RF Module
User’s Guide
RF Module User's Guide

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Part number: CM021001
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Introduction

This guide describes the RF Module, an optional add-on package for COMSOL Multiphysics® with customized physics interfaces and functionality optimized for the analysis of electromagnetic waves.

This chapter introduces you to the capabilities of this module. A summary of the physics interfaces and where you can find documentation and model examples is also included. The last section is a brief overview with links to each chapter in this guide.

- About the RF Module
- Overview of the User’s Guide
About the RF Module

In this section:

- What Can the RF Module Do?
- What Problems Can You Solve?
- The RF Module Physics Interface Guide
- Common Physics Interface and Feature Settings and Nodes
- Selecting the Study Type
- The RF Module Modeling Process
- Where Do I Access the Documentation and Application Libraries?

What Can the RF Module Do?

The RF Module solves problems in the general field of electromagnetic waves, such as RF and microwave applications, optics, and photonics. The underlying equations for electromagnetics are automatically available in all of the physics interfaces—a feature unique to COMSOL Multiphysics. This also makes nonstandard modeling easily accessible.

The module is useful for component design in virtually all areas where you find electromagnetic waves, such as:

- Antennas
- Filters, couplers, and power dividers
- Planar circuits and passive devices
- RF interconnects and packages
- Waveguides and cavity resonators
- Frequency-selective surfaces
- Meta-materials
The physics interfaces cover the following types of electromagnetics field simulations and handle time-harmonic, time-dependent, and eigenfrequency/eigenmode problems:

- In-plane, axisymmetric, and full 3D electromagnetic wave propagation
- Full vector mode analysis in 2D and 3D

Material properties include inhomogeneous and fully anisotropic materials, media with gains or losses, and complex-valued material properties. In addition to the standard postprocessing features, the module supports direct computation of S-parameters and far-field patterns. You can add ports with a wave excitation with specified power level and mode type, and add PMLs (perfectly matched layers) to simulate electromagnetic waves that propagate into an unbounded domain. For time-harmonic simulations, you can use the scattered wave or the total wave.

Using the multiphysics capabilities of COMSOL Multiphysics you can couple simulations with heat transfer, structural mechanics, fluid flow formulations, and other physical phenomena.

This module also has interfaces for circuit modeling, a SPICE interface, and support for importing ECAD drawings.

What Problems Can You Solve?

### QUASI-STATIC AND HIGH FREQUENCY MODELING

One major difference between quasi-static and high-frequency modeling is that the formulations depend on the electrical size of the structure. This dimensionless measure is the ratio between the largest distance between two points in the structure divided by the wavelength of the electromagnetic fields.

For simulations of structures with an electrical size in the range up to 1/10, quasi-static formulations are suitable. The physical assumption of these situations is that wave propagation delays are small enough to be neglected. Thus, phase shifts or phase gradients in fields are caused by materials and/or conductor arrangements being inductive or capacitive rather than being caused by propagation delays.

For electrostatic, magnetostatic, and quasi-static electromagnetics, use the AC/DC Module, a COMSOL Multiphysics add-on module for low-frequency electromagnetics.
When propagation delays become important, it is necessary to use the full Maxwell equations for high-frequency electromagnetic waves. They are appropriate for structures of electrical size 1/100 and larger. Thus, an overlapping range exists where you can use both the quasi-static and the full Maxwell physics interfaces.

Independently of the structure size, the module accommodates any case of nonlinear, inhomogeneous, or anisotropic media. It also handles materials with properties that vary as a function of time as well as frequency-dispersive materials.

*The RF Module Physics Interface Guide*

The physics interfaces in this module form a complete set of simulation tools for electromagnetic wave simulations. Add the physics interface and study type when starting to build a new model. You can add physics interfaces and studies to an existing model throughout the design process. In addition to the core physics interfaces included with the basic COMSOL Multiphysics license, the physics interfaces below are included with the RF Module and available in the indicated space dimension. All physics interfaces are available in 2D and 3D. In 2D there are in-plane formulations for problems with a planar symmetry as well as axisymmetric formulations for problems with a cylindrical symmetry. 2D mode analysis of waveguide cross sections with out-of-plane propagation is also supported.

In the *COMSOL Multiphysics Reference Manual*:

- Studies and Solvers
- The Physics Interfaces
- Creating a New Model
- For a list of all the core physics interfaces included with a COMSOL Multiphysics license, see *Physics Interface Guide*. 

ABOUT THE RF MODULE

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<th>PHYSICS INTERFACE</th>
<th>ICON</th>
<th>TAG</th>
<th>SPACE DIMENSION</th>
<th>AVAILABLE PRESET STUDY TYPE</th>
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<td>AC/DC</td>
<td></td>
<td>cir</td>
<td>Not space dependent</td>
<td>stationary; frequency domain; time dependent; small signal analysis, frequency domain</td>
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<td>Heat Transfer</td>
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<td>Electromagnetic Heating</td>
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<tr>
<td>Microwave Heating</td>
<td>—</td>
<td></td>
<td>3D, 2D, 2D axisymmetric</td>
<td>frequency-stationary; frequency-transient</td>
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<td>Radio Frequency</td>
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<tr>
<td>Electromagnetic Waves, Frequency Domain</td>
<td></td>
<td>emw</td>
<td>3D, 2D, 2D axisymmetric</td>
<td>eigenfrequency; frequency domain; frequency-domain modal; boundary mode analysis; mode analysis (2D and 2D axisymmetric models only)</td>
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<td>ewte</td>
<td>3D, 2D, 2D axisymmetric</td>
<td>time dependent</td>
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<td>tl</td>
<td>3D, 2D, 1D</td>
<td>eigenfrequency; frequency domain</td>
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1 This physics interface is a predefined multiphysics coupling that automatically adds all the physics interfaces and coupling features required.

Common Physics Interface and Feature Settings and Nodes

There are several common settings and sections available for the physics interfaces and feature nodes. Some of these sections also have similar settings or are implemented in
the same way no matter the physics interface or feature being used. There are also some physics feature nodes that display in COMSOL Multiphysics.

In each module’s documentation, only unique or extra information is included; standard information and procedures are centralized in the COMSOL Multiphysics Reference Manual.

Selecting the Study Type

To carry out different kinds of simulations for a given set of parameters in a physics interface, you can select, add, and change the Study Types at almost every stage of modeling.

COMPARING THE TIME DEPENDENT AND FREQUENCY DOMAIN STUDIES

When variations in time are present there are two main approaches to represent the time dependence. The most straightforward is to solve the problem by calculating the changes in the solution for each time step; that is, solving using the Time Dependent study (available with the Electromagnetic Waves, Transient interface). However, this approach can be time consuming if small time steps are necessary for the desired accuracy. It is necessary when the inputs are transients like turn-on and turn-off sequences.

However, if the Frequency Domain study available with the Electromagnetic Waves, Frequency Domain interface is used, this allows you to efficiently simplify and assume that all variations in time occur as sinusoidal signals. Then the problem is time-harmonic and in the frequency domain. Thus you can formulate it as a stationary problem with complex-valued solutions. The complex value represents both the amplitude and the phase of the field, while the frequency is specified as a scalar model input, usually provided by the solver. This approach is useful because, combined with Fourier analysis, it applies to all periodic signals with the exception of nonlinear
problems. Examples of typical frequency domain simulations are wave-propagation problems like waveguides and antennas.

For nonlinear problems you can apply a Frequency Domain study after a linearization of the problem, which assumes that the distortion of the sinusoidal signal is small.

Use a Time Dependent study when the nonlinear influence is strong, or if you are interested in the harmonic distortion of a sine signal. It can also be more efficient to use a Time Dependent study if you have a periodic input with many harmonics, like a square-shaped signal.

**DEFAULT FREQUENCY**

In the RF module, the Frequency Domain, the Frequency-Domain Modal, and the Eigenfrequency study step have GHz as the default frequency unit and a default frequency of 1[GHz].

*The RF Module Modeling Process*

The modeling process has these main steps, which (excluding the first step), correspond to the branches displayed in the Model Builder in the COMSOL Desktop environment.

1. Selecting the appropriate physics interface or predefined multiphysics coupling when adding a physics interface.
2. Defining component parameters and variables in the *Definitions* branch ( ).
3. Drawing or importing the component geometry in the *Geometry* branch ( ).
4. Assigning material properties to the geometry in the *Materials* branch ( ).
5. Setting up the model equations and boundary conditions in the physics interfaces branch.
6. Meshing in the *Mesh* branch ( ).
7. Setting up the study and computing the solution in the *Study* branch ( ).
8. Analyzing and visualizing the results in the *Results* branch ( ).

When using frequency related study steps, define the simulation frequency in the *Study* branch first. The frequency in the *Study* node will then be used for defining the physics-controlled mesh and in any frequency-based physics features.
Even after a model is defined, you can edit input data, equations, boundary conditions, geometry—the equations and boundary conditions are still available through associative geometry—and mesh settings. You can restart the solver, for example, using the existing solution as the initial condition or initial guess. It is also easy to add another physics interface to account for a phenomenon not previously described in a model.

Where Do I Access the Documentation and Application Libraries?

A number of internet resources have more information about COMSOL, including licensing and technical information. The electronic documentation, topic-based (or context-based) help, and the application libraries are all accessed through the COMSOL Desktop.

If you are reading the documentation as a PDF file on your computer, the blue links do not work to open an application or content referenced in a different guide. However, if you are using the Help system in COMSOL Multiphysics, these links work to open other modules (as long as you have a license), application examples, and documentation sets.

The Documentation and Online Help

The COMSOL Multiphysics Reference Manual describes the core physics interfaces and functionality included with the COMSOL Multiphysics license. This book also has instructions about how to use COMSOL Multiphysics and how to access the electronic Documentation and Help content.

Opening Topic-Based Help

The Help window is useful as it is connected to many of the features on the GUI. To learn more about a node in the Model Builder, or a window on the Desktop, click to highlight a node or window, then press F1 to open the Help window, which then
displays information about that feature (or click a node in the Model Builder followed by the Help button (\text{Help})). This is called topic-based (or context) help.

To open the Help window:

\begin{itemize}
  \item In the Model Builder, Application Builder, or Physics Builder click a node or window and then press F1.
  \item On any toolbar (for example, Home, Definitions, or Geometry), hover the mouse over a button (for example, Add Physics or Build All) and then press F1.
  \item From the File menu, click Help (\text{Help}).
  \item In the upper-right corner of the COMSOL Desktop, click the Help (\text{Help}) button.
\end{itemize}

Opening the Documentation Window

To open the Documentation window:

\begin{itemize}
  \item In the Model Builder or Physics Builder click a node or window and then press F1.
  \item On the main toolbar, click the Help (\text{Help}) button.
  \item From the main menu, select Help>Help.
\end{itemize}

To open the Documentation window:

\begin{itemize}
  \item Press Ctrl+F1.
  \item From the File menu select Help>Documentation (\text{Help}).
\end{itemize}

To open the Documentation window:

\begin{itemize}
  \item Press Ctrl+F1.
  \item On the main toolbar, click the Documentation (\text{Documentation}) button.
  \item From the main menu, select Help>Documentation.
\end{itemize}
THE APPLICATION LIBRARIES WINDOW

Each application includes documentation with the theoretical background and step-by-step instructions to create a model application. The applications are available in COMSOL as MPH-files that you can open for further investigation. You can use the step-by-step instructions and the actual applications as a template for your own modeling and applications. In most models, SI units are used to describe the relevant properties, parameters, and dimensions in most examples, but other unit systems are available.

Once the Application Libraries window is opened, you can search by name or browse under a module folder name. Click to view a summary of the application and its properties, including options to open it or a PDF document.

CONTACTING COMSOL BY EMAIL

For general product information, contact COMSOL at info@comsol.com.

To receive technical support from COMSOL for the COMSOL products, please contact your local COMSOL representative or send your questions to
support@comsol.com. An automatic notification and a case number are sent to you by email.

**COMSOL ONLINE RESOURCES**

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Overview of the User’s Guide

The RF Module User’s Guide gets you started with modeling using COMSOL Multiphysics. The information in this guide is specific to this module. Instructions how to use COMSOL in general are included with the COMSOL Multiphysics Reference Manual.

As detailed in the section Where Do I Access the Documentation and Application Libraries? this information can also be searched from the COMSOL Multiphysics software Help menu.

TABLE OF CONTENTS, GLOSSARY, AND INDEX
To help you navigate through this guide, see the Contents, Glossary, and Index.

MODELING WITH THE RF MODULE
The RF Modeling chapter familiarize you with the modeling procedures. A number of examples available through the Application Libraries window also illustrate the different aspects of the simulation process. Topics include Preparing for RF Modeling, Simplifying Geometries, and Scattered Field Formulation.

RF THEORY
The Electromagnetics Theory chapter contains a review of the basic theory of electromagnetics, starting with Maxwell’s Equations, and the theory for some Special Calculations: S-parameters, lumped port parameters, and far-field analysis. There is also a list of Electromagnetic Quantities with their SI units and symbols.

RADIO FREQUENCY
Radio Frequency Interfaces chapter describes:

• The Electromagnetic Waves, Frequency Domain Interface, which analyzes frequency domain electromagnetic waves, and uses time-harmonic and eigenfrequency or eigenmode (2D only) studies, boundary mode analysis and frequency domain modal.

• The Electromagnetic Waves, Transient Interface, which supports the Time Dependent study type.
• The Transmission Line Interface, which solves the time-harmonic transmission line equation for the electric potential.

• The Electromagnetic Waves, Time Explicit Interface, which solves a transient wave equation for both the electric and magnetic fields.

