **cyl7** only.
3 In the **Settings** window for **Difference**, locate the **Difference** section.
4 Find the **Objects to subtract** subsection. Select the **Activate selection** toggle button.
5 Select the objects **cyl2**, **cyl4**, **cyl6**, and **cyl8** only.
6 Click **Build Selected**.
7 Click the **Go to Default View** button in the **Graphics** toolbar.

Work Plane 1 (wp1)

1 In the **Geometry** toolbar, click **Work Plane**.
2 Click the **Wireframe Rendering** button in the **Graphics** toolbar.
3 In the **Settings** window for **Work Plane**, locate the **Plane Definition** section.
4 From the **Plane type** list, choose **Face parallel**.
5 On the object **dif1**, select Boundary 3 only.
6 Click **Show Work Plane**.

Work Plane 1 (wp1)>Plane Geometry

Click the **Zoom Extents** button in the **Graphics** toolbar.

Work Plane 1 (wp1)>Circle 1 (c1)

1 In the **Work Plane** toolbar, click **Circle**.
2 In the **Settings** window for **Circle**, locate the **Size and Shape** section.
3 In the **Radius** text field, type 2.
4 Click **Build Selected**.

   Last, create a circular edge to be used in the **Coil** feature as a reference edge.

5 In the **Model Builder** window, click **Geometry 1**.

Work Plane 2 (wp2)

1 In the **Geometry** toolbar, click **Work Plane**.
2 In the **Settings** window for **Work Plane**, locate the **Plane Definition** section.
3 From the **Plane type** list, choose **Face parallel**.
4 On the object **dif1**, select Boundary 13 only.
5 Click **Show Work Plane**.

Work Plane 2 (wp2)>Circle 1 (c1)

1 In the **Work Plane** toolbar, click **Circle**.
2 In the **Settings** window for **Circle**, locate the **Object Type** section.
3 From the **Type** list, choose **Curve**.
4 Locate the **Size and Shape** section. In the **Radius** text field, type 8.
5 In the **Work Plane** toolbar, click **Build All**.