The underlying theory is also included at the end of the chapter.

**ELECTRICAL CIRCUIT**

AC/DC Interfaces chapter describes The Electrical Circuit Interface, which simulates the current in a conductive and capacitive material under the influence of an electric field. All three study types (Stationary, Frequency Domain, and Time Dependent) are available. The underlying theory is also included at the end of the chapter.

**HEAT TRANSFER**

Heat Transfer Interfaces chapter describes the Microwave Heating interface, which combines the physics features of an Electromagnetic Waves, Frequency Domain interface from the RF Module with the Heat Transfer interface. The predefined interaction adds the electromagnetic losses from the electromagnetic waves as a heat source and solves frequency domain (time-harmonic) electromagnetic waves in conjunction with stationary or transient heat transfer. This physics interface is based on the assumption that the electromagnetic cycle time is short compared to the thermal time scale (adiabatic assumption). The underlying theory is also included at the end of the chapter.
The goal of this chapter is to familiarize you with the modeling procedure in the RF Module. A number of models available through the RF Module Applications Libraries also illustrate the different aspects of the simulation process.

In this chapter:
- Preparing for RF Modeling
- Simplifying Geometries
- Periodic Boundary Conditions
- Scattered Field Formulation
- Modeling with Far-Field Calculations
- S-Parameters and Ports
- Lumped Ports with Voltage Input
- Lossy Eigenvalue Calculations
- Connecting to Electrical Circuits
- Spice Import
- Reduced Order Modeling
Preparing for RF Modeling

Several modeling topics are described in this section that might not be found in ordinary textbooks on electromagnetic theory.

This section is intended to help answer questions such as:

- Which spatial dimension should I use: 3D, 2D axial symmetry, or 2D?
- Is my problem suited for time-dependent or frequency domain formulations?
- Can I use a quasi-static formulation or do I need wave propagation?
- What sources can I use to excite the fields?
- When do I need to resolve the thickness of thin shells and when can I use boundary conditions?
- What is the purpose of the model?
- What information do I want to extract from the model?

Increasing the complexity of a model to make it more accurate usually makes it more expensive to simulate. A complex model is also more difficult to manage and interpret than a simple one. Keep in mind that it can be more accurate and efficient to use several simple models instead of a single, complex one.

The Physics Interfaces and Building a COMSOL Multiphysics Model in the COMSOL Multiphysics Reference Manual
Simplifying Geometries

Most of the problems that are solved with COMSOL Multiphysics are three-dimensional (3D) in the real world. In many cases, it is sufficient to solve a two-dimensional (2D) problem that is close to or equivalent to the real problem. Furthermore, it is good practice to start a modeling project by building one or several 2D models before going to a 3D model. This is because 2D models are easier to modify and solve much faster. Thus, modeling mistakes are much easier to find when working in 2D. Once the 2D model is verified, you are in a much better position to build a 3D model.

In this section:

• 2D Models
• 3D Models
• Using Efficient Boundary Conditions
• Applying Electromagnetic Sources
• Meshing and Solving

2D Models

The text below is a guide to some of the common approximations made for 2D models. Remember that the modeling in 2D usually represents some 3D geometry under the assumption that nothing changes in the third dimension or that the field has a prescribed propagation component in the third dimension.

**Cartesian Coordinates**

In this case a cross section is viewed in the xy-plane of the actual 3D geometry. The geometry is mathematically extended to infinity in both directions along the z-axis, assuming no variation along that axis or that the field has a prescribed wave vector component along that axis. All the total flows in and out of boundaries are per unit length along the z-axis. A simplified way of looking at this is to assume that the geometry is extruded one unit length from the cross section along the z-axis. The total flow out of each boundary is then from the face created by the extruded boundary (a boundary in 2D is a line).
There are usually two approaches that lead to a 2D cross-section view of a problem. The first approach is when it is known that there is no variation of the solution in one particular dimension.

This is shown in the model *H-Bend Waveguide 2D*, where the electric field only has one component in the \( z \) direction and is constant along that axis. The second approach is when there is a problem where the influence of the finite extension in the third dimension can be neglected.

**Figure 2-1: The cross sections and their real geometry for Cartesian coordinates and cylindrical coordinates (axial symmetry).**

**AXIAL SYMMETRY (CYLINDRICAL COORDINATES)**

If the 3D geometry can be constructed by revolving a cross section around an axis, and if no variations in any variable occur when going around the axis of revolution (or that the field has a prescribed wave vector component in the direction of revolution), then use an axisymmetric physics interface. The spatial coordinates are called \( r \) and \( z \), where \( r \) is the radius. The flow at the boundaries is given per unit length along the third dimension. Because this dimension is a revolution all flows must be multiplied with \( \alpha r \), where \( \alpha \) is the revolution angle (for example, \( 2\pi \) for a full turn).
POLARIZATION IN 2D

In addition to selecting 2D or 2D axisymmetry when you start building the model, the physics interfaces (The Electromagnetic Waves, Frequency Domain Interface or The Electromagnetic Waves, Transient Interface) in the Model Builder offers a choice in the Components settings section. The available choices are Out-of-plane vector, In-plane vector, and Three-component vector. This choice determines what polarizations can be handled. For example, as you are solving for the electric field, a 2D TM (out-of-plane H field) model requires choosing In-plane vector as then the electric field components are in the modeling plane.

3D Models

Although COMSOL Multiphysics fully supports arbitrary 3D geometries, it is important to simplify the problem. This is because 3D models often require more computer power, memory, and time to solve. The extra time spent on simplifying a model is probably well spent when solving it. Below are a few issues that need to be addressed before starting to implement a 3D model in this module.

- Check if it is possible to solve the problem in 2D. Given that the necessary approximations are small, the solution is more accurate in 2D, because a much denser mesh can be used.
- Look for symmetries in the geometry and model. Many problems have planes where the solution is the same on both sides of the plane. A good way to check this is to flip the geometry around the plane, for example, by turning it up-side down around the horizontal plane. Then remove the geometry below the plane if no differences are observed between the two cases regarding geometry, materials, and sources. Boundaries created by the cross section between the geometry and this plane need a symmetry boundary condition, which is available in all 3D physics interfaces.
- There are also cases when the dependence along one direction is known, and it can be replaced by an analytical function. Use this approach either to convert 3D to 2D or to convert a layer to a boundary condition.
Using Efficient Boundary Conditions

An important technique to minimize the problem size is to use efficient boundary conditions. Truncating the geometry without introducing too large errors is one of the great challenges in modeling. Below are a few suggestions of how to do this. They apply to both 2D and 3D problems.

- Many models extend to infinity or can have regions where the solution only undergoes small changes. This problem is addressed in two related steps. First, the geometry needs to be truncated in a suitable position. Second, a suitable boundary condition needs to be applied there. For static and quasi-static models, it is often possible to assume zero fields at the open boundary, provided that this is at a sufficient distance away from the sources. For radiation problems, special low-reflecting boundary conditions need to be applied. This boundary should be in the order of a few wavelengths away from any source.

A more accurate option is to use perfectly matched layers (PMLs). PMLs are layers that absorbs all radiated waves with small reflections.

- Replace thin layers with boundary conditions where possible. There are several types of boundary conditions in COMSOL Multiphysics suitable for such replacements. For example, replace materials with high conductivity by the perfect electric conductor (PEC) boundary condition.

- Use boundary conditions for known solutions. For example, an antenna aperture can be modeled as an equivalent surface current density on a 2D face (boundary) in a 3D model.

Applying Electromagnetic Sources

Electromagnetic sources can be applied in many different ways. The typical options are boundary sources, line sources, and point sources, where point sources in 2D formulations are equivalent to line sources in 3D formulations. The way sources are imposed can have an impact on what quantities can be computed from the model. For example, a line source in an electromagnetic wave model represents a singularity and the magnetic field does not have a finite value at the position of the source. In a COMSOL Multiphysics model, the magnetic field of a line source has a finite but mesh-dependent value. In general, using volume or boundary sources is more flexible than using line sources or point sources, but the meshing of the source domains becomes more expensive.
Simplifying Geometries

The finite element method approximates the solution within each element, using some elementary shape function that can be constant, linear, or of higher order. Depending on the element order in the model, a finer or coarser mesh is required to resolve the solution. In general, there are three problem-dependent factors that determine the necessary mesh resolution:

- The first is the variation in the solution due to geometrical factors. The mesh generator automatically generates a finer mesh where there is a lot of fine geometrical details. Try to remove such details if they do not influence the solution, because they produce a lot of unnecessary mesh elements.

- The second is the skin effect or the field variation due to losses. It is easy to estimate the skin depth from the conductivity, permeability, and frequency. At least two linear elements per skin depth are required to capture the variation of the fields. If the skin depth is not studied or a very accurate measure of the dissipation loss profile is not needed, replace regions with a small skin depth with a boundary condition, thereby saving elements. If it is necessary to resolve the skin depth, the boundary layer meshing technique can be a convenient way to get a dense mesh near a boundary.

- The third and last factor is the wavelength. To resolve a wave properly, it is necessary to use about 10 linear (or five 2nd order) elements per wavelength. Keep in mind that the wavelength depends on the local material properties.

Solvers

In most cases the solver sequence generated by COMSOL Multiphysics can be used. The choice of solver is optimized for the typical case for each physics interface and study type in this module. However, in special cases tuning the solver settings can be required. This is especially important for 3D problems because they can require a large amount of memory.
Periodic Boundary Conditions

The RF Module has a dedicated Periodic Condition. The periodic condition can identify simple mappings on plane source and destination boundaries of equal shape. The destination can also be rotated with respect to the source. There are three types of periodic conditions available (only the first two for transient analysis):

- **Continuity**—The tangential components of the solution variables are equal on the source and destination.
- **Antiperiodicity**—The tangential components have opposite signs.
- **Floquet periodicity**—There is a phase shift between the tangential components. The phase shift is determined by a wave vector and the distance between the source and destination. Floquet periodicity is typically used for models involving plane waves interacting with periodic structures.

Periodic boundary conditions must have compatible meshes. This can be done automatically by enabling the **Physics-control mesh** in the setting for The Electromagnetic Waves, Frequency Domain Interface or by manually setting up the correct mesh sequence.

If more advanced periodic boundary conditions are required, for example, when there is a known rotation of the polarization from one boundary to another, see Component Couplings and Coupling Operators in the COMSOL Multiphysics Reference Manual for tools to define more general mappings between boundaries.

To learn how to use the Copy Mesh feature to ensure that the mesh on the destination boundary is identical to that on the source boundary, see Plasmonic Wire Grating: Application Library path RF_Module/Tutorials/plasmonic_wire_grating.

For an example of how to use the Physics-controlled mesh, see Fresnel Equations: Application Library path RF_Module/Verification_Examples/fresnel_equations.
In the COMSOL Multiphysics Reference Manual:

- Periodic Condition and Destination Selection
- Periodic Boundary Conditions
Scattered Field Formulation

For problems where a known background field is illuminating an object in free space it is possible to use the scattered field formulation. Since the equation of the background field is known it can be entered as a model input and does not need to be computed. Starting from the frequency-domain governing equation:

\[ \nabla \times (\mu^{-1} \nabla \times \mathbf{E}) - \omega^2 \varepsilon \mathbf{E} = 0 \]

The total electric field, \( \mathbf{E} \), can be decomposed into two components:

\[ \mathbf{E} = \mathbf{E}_{\text{total}} = \mathbf{E}_{\text{background}} + \mathbf{E}_{\text{relative}} \]

The known background field becomes a source term and the scattered field formulation thus solves for the relative electric field. A linearly polarized plane wave background field, a paraxial-approximate Gaussian beam, or a user-defined background field can be specified. When solving the scattered field formulation the total, the background, and the relative electric fields are available. The relative field is the difference between the background field and the total field. It is the relative field that contributes to the far-field calculation. For more information about the Far-Field computation, see Far-Field Calculations Theory. The benefit to this approach is that if the background field is much larger in magnitude than the scattered field, the accuracy of the simulation improves if the relative field is solved for. Another advantage is that it becomes very easy to set up a perfectly matched layer surrounding the homogeneous medium modeling domain.

The drawback to the this approach is that the relative field requires some careful interpretation. The relative electric field can conceptually be decomposed into:

\[ \mathbf{E}_{\text{relative}} = \mathbf{E}_{\text{scattered}} + \mathbf{E}_{\text{correction}} + \mathbf{E}_{\text{cancellation}} \]

The \( \mathbf{E}_{\text{scattered}} \) component is the scattered field from object. This is the field that is of interest in a scattering problem. However, the relative field may also consist of a component that represents a correction to the background field and a cancellation of the background field. The \( \mathbf{E}_{\text{correction}} \) component can be nonzero when the background field does not exactly satisfy Maxwell’s equations, such as when the paraxial Gaussian beam approximation is used for a tightly focused beam. For more information about the Gaussian beam theory, see Gaussian Beams as Background Fields. The \( \mathbf{E}_{\text{cancellation}} \) component will be nonzero and equal to \(-\mathbf{E}_{\text{background}}\) wherever the total field should be zero, such as in the interior of any perfectly shielded...
objects, or behind a relatively large shielding object. Note that this decomposition is conceptual only, it is only the relative field that is available.

An alternative of using the scattered-field formulation, is to use ports with the **Activate slit condition on interior port** setting enabled. Then the domain can be excited by the interior port and the outgoing field can be absorbed by perfectly matched layers. For more information about the **Port** feature and the **Activate slit condition on interior port** setting, see **Port Properties**.

**SCATTERED FIELDS SETTING**

The scattered-field formulation is available for **The Electromagnetic Waves, Frequency Domain Interface** under the **Settings** section. The scattered field in the analysis is called the relative electric field. The total electric field is always available, and for the scattered-field formulation this is the sum of the scattered field and the incident field.

```
Radar Cross Section: Application Library path RF_Module/
Scattering_and_RCS/radar_cross_section
```
Modeling with Far-Field Calculations

The far electromagnetic field from, for example, antennas can be calculated from the near-field solution on a boundary using far-field analysis. The antenna is located in the vicinity of the origin, while the far-field is taken at infinity but with a well-defined angular direction \((\theta, \phi)\). The far-field radiation pattern is given by evaluating the squared norm of the far-field on a sphere centered at the origin. Each coordinate on the surface of the sphere represents an angular direction.

In this section:

- Far-Field Support in the Electromagnetic Waves, Frequency Domain Interface
- The Far Field Plots

---

Far-Field Support in the Electromagnetic Waves, Frequency Domain Interface

The Electromagnetic Waves, Frequency Domain interface supports far-field analysis. To define the far-field variables use the Far-Field Calculation node. Select a domain for the far-field calculation. Then select the boundaries where the algorithm integrates the near field, and enter a name for the far electric field. Also specify if symmetry planes are used in the model when calculating the far-field variable. The symmetry planes have to coincide with one of the Cartesian coordinate planes. For each of these planes it is possible to select the type of symmetry to use, which can be of either symmetry in \(E\) (PMC) or symmetry in \(H\) (PEC). Make the choice here match the boundary condition used for the symmetry boundary. Using these settings, the parts of the geometry that are not in the model for symmetry reasons can be included in the far-field analysis.

The Far-Field Domain and the Far-Field Calculation nodes get their selections automatically, if the Perfectly Matched Layer (PML) feature has been defined before adding the Far-Field Domain feature.

For each variable name entered, the software generates functions and variables, which represent the vector components of the far electric field. The names of these variables
are constructed by appending the names of the independent variables to the name entered in the field.

For example, the name \( \text{Efar} \) is entered and the geometry is Cartesian with the independent variables \( x, y, \) and \( z \), the generated variables get the names \( \text{Efarx}, \text{Efary}, \) and \( \text{Efarz} \).

If, on the other hand, the geometry is axisymmetric with the independent variables \( r, \phi, \) and \( z \), the generated variables get the names \( \text{Efar}_r, \text{Efar}_\phi, \) and \( \text{Efar}_z \).

In 2D, the software only generates the variables for the nonzero field components. The physics interface name also appears in front of the variable names so they can vary, but typically look something like \( \text{emw.Efarz} \) and so forth.

To each of the generated variables, there is a corresponding function with the same name. This function takes the vector components of the evaluated far-field direction as arguments.

The vector components also can be interpreted as a position. For example, assume that the variables \( dx, dy, \) and \( dz \) represent the direction in which the far electric field is evaluated.

The expression

\[
\text{Efar}(dx, dy, dz)
\]

gives the value of the far electric field in this direction. To give the direction as an angle, use the expression

\[
\text{Efar}(\sin(\theta)\cos(\phi), \sin(\theta)\sin(\phi), \cos(\theta))
\]

where the variables \( \theta \) and \( \phi \) are defined to represent the angular direction \((\theta, \phi)\) in radians. The magnitude of the far field and its value in dB are also generated as the variables \( \text{normEfar} \) and \( \text{normdBfar} \), respectively.

---

**Far-Field Calculations Theory**

**The Far Field Plots**

The **Far Field** plots are available with this module to plot the value of a global variable (the far field norm, \( \text{normEfar} \) and \( \text{normdBfar} \), or components of the far field variable \( \text{Efar} \)).
The variables are plotted for a selected number of angles on a unit circle (in 2D) or a unit sphere (in 3D). The angle interval and the number of angles can be manually specified. For 2D Far Field plots also the reference direction from which the angle is measured and the normal to the plane the far field is computed for can be specified. For 3D Far Field plots you also specify an expression for the surface color.

The main advantage with the Far Field plot, as compared to making a Line Graph, is that the unit circle/sphere that you use for defining the plot directions, is not part of your geometry for the solution. Thus, the number of plotting directions is decoupled from the discretization of the solution domain.

Available variables are:

- Effective isotropic radiated power ($\text{emw}.\text{EIRP}$)
- Effective isotropic radiated power, dB ($\text{emw}.\text{EIRPdB}$)
- Far-field gain ($\text{emw}.\text{gainEfar}$)
- Far-field gain, dB ($\text{emw}.\text{gainBEfar}$)
- Far-field norm ($\text{emw}.\text{normEfar}$)
- Far-field norm, dB ($\text{emw}.\text{normdBEfar}$)
- Far-field variable, $x$ component ($\text{emw}.\text{Efarx}$)
- Far-field variable, $y$ component ($\text{emw}.\text{Efary}$)
- Far-field variable, $z$ component ($\text{emw}.\text{Efarz}$)
- Realized far-field gain ($\text{emw}.\text{rGainEfar}$)
- Realized far-field realized gain, dB ($\text{emw}.\text{rGainBEfar}$)

Additional variables are provided for 3D models.

- Axial ratio ($\text{emw}.\text{axialRatio}$)
- Axial ratio, dB ($\text{emw}.\text{axialRatiodB}$)
- Bistatic radar cross section ($\text{emw}.\text{bRCS3D}$)
- Far-field variable, phi component ($\text{emw}.\text{Efarphi}$)
- Far-field variable, theta component ($\text{emw}.\text{Efartheta}$)
Additional variable is provided for 2D models.

- Bistatic radar cross section per unit length ($\omega_{\text{B}, \text{RCS2D}}$)

<table>
<thead>
<tr>
<th>Example</th>
<th>Description</th>
<th>Application Library path</th>
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</thead>
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<tr>
<td>2D example with a Polar Plot Group—Radar Cross Section:</td>
<td>Application Library path RF_Module/Scattering_and_RCS/radar_cross_section.</td>
<td></td>
</tr>
<tr>
<td>2D axisymmetric example with a Polar Plot Group and a 3D Plot Group—Conical Antenna:</td>
<td>Application Library path RF_Module/Antennas/conical_antenna.</td>
<td></td>
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</tbody>
</table>

- Far-Field Support in the Electromagnetic Waves, Frequency Domain Interface
- Far Field in the COMSOL Multiphysics Reference Manual
S-Parameters and Ports

In this section:

- S-Parameters in Terms of Voltage and Electric Field
- S-Parameter Calculations
- S-Parameter Variables
- Port Sweeps/Manual Terminal Sweeps and Touchstone Export

S-Parameters in Terms of Voltage and Electric Field

Scattering parameters (or S-parameters) are complex-valued, frequency dependent matrices describing the transmission and reflection of electromagnetic waves at different ports of devices like filters, antennas, waveguide transitions, and transmission lines. S-parameters originate from transmission-line theory and are defined in terms of transmitted and reflected voltage waves. All ports are assumed to be connected to matched loads/feeds, that is, there is no reflection directly at a port.

For a device with \( n \) ports, the S-parameters are

\[
S = \begin{bmatrix}
S_{11} & S_{12} & \cdots & S_{1n} \\
S_{21} & S_{22} & \cdots & \cdots \\
\vdots & \vdots & \ddots & \cdots \\
S_{n1} & \cdots & \cdots & S_{nn}
\end{bmatrix}
\]

where \( S_{11} \) is the voltage reflection coefficient at port 1, \( S_{21} \) is the voltage transmission coefficient from port 1 to port 2, and so on. The time average power reflection/transmission coefficients are obtained as \( |S_{ij}|^2 \).

Now, for high-frequency problems, voltage is not a well-defined entity, and it is necessary to define the scattering parameters in terms of the electric field.

For details on how COMSOL Multiphysics calculates the S-parameters, see S-Parameter Calculations.
S-Parameter Calculations

The RF interfaces have built-in support for S-parameter calculations. Use a Port boundary feature for each port in the model. For connecting transmission lines and other lumped feeds, use a lumped port that approximates a connecting transmission lines or a voltage source with a known internal impedance. The lumped port should only be used when the port width is much smaller than the wavelength.

- For more details about lumped ports, see Lumped Ports with Voltage Input.
- See Port and Lumped Port for instructions to set up a model.

For a detailed description of how to model numerical ports with a boundary mode analysis, see Waveguide Adapter: Application Library path RF_Module/Transmission_Lines_and_Waveguides/waveguide_adapter.

S-Parameter Variables

This module automatically generates variables for the S-parameters. The port names (use numbers for sweeps to work correctly) determine the variable names. If, for example, there are two ports with the numbers 1 and 2 and Port 1 is the inport, the software generates the variables $S_{11}$ and $S_{21}$. $S_{11}$ is the S-parameter for the reflected wave and $S_{21}$ is the S-parameter for the transmitted wave. For convenience, two variables for the S-parameters on a dB scale, $S_{11dB}$ and $S_{21dB}$, are also defined using the following relation:

$$S_{11dB} = 20 \log_{10}(|S_{11}|)$$

The model and physics interface names also appear in front of the variable names so they can vary. The S-parameter variables are added to the predefined quantities in appropriate plot lists.

Port Sweeps/Manual Terminal Sweeps and Touchstone Export

The Port Sweep Settings section in the Electromagnetic Waves interface describes how to cycle through the ports, compute the entire S-matrix and export it to a Touchstone file.
H-Bend Waveguide 3D: Application Library path RF_Module/
Transmission_Lines_and_Waveguides/h_bend_waveguide_3d
Lumped Ports with Voltage Input

In this section:
- About Lumped Ports
- Lumped Port Parameters
- Lumped Ports in the RF Module

About Lumped Ports

The ports described in the S-Parameters and Ports section require a detailed specification of the mode, including the propagation constant and field profile. In situations with the mode being TEM, a lumped port might be a better choice. It also allows for connecting to an electrical circuit. It is not as accurate as the ordinary port in terms of calculating S-parameters, but it is easier to use. Lumped ports are used to model a connecting transmission line or as a voltage or current source applied between electrodes. For example, apply a lumped port directly to a printed circuit board or to the transmission line feed of a device. The lumped port must be applied between two metallic objects separated by a distance much smaller than the wavelength, that is a local quasi-static approximation must be justified. This is because the concept of port or gap voltage breaks down unless the gap is much smaller than the local wavelength.

A lumped port specified as an input port calculates the impedance, $Z_{\text{port}}$, and $S_{11}$ S-parameter for that port. The parameters are directly given by the relations

$$Z_{\text{port}} = \frac{V_{\text{port}}}{I_{\text{port}}}$$

$$S_{11} = \frac{V_{\text{port}} - V_{\text{in}}}{V_{\text{in}}}$$

where $V_{\text{port}}$ is the extracted voltage for the port given by the electric field line integral between the terminals averaged over the entire port. The current $I_{\text{port}}$ is the averaged total current over all cross sections parallel to the terminals. Ports not specified as input ports only return the extracted voltage and current.
**Lumped Port Parameters**

In transmission line theory voltages and currents are dealt with rather than electric and magnetic fields, so the lumped port provides an interface between them. The requirement on a lumped port is that the feed point must be similar to a transmission line feed, so its gap must be much less than the wavelength. It is then possible to define the electric field from the voltage as

\[ V = \int_{h} E \cdot dl = \int_{h} (E \cdot a_h) dl \]

where \( h \) is a line between the terminals at the beginning of the transmission line, and the integration is going from positive (phase) \( V \) to ground. The current is positive going into the terminal at positive \( V \).

The transmission line current can be represented with a surface current at the lumped port boundary directed opposite to the electric field.

The impedance of a transmission line is defined as

\[ Z = \frac{V}{I} \]

and in analogy to this an equivalent surface impedance is defined at the lumped port boundary

\[ \eta = \frac{E \cdot a_h}{J_s \cdot (-a_h)} \]

To calculate the surface current density from the current, integrate along the width, \( w \), of the transmission line

\[ I = \int_{w} (n \times J_s) \cdot dl = -\int_{w} (J_s \cdot a_h) dl \]
where the integration is taken in the direction of $\mathbf{a}_h \times \mathbf{n}$. This gives the following relation between the transmission line impedance and the surface impedance

\[
Z = \frac{V}{I} = \frac{\int (\mathbf{E} \cdot \mathbf{a}_h) dl}{\int (\mathbf{J}_s \cdot \mathbf{a}_h) dl} = \frac{\eta h}{w} \Rightarrow \eta = \frac{Z w}{h}
\]

where the last approximation assumed that the electric field is constant over the integrations. A similar relationship can be derived for coaxial cables

\[
\eta = Z \frac{2\pi}{\ln \frac{b}{a}}
\]

The transfer equations above are used in an impedance type boundary condition, relating surface current density to tangential electric field via the surface impedance.

\[
\mathbf{n} \times (\mathbf{H}_1 - \mathbf{H}_2) + \frac{1}{\eta} \mathbf{n} \times (\mathbf{E} \times \mathbf{n}) = \frac{1}{\eta} \mathbf{n} \times (\mathbf{E}_0 \times \mathbf{n})
\]

where $\mathbf{E}$ is the total field and $\mathbf{E}_0$ the incident field, corresponding to the total voltage, $V$, and incident voltage, $V_0$, at the port.

---

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When using the lumped port as a circuit port, the port voltage is fed as input to the circuit and the current computed by the circuit is applied as a uniform current density, that is as a surface current condition. Thus, an open (unconnected) circuit port is just a continuity condition.

### Lumped Ports in the RF Module

Not all models can use lumped ports due to the polarization of the fields and how sources are specified. For the physics interfaces and study types that support the lumped port, the Lumped Port is available as a boundary feature. See Lumped Port for instructions to set up this feature.
LUMPED PORT VARIABLES

Each lumped port generates variables that are accessible to the user. Apart from the S-parameter, a lumped port condition also generates the following variables.

<table>
<thead>
<tr>
<th>NAME</th>
<th>DESCRIPTION</th>
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<tr>
<td>Vport</td>
<td>Extracted port voltage</td>
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<tr>
<td>Iport</td>
<td>Port current</td>
</tr>
<tr>
<td>Zport</td>
<td>Port impedance</td>
</tr>
</tbody>
</table>

For example, a lumped port with port number 1, defined in the first geometry, for the Electromagnetic Waves interface with the tag emw, defines the port impedance variable emw.Zport_1.

RF Coil: Application Library path RF_Module/Passive_Devices/rf_coil
Lossy Eigenvalue Calculations

In mode analysis and eigenfrequency analysis, it is usually the primary goal to find a propagation constant or an eigenfrequency. These quantities are often real valued although it is not necessary. If the analysis involves some lossy part, like a nonzero conductivity or an open boundary, the eigenvalue is complex. In such situations, the eigenvalue is interpreted as two parts (1) the propagation constant or eigenfrequency and (2) the damping in space and time.

In this section:
- Eigenfrequency Analysis
- Mode Analysis

Eigenfrequency Analysis

The eigenfrequency analysis solves for the eigenfrequency of a model. The time-harmonic representation of the fields is more general and includes a complex parameter in the phase

\[ \mathbf{E}(\mathbf{r}, t) = \text{Re}(\mathbf{E}(\mathbf{r}, t)e^{j\omega t}) = \text{Re}(\mathbf{E}(\mathbf{r})e^{-\lambda t}) \]

where the eigenvalue, \((-\lambda) = -\delta + j\omega\), has an imaginary part representing the eigenfrequency, and a real part responsible for the damping. It is often more common to use the quality factor or Q-factor, which is derived from the eigenfrequency and damping

\[ Q_{\text{fact}} = \frac{\omega}{2|\delta|} \]
**VARIABLES AFFECTED BY EIGENFREQUENCY ANALYSIS**

The following list shows the variables that the eigenfrequency analysis affects:

<table>
<thead>
<tr>
<th>NAME</th>
<th>EXPRESSION</th>
<th>CAN BE COMPLEX</th>
<th>DESCRIPTION</th>
</tr>
</thead>
<tbody>
<tr>
<td>omega</td>
<td><code>imag(-lambda)</code></td>
<td>No</td>
<td>Angular frequency</td>
</tr>
<tr>
<td><code>damp</code></td>
<td><code>real(lambda)</code></td>
<td>No</td>
<td>Damping in time</td>
</tr>
<tr>
<td>Qfact</td>
<td><code>0.5*omega/abs(damp)</code></td>
<td>No</td>
<td>Quality factor</td>
</tr>
<tr>
<td><code>nu</code></td>
<td><code>omega/(2*pi)</code></td>
<td>No</td>
<td>Frequency</td>
</tr>
</tbody>
</table>

**NONLINEAR EIGENFREQUENCY PROBLEMS**

For some combinations of formulation, material parameters, and boundary conditions, the eigenfrequency problem can be nonlinear, which means that the eigenvalue enters the equations in another form than the expected second-order polynomial form. The following table lists those combinations:

<table>
<thead>
<tr>
<th>SOLVE FOR</th>
<th>CRITERION</th>
<th>BOUNDARY CONDITION</th>
</tr>
</thead>
<tbody>
<tr>
<td>E</td>
<td>Nonzero conductivity</td>
<td>Impedance boundary condition</td>
</tr>
<tr>
<td>E</td>
<td>Nonzero conductivity at adjacent domain</td>
<td>Scattering boundary condition</td>
</tr>
<tr>
<td>E</td>
<td>Analytical ports</td>
<td>Port boundary condition</td>
</tr>
</tbody>
</table>

These situations may require special treatment, especially since it can lead to “singular matrix” or “undefined value” messages if not treated correctly. Under normal circumstances, the automatically generated solver settings should handle the cases described in the table above. However, the following discussion provide some background to the problem of defining the eigenvalue linearization point. The complication is not only the nonlinearity itself, it is also the way it enters the equations. For example the impedance boundary conditions with nonzero boundary conductivity has the term

\[-(\lambda)\left[\frac{\varepsilon_0\varepsilon_{rbdn}\sqrt{\mu_{rbdn}}}{{\sqrt{\mu_0\varepsilon_{rbdn}}} + \mu_{rbdn} + \frac{\varepsilon_{rbdn}}{(\lambda)^2\varepsilon_0} - (\lambda)\varepsilon_0}\right]\]

where \((-\lambda) = -\delta + j\omega\). When the solver starts to solve the eigenfrequency problem it linearizes the entire formulation with respect to the eigenvalue around a certain linearization point. By default this linearization point is set to the value provided to the **Search for eigenvalues around** field, for the three cases listed in the table above.

Normally, this should be a good value for the linearization point. For instance, for the
impedance boundary condition, this avoids setting the eigenvalue \( \lambda \) to zero in the denominator in the equation above. For other cases than those listed in the table above, the default linearization point is zero.

If the default values for the linearization point is not suitable for your particular problem, you can manually provide a “good” linearization point for the eigenvalue solver. Do this in the Eigenvalue node (not the Eigenfrequency node) under the Solver Sequence node in the Study branch of the Model Builder. A solver sequence can be generated first. In the Linearization Point section, select the Transform point check box and enter a suitable value in the Point field. For example, if it is known that the eigenfrequency is close to 1 GHz, enter the eigenvalue 1[GHz] in the field.

In many cases it is enough to specify a good linearization point and then solve the problem once. If a more accurate eigenvalue is needed, an iterative scheme is necessary:

1. Specify that the eigenvalue solver only searches for one eigenvalue. Do this either for an existing solver sequence in the Eigenvalue node or, before generating a solver sequence, in the Eigenfrequency node.
2. Solve the problem with a “good” linearization point. As the eigenvalue shifts, use the same value with the real part removed from the eigenvalue or, equivalently, use the real part of the eigenfrequency.
3. Extract the eigenvalue from the solution and update the linearization point and the shift.
4. Repeat until the eigenvalue does not change more than a desired tolerance.

**Mode Analysis**

In mode analysis and boundary mode analysis COMSOL Multiphysics solves for the propagation constant. The time-harmonic representation is almost the same as for the eigenfrequency analysis, but with a known propagation in the out-of-plane direction

\[
\mathbf{E}(r, t) = \text{Re}(\mathbf{E}(r)e^{j\omega t - jkz}) = \text{Re}(\mathbf{E}(r)e^{j\omega t - \alpha z})
\]
The spatial parameter, $\alpha = \delta_z + j\beta = -\lambda$, can have a real part and an imaginary part. The propagation constant is equal to the imaginary part, and the real part, $\delta_z$, represents the damping along the propagation direction.

**Variables Influenced by Mode Analysis**

The following table lists the variables that are influenced by the mode analysis:

<table>
<thead>
<tr>
<th>NAME</th>
<th>EXPRESSION</th>
<th>CAN BE COMPLEX</th>
<th>DESCRIPTION</th>
</tr>
</thead>
<tbody>
<tr>
<td>beta</td>
<td>imag(-lambda)</td>
<td>No</td>
<td>Propagation constant</td>
</tr>
<tr>
<td>dampz</td>
<td>real(-lambda)</td>
<td>No</td>
<td>Attenuation constant</td>
</tr>
<tr>
<td>dampzdB</td>
<td>20<em>log10(exp(1))</em></td>
<td>No</td>
<td>Attenuation per meter in dB</td>
</tr>
<tr>
<td>neff</td>
<td>j*lambda/k0</td>
<td>Yes</td>
<td>Effective mode index</td>
</tr>
</tbody>
</table>

For an example of Boundary Mode Analysis, see the model *Polarized Circular Ports*: Application Library path `RF_Module/Tutorials/ polarized_circular_ports`.

- For a list of the studies available by physics interface, see *The RF Module Physics Interface Guide*
- *Studies and Solvers* in the *COMSOL Multiphysics Reference Manual*
Connecting to Electrical Circuits

In this section:

- About Connecting Electrical Circuits to Physics Interfaces
- Connecting Electrical Circuits Using Predefined Couplings
- Connecting Electrical Circuits by User-Defined Couplings
- Solving
- Postprocessing

Connecting a 3D Electromagnetic Wave Model to an Electrical Circuit: Application Library path RF_Module/Transmission_Lines_and_Waveguides/coaxial_cable_circuit

About Connecting Electrical Circuits to Physics Interfaces

This section describes the various ways electrical circuits can be connected to other physics interfaces in COMSOL Multiphysics. If you are not familiar with circuit modeling, it is recommended that you review the Theory for the Electrical Circuit Interface.

In general electrical circuits connect to other physics interfaces via one or more of three special circuit features:

- External I vs. U
- External U vs. I
- External I-Terminal
These features either accept a voltage measurement from the connecting non-circuit physics interface and return a current from the Electrical Circuit interface or the other way around.

The “External” features are considered “ideal” current or voltage sources by the Electrical Circuit interface. Hence, you cannot connect them directly in parallel (voltage sources) or in series (current sources) with other ideal sources. This results in the error message *The DAE is structurally inconsistent*. A workaround is to provide a suitable parallel or series resistor, which can be tuned to minimize its influence on the results.

**Connecting Electrical Circuits Using Predefined Couplings**

In addition to these circuit features, interfaces in the AC/DC Module, RF Module, MEMS Module, Plasma Module, and Semiconductor Module (the modules that include the Electrical Circuit interface) also contain features that provide couplings to the Electrical Circuit interface by accepting a voltage or a current from one of the specific circuit features (External I vs. U, External U vs. I, and External I-Terminal).

This coupling is typically activated when:

- A choice is made in the *Settings* window for the non-circuit physics interface feature, which then announces (that is, includes) the coupling to the Electrical Circuit interface. Its voltage or current is then included to make it visible to the connecting circuit feature.

- A voltage or current that has been announced (that is, included) is selected in a feature node’s *Settings* window.

These circuit connections are supported in Lumped Ports.

**Connecting Electrical Circuits by User-Defined Couplings**

A more general way to connect a physics interface to the Electrical Circuit interface is to:

- Apply the voltage or current from the connecting “External” circuit feature as an excitation in the non-circuit physics interface.
• Define your own voltage or current measurement in the non-circuit physics interface using variables, coupling operators and so forth.

• In the Settings window for the Electrical Circuit interface feature, selecting the User-defined option and entering the name of the variable or expression using coupling operators defined in the previous step.

**DETERMINING A CURRENT OR VOLTAGE VARIABLE NAME**

To determine a current or voltage variable name, look at the Dependent Variables node under the Study node. To do this:

1. In the Model Builder, right-click the Study node and select Show Default Solver.

2. Expand the Solver>Dependent Variables node and click the state node, in this example, Current through device R1 (comp1.currents). The variable name is shown on the Settings window for State.

![Diagram showing Model Builder and Settings windows](image)

Typically, voltage variables are named `cir.Xn_v` and current variables `cir.Xn_i`, where `n` is the “External” device number—1, 2, and so on.
**Solving**

Some modeling errors lead to the error message *The DAE is structurally inconsistent*, being displayed when solving. This typically occurs from having an open current loop, from connecting voltage sources in parallel, or connecting current sources in series.

In this respect, the predefined coupling features are also treated as (ideal) voltage or current sources. The remedy is to close current loops and to connect resistors in series with voltage sources or in parallel with current sources.

**Postprocessing**

The Electrical Circuits interface, unlike most of the other physics interfaces, solves for a relatively large number of Global dependent variables (such as voltages and currents), instead of solving for a few space-varying fields (such as temperature or displacement). For this reason, the Electrical Circuit interface does not provide default plots when computing a Study.

The physics interface defines a number of variables that can be used in postprocessing. All variables defined by the Electrical Circuit interface are of a global scope, and can be evaluated in a *Global Evaluation* node (under *Derived Values*). In addition, the time evolution or dependency on a parameter can be plotted in a *Global* plot (under a *1D Plot Group* node).

The physics interface defines a Node voltage variable for each electrical node in the circuit, with name *cir.v_name*, where *cir* is the physics interface Label and *<name>* is the node Name. For each two pin component, the physics interface also defines variables containing the voltage across it and the current flowing through it.

In the *COMSOL Multiphysics Reference Manual*:

- *Derived Values and Tables* and *Global Evaluation*
- *Plot Groups and Plots* and *Global*
SPICE Import

The circuit definition in COMSOL Multiphysics adheres to the SPICE format developed at the University of California, Berkeley (Ref. 1). SPICE netlists can be imported and the corresponding circuit nodes are generated in the COMSOL Multiphysics model. Most circuit simulators can export to this format or some version of it.

The Electrical Circuit interface supports the following device models:

<table>
<thead>
<tr>
<th>STATEMENT</th>
<th>DEVICE MODEL</th>
</tr>
</thead>
<tbody>
<tr>
<td>R</td>
<td>Resistor</td>
</tr>
<tr>
<td>C</td>
<td>Capacitor</td>
</tr>
<tr>
<td>L</td>
<td>Inductor</td>
</tr>
<tr>
<td>V</td>
<td>Voltage Source</td>
</tr>
<tr>
<td>I</td>
<td>Current Source</td>
</tr>
<tr>
<td>E</td>
<td>Voltage-Controlled Voltage Source</td>
</tr>
<tr>
<td>F</td>
<td>Current-Controlled Current Source</td>
</tr>
<tr>
<td>G</td>
<td>Voltage-Controlled Current Source</td>
</tr>
<tr>
<td>H</td>
<td>Current-Controlled Voltage Source</td>
</tr>
<tr>
<td>D</td>
<td>Diode</td>
</tr>
<tr>
<td>Q</td>
<td>NPN BJT and PNP BJT</td>
</tr>
<tr>
<td>M</td>
<td>n-Channel MOSFET and p-Channel MOSFET</td>
</tr>
<tr>
<td>X</td>
<td>Subcircuit Instance</td>
</tr>
</tbody>
</table>

Statements corresponding to multiple devices are resolved by parsing the associated .model statement. The physics interface also supports the .subckt statement, which is represented in COMSOL by a Subcircuit Definition node, and the .include statement. SPICE commands are interpreted case-insensitively. The statement defining each device is also interpreted as the Device name.

According to SPICE specification, the first line in the netlist file is assumed to be the title of the netlist and it is ignored by the parser.
SPICE Export

The SPICE Export functionality creates a SPICE netlist file containing a description of the circuit represented by the physics interface. This functionality can be accessed from the physics interface context menu (right-click the physics interface node and select Export SPICE Netlist). After specifying a file name, the circuit is exported and messages from the export process display in the Messages window. During the export process, a series of operations are performed:

- In order to avoid conflicts, each component must be identified by a unique Device name. If one or more components have the same device name, the export operation fails and an error message is displayed. All characters in a Device name that are not letters, digits or underscores are replaced by underscores.

- According to the SPICE specification, each circuit must have a node with name 0, which is assumed to be the only ground node. When exporting a circuit, any node with name 0 that is not connected to a Ground component is exported with a different node name. All nodes that are connected to a Ground components are exported as a merged node with name 0. The Messages window shows a log message if these operations are performed, showing the name of the renamed or merged nodes.

- All characters in node names that are not letters, digits or underscores are replaced by underscores.

- Some components (most notably, the External components used to couple to other physics interfaces) cannot be exported to a SPICE netlist. These components are ignored during the export process, and a message is shown in the Messages window. Note that this can change the exported circuit, since some components are then missing.

- Subcircuit definitions are added as .subckt statements in the netlist. Semiconductor devices (such as MOSFETs, BJTs, and diodes) are exported as a SPICE device with a corresponding .model statement.

The title of the exported netlist file is the model’s file name, and the time, date and the version of COMSOL Multiphysics is added as a comment in the netlist file.

Reference for SPICE Import and Export

1. http://bwrc.eecs.berkeley.edu/Classes/IcBook/SPICE/
**Reduced Order Modeling**

When designing bandpass-filter type high-Q devices in the frequency domain, it may be necessary to apply many frequency samples to describe the passband accurately. The reduced order modeling technique can help accelerate the modeling of such devices as a fine frequency resolution can be used for a modest simulation time.

Two simulation methods: the asymptotic waveform evaluation (AWE) and frequency-domain modal methods, both are designed to help overcome the conventional issue of a longer simulation time when using a very fine frequency resolution or running a very wide band simulation. The AWE is efficient when it comes to describing smooth frequency responses with a single resonance or no resonance at all. The frequency-domain modal method, meanwhile, is useful for quickly analyzing multistage filters or filters of a high number of elements that have multiple resonances in a target passband.

In this section:
- Asymptotic Waveform Evaluation (AWE) Method
- Frequency-Domain Modal Method

| AWE Solver and Frequency-Domain Modal in the COMSOL Multiphysics Reference Manual |

**Asymptotic Waveform Evaluation (AWE) Method**

The AWE method is very useful when simulating resonant circuits, especially single-resonance bandpass-filter type devices with many frequency points. When using the AWE option, the simulation time with a much finer frequency resolution can be almost the same as a coarse resolution regular Frequency Domain simulation.

**AWE CONFIGURATION IN FREQUENCY DOMAIN STUDY STEP SETTINGS**

The solver performs fast adaptive frequency sweeping using an asymptotic waveform evaluation method. To trigger the AWE, the following steps are required:

- Expand **Study Extensions section** in **Frequency Domain** study step settings.
• Click the **Use asymptotic waveform evaluation** check box in the *Study Extensions* section.

• Specify the user input for the **AWE expressions**.

**CHOOSING APPROPRIATE AWE EXPRESSION**

The simulation time may vary depending on the user input for the **AWE expressions**. Any model variable works as an AWE expression, so long as it has a smooth curve shape like a Gaussian pulse as a function of frequency. The absolute value of $S_{21}$, $\text{abs}(\text{emw.S21})$, often works as the input for the AWE expression in the case of two-port bandpass filters. For one-port devices like antennas, the absolute value of $S_{11}$ is a good choice. If the frequency response of the AWE expression contains an infinite gradient — the case for the $S_{11}$ value of an antenna with excellent impedance matching at a single frequency point — the simulation will take longer to complete because it requires many data points to describe the sharp dip.

**DATA MANAGEMENT**

With a very fine frequency step simulation, the solutions contain a lot of data. As a result, the model file size will increase tremendously when it is saved. By selecting the **Store fields in output** check box in the *Values of Dependent Variables* section of the *Frequency Domain* study step settings, it is possible to define for what part of the model the computed solution should be saved. When only S-parameters are of interest, it is not necessary to store all of the field solutions. Instead, only store the field on the selections for the port and lumped port boundaries, as those will be used for the S-parameter calculations.

In the *Values of Dependent Variables* section, change the selection in the **Store fields in output** combo box from **All** to **For selections** and then add the explicit selections that include the port and lumped port boundaries. The explicit selection can be easily created from the port and lumped port feature by clicking **Create Selection** icon in the **Boundary Selection** settings once the selection is specified.

**Frequency-Domain Modal Method**

Bandpass-frequency responses of a passive circuit often result from a combination of multiple resonances. **Eigenfrequency** analysis is used for capturing the resonance
frequencies of a device. In a subsequent step, the information from the Eigenfrequency solutions can be reused in a Frequency-Domain Modal study to generate a solution for the driven problem.

**Eigenfrequency Study Step Settings**

To perform a Frequency-Domain Modal analysis, it is necessary to configure the Eigenfrequency study step properly. As the output of the Eigenfrequency study may include unphysical results (so called spurious modes), appropriate settings help refine the Eigenfrequency study results.

- Set Eigenfrequency search method to Manual
- Adjust Desired number of eigenfrequencies if necessary.
- Set Search for eigenfrequencies around to the estimate of the lowest passband frequency.
- Set Eigenfrequency search method around shift to Larger real part.

**Data Management**

The Store fields in output check box in the Values of Dependent Variables section can be applied to the Frequency-Domain Modal study — if you are interested only in S-parameters. By storing solutions only on port or lumped port boundaries, the saved model file size will decrease a lot.

---

Note that the phase of the computed S-parameters in the Frequency-Domain Modal study can be different from that of the regular frequency sweep model due to that all relevant eigenmodes might not be included in the simulation. It is recommended to perform an initial eigenfrequency investigation, to find all relevant eigenfrequencies contributing to the frequency response. If not all relevant eigenfrequencies are included in the simulation, the results are compatible only for phase-independent S-parameter values, such as dB-scaled, absolute value, reflectivity, or transmittivity.
Cascaded Rectangular Cavity Filter: Application Library path
RF_Module/Filters/cascaded_cavity_filter

Coupled-Line Bandpass Filter: Application Library path
RF_Module/Filters/coupled_line_filter

Coplanar Waveguide Bandpass Filter: Application Library path
RF_Module/Filters/cpw_bandpass_filter
This chapter contains a review of the basic theory of electromagnetics, starting with Maxwell’s equations, and the theory for some special calculations: S-parameters, lumped port parameters, and far-field analysis. There is also a list of electromagnetic quantities with their SI units and symbols.

In this chapter:

- Maxwell’s Equations
- Special Calculations
- Electromagnetic Quantities

See also:

- Theory for the Electromagnetic Waves Interfaces
- Theory for the Electrical Circuit Interface
Maxwell’s Equations

In this section:

- Introduction to Maxwell’s Equations
- Constitutive Relations
- Potentials
- Electromagnetic Energy
- Material Properties
- Boundary and Interface Conditions
- Phasors

Introduction to Maxwell’s Equations

Electromagnetic analysis on a macroscopic level involves solving Maxwell’s equations subject to certain boundary conditions. Maxwell’s equations are a set of equations, written in differential or integral form, stating the relationships between the fundamental electromagnetic quantities. These quantities are the:

- Electric field intensity \( \mathbf{E} \)
- Electric displacement or electric flux density \( \mathbf{D} \)
- Magnetic field intensity \( \mathbf{H} \)
- Magnetic flux density \( \mathbf{B} \)
- Current density \( \mathbf{J} \)
- Electric charge density \( \rho \)

The equations can be formulated in differential or integral form. The differential form are presented here, because it leads to differential equations that the finite element method can handle. For general time-varying fields, Maxwell’s equations can be written as

\[
\nabla \times \mathbf{H} = \mathbf{J} + \frac{\partial \mathbf{D}}{\partial t} \\
\n\nabla \times \mathbf{E} = \frac{\partial \mathbf{B}}{\partial t} \\
\n\n\nabla \cdot \mathbf{D} = \rho \\
\n\n\nabla \cdot \mathbf{B} = 0
\]
The first two equations are also referred to as Maxwell-Ampère’s law and Faraday’s law, respectively. Equation three and four are two forms of Gauss’ law, the electric and magnetic form, respectively.

Another fundamental equation is the equation of continuity, which can be written as

\[ \nabla \cdot \mathbf{J} = \frac{\partial \mathbf{P}}{\partial t} \]

Out of the five equations mentioned, only three are independent. The first two combined with either the electric form of Gauss’ law or the equation of continuity form such an independent system.

**Constitutive Relations**

To obtain a closed system, the constitutive relations describing the macroscopic properties of the medium, are included. They are given as

\[ \mathbf{D} = \varepsilon_0 \mathbf{E} + \mathbf{P} \]

\[ \mathbf{B} = \mu_0 \left( \mathbf{H} + \mathbf{M} \right) \]

\[ \mathbf{J} = \sigma \mathbf{E} \]

Here \( \varepsilon_0 \) is the permittivity of vacuum, \( \mu_0 \) is the permeability of vacuum, and \( \sigma \) the electrical conductivity. In the SI system, the permeability of a vacuum is chosen to be \( 4\pi \times 10^{-7} \) H/m. The velocity of an electromagnetic wave in a vacuum is given as \( c_0 \) and the permittivity of a vacuum is derived from the relation

\[ \varepsilon_0 = \frac{1}{c^2_0 \mu_0} = 8.854 \times 10^{-12} \text{ F/m} = \frac{1}{36\pi} \times 10^{-9} \text{ F/m} \]

The electric polarization vector \( \mathbf{P} \) describes how the material is polarized when an electric field \( \mathbf{E} \) is present. It can be interpreted as the volume density of electric dipole moments. \( \mathbf{P} \) is generally a function of \( \mathbf{E} \). Some materials can have a nonzero \( \mathbf{P} \) also when there is no electric field present.

The magnetization vector \( \mathbf{M} \) similarly describes how the material is magnetized when a magnetic field \( \mathbf{H} \) is present. It can be interpreted as the volume density of magnetic dipole moments. \( \mathbf{M} \) is generally a function of \( \mathbf{H} \). Permanent magnets, however, have a nonzero \( \mathbf{M} \) also when there is no magnetic field present.

For linear materials, the polarization is directly proportional to the electric field,

\[ \mathbf{P} = \varepsilon_0 \chi_e \mathbf{E}, \] where \( \chi_e \) is the electric susceptibility. Similarly in linear materials, the
magnetization is directly proportional to the magnetic field, \( \mathbf{M} = \chi_m \mathbf{H} \), where \( \chi_m \) is the magnetic susceptibility. For such materials, the constitutive relations can be written

\[
\begin{align*}
\mathbf{D} &= \varepsilon_0 (1 + \chi_e) \mathbf{E} = \varepsilon_0 \varepsilon_r \mathbf{E} = \varepsilon \mathbf{E} \\
\mathbf{B} &= \mu_0 (1 + \chi_m) \mathbf{H} = \mu_0 \mu_r \mathbf{H} = \mu \mathbf{H}
\end{align*}
\]

The parameter \( \varepsilon_e \) is the relative permittivity and \( \mu_r \) is the relative permeability of the material. These are usually scalar properties but they can, for a general anisotropic material, be 3-by-3 tensors. The properties \( \varepsilon \) and \( \mu \) (without subscripts) are the permittivity and permeability of the material.

**GENERALIZED CONSTITUTIVE RELATIONS**

Generalized forms of the constitutive relations are well suited for modeling nonlinear materials. The relation used for the electric fields is

\[ \mathbf{D} = \varepsilon_0 \varepsilon_r \mathbf{E} + \mathbf{D}_r \]

The field \( \mathbf{D}_r \) is the *remanent displacement*, which is the displacement when no electric field is present.

Similarly, a generalized form of the constitutive relation for the magnetic field is

\[ \mathbf{B} = \mu_0 \mu_r \mathbf{H} + \mathbf{B}_r \]

where \( \mathbf{B}_r \) is the *remanent magnetic flux density*, which is the magnetic flux density when no magnetic field is present.

The relation defining the current density is generalized by introducing an externally generated current \( \mathbf{J}^e \). The resulting constitutive relation is

\[ \mathbf{J} = \sigma \mathbf{E} + \mathbf{J}^e \]

**Potentials**

Under certain circumstances it can be helpful to formulate the problems in terms of the electric scalar potential \( V \) and the magnetic vector potential \( \mathbf{A} \). They are given by the equalities

\[
\begin{align*}
\mathbf{B} &= \nabla \times \mathbf{A} \\
\mathbf{E} &= -\nabla V - \frac{\partial \mathbf{A}}{\partial t}
\end{align*}
\]
The defining equation for the magnetic vector potential is a direct consequence of the magnetic Gauss' law. The electric potential results from Faraday’s law.

**Electromagnetic Energy**

The electric and magnetic energies are defined as

\[
W_e = \int_V \left( \int_0^T \mathbf{E} \cdot \frac{\partial \mathbf{D}}{\partial t} \right) dV = \int_V \left( \int_0^T \mathbf{E} \cdot \frac{\partial \mathbf{D}}{\partial t} \right) dV
\]

\[
W_m = \int_V \left( \int_0^T \mathbf{H} \cdot \frac{\partial \mathbf{B}}{\partial t} \right) dV = \int_V \left( \int_0^T \mathbf{H} \cdot \frac{\partial \mathbf{B}}{\partial t} \right) dV
\]

The time derivatives of these expressions are the electric and magnetic power

\[
P_e = \int_V \mathbf{E} \cdot \frac{\partial \mathbf{D}}{\partial t} dV
\]

\[
P_m = \int_V \mathbf{H} \cdot \frac{\partial \mathbf{B}}{\partial t} dV
\]

These quantities are related to the resistive and radiative energy, or energy loss, through Poynting’s theorem (Ref. 3)

\[
-\int_V \left( \mathbf{E} \cdot \frac{\partial \mathbf{D}}{\partial t} + \mathbf{H} \cdot \frac{\partial \mathbf{B}}{\partial t} \right) dV = \int_V \mathbf{J} \cdot \mathbf{E} dV + \oint_S \left( \mathbf{E} \times \mathbf{H} \right) \cdot \mathbf{n} dS
\]

where \( V \) is the computation domain and \( S \) is the closed boundary of \( V \).

The first term on the right-hand side represents the resistive losses,

\[
P_h = \int_V \mathbf{J} \cdot \mathbf{E} dV
\]

which result in heat dissipation in the material. (The current density \( \mathbf{J} \) in this expression is the one appearing in Maxwell-Ampère’s law.)

The second term on the right-hand side of Poynting’s theorem represents the radiative losses,

\[
P_r = \oint_S \left( \mathbf{E} \times \mathbf{H} \right) \cdot \mathbf{n} dS
\]

The quantity \( S = \mathbf{E} \times \mathbf{H} \) is called the Poynting vector.

Under the assumption the material is linear and isotropic, it holds that
By interchanging the order of differentiation and integration (justified by the fact that
the volume is constant and the assumption that the fields are continuous in time), this
equation results:

\[
\begin{align*}
\mathbf{E} \cdot \frac{\partial \mathbf{D}}{\partial t} &= \varepsilon \mathbf{E} \cdot \frac{\partial \mathbf{E}}{\partial t} = \frac{\partial}{\partial t} \left( \frac{1}{2} \varepsilon \mathbf{E} \cdot \mathbf{E} \right) \\
\mathbf{H} \cdot \frac{\partial \mathbf{B}}{\partial t} &= \frac{1}{\mu} \mathbf{B} \cdot \frac{\partial \mathbf{B}}{\partial t} = \frac{\partial}{\partial t} \left( \frac{1}{2\mu} \mathbf{B} \cdot \mathbf{B} \right)
\end{align*}
\]

The integrand of the left-hand side is the total electromagnetic energy density

\[
w = w_e + w_m = \frac{1}{2} \varepsilon \mathbf{E} \cdot \mathbf{E} + \frac{1}{2\mu} \mathbf{B} \cdot \mathbf{B}
\]

**Material Properties**

Until now, there has only been a formal introduction of the constitutive relations. These seemingly simple relations can be quite complicated at times. There are four main groups of materials where they require some consideration. A given material can belong to one or more of these groups. The groups are:

- Inhomogeneous materials
- Anisotropic materials
- Nonlinear materials
- Dispersive materials

The least complicated of the groups above is that of the inhomogeneous materials. An inhomogeneous medium is one where the constitutive parameters vary with the space coordinates, so that different field properties prevail at different parts of the material structure.

For anisotropic materials, the field relations at any point are different for different directions of propagation. This means that a 3-by-3 tensor is required to properly define the constitutive relations. If this tensor is symmetric, the material is often referred to as *reciprocal*. In these cases, the coordinate system can be rotated in such a way that a diagonal matrix is obtained. If two of the diagonal entries are equal, the material is *uniaxially anisotropic*. If none of the elements have the same value, the material is *biaxially anisotropic* (Ref. 2). An example where anisotropic parameters
Nonlinearity is the effect of variations in permittivity or permeability with the intensity of the electromagnetic field. This also includes hysteresis effects, where not only the current field intensities influence the physical properties of the material, but also the history of the field distribution.

Finally, dispersion describes changes in the velocity of the wave with wavelength. In the frequency domain, dispersion is expressed by a frequency dependence in the constitutive laws.

**MATERIAL PROPERTIES AND THE MATERIAL BROWSER**

All interfaces in the RF Module support the use of the COMSOL Multiphysics material database libraries. The electromagnetic material properties that can be stored in the materials database are:

- The electrical conductivity
- The relative permittivity
- The relative permeability
- The refractive index

The physics-specific domain material properties are by default taken from the material specification. The material properties are inputs to material laws or constitutive relations that are defined on the feature level below the physics interface node in the model tree. There is one editable default domain feature (wave equation) that initially represents a linear isotropic material. Domains with different material laws are specified by adding additional features. Some of the domain parameters can either be a scalar or a matrix (tensor) depending on whether the material is isotropic or anisotropic.

In a similar way, boundary, edge, and point settings are specified by adding the corresponding features. A certain feature might require one or several fields to be specified, while others generate the conditions without user-specified fields.

Materials and Modeling Anisotropic Materials in the *COMSOL Multiphysics Reference Manual*
Boundary and Interface Conditions

To get a full description of an electromagnetic problem, specify boundary conditions at material interfaces and physical boundaries. At interfaces between two media, the boundary conditions can be expressed mathematically as

\[
\begin{align*}
\mathbf{n}_2 \times (\mathbf{E}_1 - \mathbf{E}_2) &= 0 \\
\mathbf{n}_2 \cdot (\mathbf{D}_1 - \mathbf{D}_2) &= \rho_s \\
\mathbf{n}_2 \times (\mathbf{H}_1 - \mathbf{H}_2) &= \mathbf{J}_s \\
\mathbf{n}_2 \cdot (\mathbf{B}_1 - \mathbf{B}_2) &= 0
\end{align*}
\]

where \(\rho_s\) and \(\mathbf{J}_s\) denote surface charge density and surface current density, respectively, and \(\mathbf{n}_2\) is the outward normal from medium 2. Of these four conditions, only two are independent. One of the first and the fourth equations, together with one of the second and third equations, form a set of two independent conditions.

A consequence of the above is the interface condition for the current density,

\[
\mathbf{n}_2 \cdot (\mathbf{J}_1 - \mathbf{J}_2) = -\frac{\partial \rho_s}{\partial t}
\]

Interface Between a Dielectric and a Perfect Conductor

A perfect conductor has infinite electrical conductivity and thus no internal electric field. Otherwise, it would produce an infinite current density according to the third fundamental constitutive relation. At an interface between a dielectric and a perfect conductor, the boundary conditions for the \(\mathbf{E}\) and \(\mathbf{D}\) fields are simplified. If, say, subscript 1 corresponds to the perfect conductor, then \(\mathbf{D}_1 = 0\) and \(\mathbf{E}_1 = 0\) in the relations above. For the general time-varying case, it holds that \(\mathbf{B}_1 = 0\) and \(\mathbf{H}_1 = 0\) as well (as a consequence of Maxwell’s equations). What remains is the following set of boundary conditions for time-varying fields in the dielectric medium.

\[
\begin{align*}
-\mathbf{n}_2 \times \mathbf{E}_2 &= 0 \\
-\mathbf{n}_2 \times \mathbf{H}_2 &= \mathbf{J}_s \\
-\mathbf{n}_2 \cdot \mathbf{D}_2 &= \rho_s \\
-\mathbf{n}_2 \cdot \mathbf{B}_2 &= 0
\end{align*}
\]

Phasors

Whenever a problem is time-harmonic the fields can be written in the form...
Instead of using a cosine function for the time dependence, it is more convenient to use an exponential function, by writing the field as

\[ \mathbf{E}(\mathbf{r}, t) = \mathbf{E}(\mathbf{r}) \cos(\omega t + \phi) \]

The field \( \mathbf{E}(\mathbf{r}) \) is a *phasor* (phase vector), which contains amplitude and phase information of the field but is independent of \( t \). One thing that makes the use of phasors suitable is that a time derivative corresponds to a multiplication by \( j\omega \),

\[ \frac{\partial \mathbf{E}}{\partial t} = \text{Re}(j\omega \mathbf{E}(\mathbf{r})e^{j\omega t}) \]

This means that an equation for the phasor can be derived from a time-dependent equation by replacing the time derivatives by a factor \( j\omega \). All time-harmonic equations in this module are expressed as equations for the phasors. (The tilde is dropped from the variable denoting the phasor.).

---

When looking at the solution of a time-harmonic equation, it is important to remember that the field that has been calculated is a phasor and not a physical field.

For example, all plot functions visualize \( \text{Re}(\mathbf{E}(\mathbf{r})) \) by default, which is \( \mathbf{E} \) at time \( t = 0 \). To obtain the solution at a given time, specify a phase factor when evaluating and visualizing the results.
**Special Calculations**

In this section:
- **S-Parameter Calculations**
- **Far-Field Calculations Theory**
- **References**

---

**S-Parameter Calculations**

For high-frequency problems, voltage is not a well-defined entity, and it is necessary to define the scattering parameters (S-parameter) in terms of the electric field. To convert an electric field pattern on a port to a scalar complex number corresponding to the voltage in transmission line theory an eigenmode expansion of the electromagnetic fields on the ports needs to be performed. Assume that an eigenmode analysis has been performed on the ports 1, 2, 3, … and that the electric field patterns \( E_1, E_2, E_3, \ldots \) of the fundamental modes on these ports are known. Further, assume that the fields are normalized with respect to the integral of the power flow across each port cross section, respectively. This normalization is frequency dependent unless TEM modes are being dealt with. The port excitation is applied using the fundamental eigenmode, the mode with subscript 1. The computed electric field \( E_c \) on the port consists of the excitation plus the reflected field. That is, on the port boundary where there is an incident wave, the computed field can be expanded in terms of the mode fields as

\[
E_c = E_1 + \sum_{i=1} S_{i1} E_i,
\]

whereas on all other port boundaries, the computed field is given by

\[
E_c = \sum_{i=1} S_{i1} E_i
\]
The $S$-parameter for the mode with index $k$ is then given by multiplying with the conjugate of the mode field for mode $k$ and integrating over the port boundary. Since the mode fields for the different modes are orthogonal, the following relations are obtained for the $S$-parameters:

\[
S_{11} = \frac{\int \left( (E_c - E_1) \cdot E_1^* \right) dA_1}{\int (E_1 \cdot E_1^*) dA_1}
\]

\[
S_{21} = \frac{\int (E_2 \cdot E_2^*) dA_2}{\int (E_c \cdot E_2^*) dA_2}
\]

\[
S_{31} = \frac{\int (E_3 \cdot E_3^*) dA_3}{\int (E_c \cdot E_3^*) dA_3}
\]

and so on. To get $S_{22}$ and $S_{12}$, excite port number 2 in the same way.

**POWER FLOW NORMALIZATION**

The fields $E_1, E_2, E_3, \ldots$ should be normalized such that they represent the same power flow through the respective ports. The power flow is given by the time-average Poynting vector,

\[ S_{av} = \frac{1}{2} \text{Re}(E \times H^*) \]

The amount of power flowing out of a port is given by the normal component of the Poynting vector,

\[ \mathbf{n} \cdot S_{av} = \mathbf{n} \cdot \frac{1}{2} \text{Re}(E \times H^*) \]

Below the *cutoff frequency*, the power flow is zero, which implies that it is not possible to normalize the field with respect to the power flow below the cutoff frequency. But in this region the $S$-parameters are trivial and do not need to be calculated.

In the following subsections the power flow is expressed directly in terms of the electric field for TE, TM, and TEM waves.
CHAPTER 3: ELECTROMAGNETICS THEORY

**TE Waves**

For TE waves it holds that

\[ E = -Z_{TE}(n \times H) \]

where \( Z_{TE} \) is the wave impedance

\[ Z_{TE} = \frac{\omega \mu}{\beta} \]

\( \omega \) is the angular frequency of the wave, \( \mu \) the permeability, and \( \beta \) the propagation constant. The power flow then becomes

\[ n \cdot S_{av} = \frac{1}{2} n \cdot \text{Re}(E \times H^*) = -\frac{1}{2} \text{Re}(E \cdot (n \times H^*)) = \frac{1}{2Z_{TE}}|E|^2 \]

**TM Waves**

For TM waves it holds that

\[ H = \frac{1}{Z_{TM}}(n \times E) \]

where \( Z_{TM} \) is the wave impedance

\[ Z_{TM} = \frac{\beta}{\omega \varepsilon} \]

and \( \varepsilon \) is the permittivity. The power flow then becomes

\[ n \cdot S_{av} = \frac{1}{2} n \cdot \text{Re}(E \times H^*) = \frac{1}{2Z_{TM}}(n \cdot \text{Re}(E \times (n \times E^*))) = \frac{1}{2Z_{TM}}|n \times E|^2 \]

**TEM Waves**

For TEM waves it holds that

\[ H = \frac{1}{Z_{TEM}}(n \times E) \]

where \( Z_{TEM} \) is the wave impedance

\[ Z_{TEM} = \sqrt{\frac{\mu}{\varepsilon}} \]
The power flow then becomes

$$\mathbf{n} \cdot S_{av} = \frac{1}{2} \mathbf{n} \cdot \text{Re}(\mathbf{E} \times \mathbf{H}^*) = \frac{1}{2Z_{\text{TEM}}} |\mathbf{n} \times \mathbf{E}|^2 = \frac{1}{2Z_{\text{TEM}}} |\mathbf{E}|^2$$

where the last equality holds because the electric field is tangential to the port.

**Far-Field Calculations Theory**

The far electromagnetic field from, for example, antennas can be calculated from the near field using the Stratton-Chu formula. In 3D, this is:

$$\mathbf{E}_p = \frac{jk}{4\pi} \mathbf{r}_0 \times \int \left[ \mathbf{n} \times \mathbf{E} - \eta \mathbf{r}_0 \times (\mathbf{n} \times \mathbf{H}) \right] \exp(jk\mathbf{r} \cdot \mathbf{r}_0) dS$$

and in 2D it looks slightly different:

$$\mathbf{E}_p = \frac{jk}{4\pi} \mathbf{r}_0 \times \int \left[ \mathbf{n} \times \mathbf{E} - \eta \mathbf{r}_0 \times (\mathbf{n} \times \mathbf{H}) \right] \exp(jk\mathbf{r} \cdot \mathbf{r}_0) dS$$

In both cases the integration is performed on a closed boundary. In the scattered field formulation, where the total electric field is the sum of the background field and the scattered field, the far-field only gets contributions from the scattered field, since the contributions from the background field cancel out when integrated over all parts of the closed boundary.

For scattering problems, the far field in COMSOL Multiphysics is identical to what in physics is known as the “scattering amplitude”.

The antenna is located in the vicinity of the origin, while the far-field point $\mathbf{p}$ is taken at infinity but with a well-defined angular position $(\theta, \phi)$.

In the above formulas,

- $\mathbf{E}$ and $\mathbf{H}$ are the fields on the “aperture”—the surface $S$ enclosing the antenna.
- $\mathbf{r}_0$ is the unit vector pointing from the origin to the field point $\mathbf{p}$. If the field points lie on a spherical surface $S'$, $\mathbf{r}_0$ is the unit normal to $S'$.
- $\mathbf{n}$ is the unit normal to the surface $S$.
- $\eta$ is the impedance:
  $$\eta = \sqrt{\mu / \varepsilon}$$
- $k$ is the wave number.
• \( \lambda \) is the wavelength.
• \( \mathbf{r} \) is the radius vector (not a unit vector) of the surface \( S \).
• \( \mathbf{E}_p \) is the calculated far field in the direction from the origin towards point \( p \).

Thus the unit vector \( \mathbf{r}_0 \) can be interpreted as the direction defined by the angular position \((\theta, \varphi)\) and \( \mathbf{E}_p \) is the far field in this direction.

Because the far field is calculated in free space, the magnetic field at the far-field point is given by

\[
\mathbf{H}_p = \frac{\mathbf{r}_0 \times \mathbf{E}_p}{\eta_0}
\]

The Poynting vector gives the power flow of the far field:

\[
\mathbf{r}_0 \cdot \mathbf{S} = \mathbf{r}_0 \cdot \text{Re}(\mathbf{E}_p^* \times \mathbf{H}_p^*) - |\mathbf{E}_p|^2
\]

Thus the relative far-field radiation pattern is given by plotting \( |\mathbf{E}_p|^2 \).

References

Table 3-1 shows the symbol and SI unit for most of the physical quantities that are included with this module.

<table>
<thead>
<tr>
<th>QUANTITY</th>
<th>SYMBOL</th>
<th>UNIT</th>
<th>ABBREVIATION</th>
</tr>
</thead>
<tbody>
<tr>
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<td>radian/second</td>
<td>rad/s</td>
</tr>
<tr>
<td>Attenuation constant</td>
<td>( \alpha )</td>
<td>meter(^{-1})</td>
<td>m(^{-1})</td>
</tr>
<tr>
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<td>( C )</td>
<td>farad</td>
<td>F</td>
</tr>
<tr>
<td>Charge</td>
<td>( q )</td>
<td>coulomb</td>
<td>C</td>
</tr>
<tr>
<td>Charge density (surface)</td>
<td>( \rho_s )</td>
<td>coulomb/meter(^2)</td>
<td>C/m(^2)</td>
</tr>
<tr>
<td>Charge density (volume)</td>
<td>( \rho )</td>
<td>coulomb/meter(^3)</td>
<td>C/m(^3)</td>
</tr>
<tr>
<td>Current</td>
<td>( I )</td>
<td>ampere</td>
<td>A</td>
</tr>
<tr>
<td>Current density (surface)</td>
<td>( J_s )</td>
<td>ampere/meter</td>
<td>A/m</td>
</tr>
<tr>
<td>Current density (volume)</td>
<td>( J )</td>
<td>ampere/meter(^2)</td>
<td>A/m(^2)</td>
</tr>
<tr>
<td>Electric displacement</td>
<td>( D )</td>
<td>coulomb/meter(^2)</td>
<td>C/m(^2)</td>
</tr>
<tr>
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<td>( E )</td>
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<td>V/m</td>
</tr>
<tr>
<td>Electric potential</td>
<td>( V )</td>
<td>volt</td>
<td>V</td>
</tr>
<tr>
<td>Electric susceptibility</td>
<td>( \chi_e )</td>
<td>(dimensionless)</td>
<td>–</td>
</tr>
<tr>
<td>Electrical conductivity</td>
<td>( \sigma )</td>
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<td>S/m</td>
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<tr>
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<td>( W )</td>
<td>joule/meter(^3)</td>
<td>J/m(^3)</td>
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<td>newton</td>
<td>N</td>
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<td>Hz</td>
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<td>( \Omega )</td>
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<td>henry</td>
<td>H</td>
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<td>Wb/m</td>
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<td>F/m</td>
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<td>W/m$^2$</td>
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<td>rad/m</td>
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<td>–</td>
</tr>
<tr>
<td>Relative permittivity</td>
<td>$\varepsilon_r$</td>
<td>(dimensionless)</td>
<td>–</td>
</tr>
<tr>
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<td>ohm</td>
<td>$\Omega$</td>
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<td>W/m$^3$</td>
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This chapter discusses the physics interfaces found under the Radio Frequency branch.

In this chapter:

- The Electromagnetic Waves, Frequency Domain Interface
- The Electromagnetic Waves, Transient Interface
- The Transmission Line Interface
- The Electromagnetic Waves, Time Explicit Interface
- Theory for the Electromagnetic Waves Interfaces
- Theory for the Transmission Line Interface
- Theory for the Electromagnetic Waves, Time Explicit Interface
The Electromagnetic Waves, Frequency Domain Interface

The Electromagnetic Waves, Frequency Domain (emw) interface, found under the Radio Frequency branch when adding a physics interface, is used to solve for time-harmonic electromagnetic field distributions.

For this physics interface, the maximum mesh element size should be limited to a fraction of the wavelength. The domain size that can be simulated thus scales with the amount of available computer memory and the wavelength. The physics interface supports the study types Frequency Domain, Eigenfrequency, Mode Analysis, and Boundary Mode Analysis. The Frequency Domain study type is used for source driven simulations for a single frequency or a sequence of frequencies. The Eigenfrequency study type is used to find resonance frequencies and their associated eigenmodes in resonant cavities.

This physics interface solves the time-harmonic wave equation for the electric field.

When this physics interface is added, these default nodes are also added to the Model Builder — Wave Equation, Electric, Perfect Electric Conductor, and Initial Values. Then, from the Physics toolbar, add other nodes that implement, for example, boundary conditions. You can also right-click Electromagnetic Waves, Frequency Domain to select physics features from the context menu.

The Mode analysis study type is applicable only for 2D and 2D axisymmetric cross-sections of waveguides and transmission lines where it is used to find allowed propagating modes. Boundary mode analysis is used for the same purpose in 2D, 2D axisymmetry, and 3D and applies to boundaries representing waveguide ports.

**SETTINGS**

The Label is the default physics interface name.

The Name is used primarily as a scope prefix for variables defined by the physics interface. Refer to such physics interface variables in expressions using the pattern `<name>.<variable_name>`. In order to distinguish between variables belonging to different physics interfaces, the name string must be unique. Only letters, numbers, and underscores (_) are permitted in the Name field. The first character must be a letter.

The default Name (for the first physics interface in the model) is `emw`. 
** SETTINGS**

From the **Formulation** list, select whether to solve for the **Full field** (the default) or the **Scattered field**.

For **Scattered field** select a **Background wave type** — **User defined** (the default), **Gaussian beam**, or **Linearly polarized plane wave**.

**User Defined**

Enter the component expressions for the **Background electric field** $E_b$ (SI unit: V/m). The entered expressions must be differentiable.

---

**Gaussian Beam**

The Gaussian beam background field is a solution to the paraxial wave equation, which is an approximation to the Helmholtz equation solved for by the **Electromagnetic Waves, Frequency Domain (emw)** interface. The approximation is valid for Gaussian beams that have a beam radius that is much larger than the wavelength. Since the paraxial Gaussian beam background field is an approximation to the Helmholtz equation, for tightly focused beams, you can get a nonzero scattered field solution, even if you do not have any scatterers. For more information about the Gaussian beam theory, see **Gaussian Beams as Background Fields**.

- Select a **Beam orientation**: Along the $x$-axis (the default), Along the $y$-axis, or for 3D components, Along the $z$-axis.
- Enter a **Beam radius** $w_0$ (SI unit: m). The default is $20\pi/\varepsilon_0\mu_0 k_0$ m (10 vacuum wavelengths).
- Enter a **Focal plane along the axis** $p_0$ (SI unit: m). The default is 0 m.
- Enter the component expressions for the **Background electric field amplitude**, Gaussian beam $E_{bg0}$ (SI unit: V/m).
- Enter a **Wave number** $k$ (SI unit: rad/m). The default is $\varepsilon_0\mu_0 k_0$ rad/m. The wave number must evaluate to a value that is the same for all the domains the scattered field is applied to. Setting the **Wave number** $k$ to a positive value, means that the wave is propagating in the positive $x$-, $y$-, or $z$-axis direction, whereas setting the **Wave number** $k$ to a negative value means that the wave is propagating in the negative $x$-, $y$-, or $z$-axis direction.

Notice that expressions including coupling operators are not differentiable and cannot be used as background fields.
**Linearly Polarized Plane Wave**

The initial background wave is predefined as \( \mathbf{E}_0 = \exp(-j k_0 x) \mathbf{z} \). This field is transformed by three successive rotations along the roll, pitch, and yaw angles, in that order. For a graphic representation of the initial background field and the definition of the three rotations, c.f. Figure 4-1 below.

- Enter an **Electric field amplitude** \( E_0 \) (SI unit: V/m). The default is 1 V/m.
- Enter a **Roll angle** (SI unit: rad), which is a right-handed rotation with respect to the +x-direction. The default is 0 rad, corresponding to polarization along the +z-direction.
- Enter a **Pitch angle** (SI unit: rad), which is a right-handed rotation with respect to the +y-direction. The default is 0 rad, corresponding to the initial direction of propagation pointing in the +x-direction.
- Enter a **Yaw angle** (SI unit: rad), which is a right-handed rotation with respect to the +z-direction.
- Enter a **Wave number** \( k \) (SI unit: rad/m). The default is \( \omega_0 k_0 \) rad/m. The wave number must evaluate to a value that is the same for the domains the scattered field is applied to.

![Figure 4-1: Schematic of the directions for the wave vector \( \mathbf{k} \), the electric field \( \mathbf{E}_0 \), and the roll, pitch and yaw rotations. The top image represents an initial wave propagating in the x direction with a polarization along the z direction.](image)

**Electric Field Components Solved For**

This section is available for 2D and 2D axisymmetric components.
Select the Electric field components solved for — Three-component vector, Out-of-plane vector, or In-plane vector. Select:

- **Three-component vector** (the default) to solve using a full three-component vector for the electric field $\mathbf{E}$.
- **Out-of-plane vector** to solve for the electric field vector component perpendicular to the modeling plane, assuming that there is no electric field in the plane.
- **In-plane vector** to solve for the electric field vector components in the modeling plane assuming that there is no electric field perpendicular to the plane.

**Out-of-plane wave number**

This section is available for 2D and 2D axisymmetric components, when solving for Three-component vector or In-plane vector.

For 2D components, assign a wave vector component to the Out-of-plane wave number field. For 2D axisymmetric components, assign an integer constant or an integer parameter expression to the Azimuthal mode number field.

**Physics-controlled mesh**

Select the **Enable** check box to use a physics-controlled mesh for the electromagnetic problem. When selected, this invokes a parameter for the maximum mesh element size in free space. The physics-controlled mesh automatically scales the maximum mesh element size as the wavelength changes in different dielectric and magnetic regions. If the model is configured by any periodic conditions, identical meshes are generated on each pair of periodic boundaries. Perfectly matched layers are built with a structured mesh, specifically, a swept mesh in 3D and a mapped mesh in 2D.

When **Enable** is selected, choose one of the three options for the Maximum mesh element size control parameter — **User defined** (the default), **Frequency**, or **Wavelength**.

For the option **User defined**, enter a suitable Maximum element size in free space. For example, 1/5 of the vacuum wavelength or smaller. When **Frequency** is selected, enter the highest frequency intended to be used during the simulation. The maximum mesh element size in free space is 1/5 of the vacuum wavelength for the entered frequency.

For the **Wavelength** option, enter the smallest vacuum wavelength intended to be used during the simulation. The maximum mesh element size in free space is 1/5 of the entered wavelength.

When **Resolve wave in lossy media** is selected, the outer boundaries of lossy media domains are meshed with a maximum mesh element size in free space given by the minimum value of half a skin depth and 1/5 of the vacuum wavelength.
The maximum mesh element size in dielectric media is that in free space divided by the square root of the product of the relative permittivity and permeability.

**ANALYSIS METHODOLOGY**

From the Methodology options list, select one of three solver configurations: Robust, Intermediate, or Fast (the default).

The settings of each methodology option are found in Solver Configurations and the subsidiary nodes.

**PORT SWEEP SETTINGS**

Select the Activate port sweep check box to switch on the port sweep. When selected, this invokes a parametric sweep over the ports in addition to the automatically generated frequency sweep. The generated lumped parameters are in the form of an S-parameter matrix.

For Activate port sweep enter a Sweep parameter name to assign a specific name to the parameter that controls the port number solved for during the sweep. Before making the port sweep, the parameter must also have been added to the list of parameters in the Parameters section of the Parameters node under the Global Definitions node.

For this physics interface, the S-parameters are subject to Touchstone file export. Click Browse to locate the file, or enter a file name and path. Select an Output format: Magnitude angle, Magnitude (dB) angle, or Real imaginary.

Enter a Reference impedance, Touchstone file export $Z_{ref}$ (SI unit: $\Omega$). The default is 50 $\Omega$. 

**DEPENDENT VARIABLES**

The dependent variables (field variables) are for the Electric field $\mathbf{E}$ and its components (in the Electric field components fields). The name can be changed but the names of fields and dependent variables must be unique within a model.

**DISCRETIZATION**

To display this section, click the Show button ( ) and select Discretization.
The Electromagnetic Waves, Frequency Domain Interface has these domain, boundary, edge, point, and pair nodes and subnodes, listed in alphabetical order, are available from the Physics ribbon toolbar (Windows users), Physics context menu (Mac or Linux users), or right-click to access the context menu (all users).

**Domain**

- Archie’s Law
- Divergence Constraint
- External Current Density
- Far-Field Calculation
- Far-Field Domain

**Boundary Conditions**

With no surface currents present the boundary conditions

\[
\mathbf{n}_2 \times (\mathbf{E}_1 - \mathbf{E}_2) = \mathbf{0} \\
\mathbf{n}_2 \times (\mathbf{H}_1 - \mathbf{H}_2) = \mathbf{0}
\]

need to be fulfilled. Because E is being solved for, the tangential component of the electric field is always continuous, and thus the first condition is automatically fulfilled. The second condition is equivalent to the natural boundary condition

\[
-\mathbf{n} \times (\mu_r^{-1} \nabla \times \mathbf{E})_1 - (\mu_r^{-1} \nabla \times \mathbf{E})_2 = \mathbf{n} \times j\omega\mu_0(\mathbf{H}_1 - \mathbf{H}_2) = \mathbf{0}
\]
and is therefore also fulfilled. These conditions are available (listed in alphabetical order):

- Diffraction Order
- Electric Field
- Impedance Boundary Condition
- Lumped Element
- Lumped Port
- Magnetic Field
- Perfect Electric Conductor
- Perfect Magnetic Conductor
- Periodic Condition
- Port
- Scattering Boundary Condition
- Surface Current Density
- Surface Roughness
- Transition Boundary Condition

**ED G E, P O I N T, A N D P A I R**

- Circular Port Reference Axis
- Edge Current
- Electric Field
- Electric Point Dipole
- Integration Line for Current
- Integration Line for Voltage
- Line Current (Out-of-Plane)
- Magnetic Current
- Magnetic Point Dipole
- Perfect Electric Conductor
- Perfect Magnetic Conductor
- Periodic Port Reference Point
- Surface Current Density

For 2D axisymmetric components, COMSOL Multiphysics takes the axial symmetry boundaries \((r = 0)\) into account and automatically adds an **Axial Symmetry** node to the component that is valid on the axial symmetry boundaries only.

In the *COMSOL Multiphysics Reference Manual* see Table 2-3 for links to common sections and Table 2-4 to common feature nodes. You can also search for information: press F1 to open the Help window or Ctrl+F1 to open the Documentation window.
Wave Equation, Electric

**Wave Equation, Electric** is the main feature node for this physics interface. The governing equation can be written in the form

$$\nabla \times (\mu_r^{-1}\nabla \times \mathbf{E}) - k_0^2\varepsilon_{re}\mathbf{E} = \mathbf{0}$$

for the time-harmonic and eigenfrequency problems. The wave number of free space $k_0$ is defined as

$$k_0 = \frac{\omega\mu_0}{c_0} = \frac{\omega}{c_0}$$

where $c_0$ is the speed of light in vacuum.

In 2D the electric field varies with the out-of-plane wave number $k_z$ as

$$\mathbf{E}(x, y, z) = \mathbf{E}(x, y) \exp(-i k_z z).$$

The wave equation is thereby rewritten as

$$(\nabla - i k_z \mathbf{z}) \times [\mu_r^{-1}(\nabla - i k_z \mathbf{z}) \times \tilde{\mathbf{E}}] - k_0^2\varepsilon_{re}\tilde{\mathbf{E}} = \mathbf{0},$$

where $\mathbf{z}$ is the unit vector in the out-of-plane $z$-direction.

Notice that the ansatz above just explains how the wave equation is modified when the out-of-plane wave vector component $k_z$ is not zero. As an example, for a plane wave with a nonzero out-of-plane wave vector component, the electric field is of course given by

$$\mathbf{E}(x, y, z) = \mathbf{E}(x, y) \exp(-i k_z z) = A \exp(-i(k_x x + k_y y + k_z z)),$$

where $A$ is a constant amplitude and $k_x$, $k_y$, and $k_z$ are the wave vector components.

In 2D axisymmetry, the electric field varies with the azimuthal mode number $m$ as

$$\mathbf{E}(r, \phi, z) = \mathbf{E}(r, z) \exp(-i m \phi).$$

For this case, the wave equation is rewritten as

$$(\nabla - i \frac{m}{r} \phi) \times [\mu_r^{-1}(\nabla - i \frac{m}{r} \phi) \times \tilde{\mathbf{E}}] - k_0^2\varepsilon_{re}\tilde{\mathbf{E}} = \mathbf{0},$$
where \( \mathbf{\varphi} \) is the unit vector in the out-of-plane \( \varphi \)-direction.

When solving the equations as an eigenfrequency problem the eigenvalue is the complex eigenfrequency \( \lambda = -j\omega + \delta \), where \( \delta \) is the damping of the solution. The Q-factor is given from the eigenvalue by the formula

\[
Q_{\text{fact}} = \frac{\omega}{2|\lambda|}
\]

Using the relation \( \varepsilon_r = n^2 \), where \( n \) is the refractive index, the equation can alternatively be written

\[
\nabla \times (\nabla \times \mathbf{E}) - k_0^2 n^2 \mathbf{E} = 0
\]

When the equation is written using the refractive index, the assumption is that \( \mu_r = 1 \) and \( \sigma = 0 \) and only the constitutive relations for linear materials are available. When solving for the scattered field the same equations are used but \( \mathbf{E} = \mathbf{E}_{\text{sc}} + \mathbf{E}_i \) and \( \mathbf{E}_{\text{sc}} \) is the dependent variable.

The Divergence Constraint subnode is available from the context menu (right-click the parent node) or from the Physics toolbar, Attributes menu.

**ELECTRIC DISPLACEMENT FIELD**

Select an Electric displacement field model — Relative permittivity (the default), Refractive index, Loss tangent, loss angle, Loss tangent, dissipation factor, Dielectric loss, Drude-Lorentz dispersion model, or Debye dispersion model.

**Relative Permittivity**

When Relative permittivity is selected, the default Relative permittivity \( \varepsilon_r \) takes values From material. Select Porous media to add a Porous Media subnode, or for User defined select Isotropic, Diagonal, Symmetric, or Anisotropic and enter values or expressions in the field or matrix.

**Refractive Index**

When Refractive index is selected, the default Refractive index \( n \) and Refractive index, imaginary part \( k \) take the values From material. To specify the real and imaginary parts of the refractive index and assume a relative permeability of unity and zero.
conductivity, for one or both of the options, select **User defined** then choose **Isotropic** or **Diagonal**. Enter values or expressions in the field or matrix.

The diagonal components of the input refractive index matrix correspond to the semi-axes of the so called index ellipsoid. You can orient the index ellipsoid by first creating a suitably oriented coordinate system below the **Definitions** node for the model component. Then select the created coordinate system in the **Coordinate system** setting in the **Coordinate System Selection** section in the settings for the **Wave Equation, Electric** feature.

Beware of the time-harmonic sign convention requiring a lossy material having a negative imaginary part of the refractive index (see **Introducing Losses in the Frequency Domain**).

**Loss Tangent, Loss Angle**
When **Loss tangent, loss angle** is selected, the default **Relative permittivity (real part)** \( \varepsilon' \) and **Loss tangent, loss angle** \( \delta \) take values **From material**. For **User defined** select **Isotropic**, **Diagonal**, **Symmetric**, or **Anisotropic** and enter values or expressions in the field or matrix. Then if **User defined** is selected for **Loss tangent, loss angle** \( \delta \), enter a value to specify a loss angle for dielectric losses. This assumes zero conductivity.

**Loss Tangent, Dissipation Factor**
When **Loss tangent, dissipation factor** is selected, the default **Relative permittivity (real part)** \( \varepsilon' \) and **Loss tangent, dissipation factor** \( \tan \delta \) take values **From material**. For **User defined** select **Isotropic**, **Diagonal**, **Symmetric**, or **Anisotropic** and enter values or expressions in the field or matrix. Then if **User defined** is selected for **Loss tangent, dissipation factor** \( \tan \delta \), enter a value to specify a dissipation for dielectric losses. This assumes zero conductivity.

**Dielectric Loss**
When **Dielectric loss** is selected, the default **Relative permittivity** \( \varepsilon' \) and **Relative permittivity (imaginary part)** \( \varepsilon'' \) take values **From material**. For **User defined** select
Isotropic, Diagonal, Symmetric, or Anisotropic and enter values or expressions in the field or matrix.

Drude-Lorentz Dispersion Model

The Drude-Lorentz dispersion model is defined by the equation

\[
\varepsilon_r(\omega) = \varepsilon_\infty + \sum_{j=1}^{M} \frac{f_j \omega_P^2}{\omega_0^2 - \omega^2 + i \Gamma_j \omega}
\]

where \( \varepsilon_\infty \) is the high-frequency contribution to the relative permittivity, \( \omega_P \) is the plasma frequency, \( f_j \) is the oscillator strength, \( \omega_0 \) is the resonance frequency, and \( \Gamma_j \) is the damping coefficient.

When Drude-Lorentz dispersion model is selected, the default Relative permittivity, high frequency \( \varepsilon_\infty \) (dimensionless) takes its value From material. For User defined select Isotropic, Diagonal, Symmetric, or Anisotropic and enter a value or expression in the field or matrix.

Enter a Plasma frequency \( \omega_\infty \) (SI unit: rad/s). The default is 0 rad/s.

In the table, enter values or expressions in the columns for the Oscillator strength, Resonance frequency (rad/s), and Damping in time (rad/s).

Debye Dispersion Model

The Debye dispersion model is given by

\[
\varepsilon(\omega) = \varepsilon_\infty + \sum_k \frac{\Delta \varepsilon_k}{1 + i \omega \tau_k}
\]

where \( \varepsilon_\infty \) is the high-frequency contribution to the relative permittivity, \( \Delta \varepsilon_k \) is the contribution to the relative permittivity, and \( \tau_k \) is the relaxation time.

When Debye dispersion model is selected, the default Relative permittivity, high frequency \( \varepsilon_\infty \) (dimensionless) takes its value From material. For User defined select Isotropic, Diagonal, Symmetric, or Anisotropic and enter a value or expression in the field or matrix.

Beware of the time-harmonic sign convention requiring a lossy material having a negative imaginary part of the relative permittivity (see Introducing Losses in the Frequency Domain).
In the table, enter values or expressions in the columns for the Relative permittivity contribution and Relaxation time (s).

**Magnetic Field**
Select the Constitutive relation — Relative permeability (the default) or Magnetic losses.

- For Relative permeability the relative permeability \( \mu_r \) uses values From material. For User defined select Isotropic, Diagonal, Symmetric, or Anisotropic based on the characteristics of the magnetic field, and then enter values or expressions in the field or matrix. If Porous media is selected, the Porous Media subnode is available from the context menu (right-click the parent node) or from the Physics toolbar, Attributes menu.
- For Magnetic losses the default values for Relative permeability (real part) \( \mu' \) and Relative permeability (imaginary part) \( \mu'' \) are taken From material. For User defined enter different values.

**Conduction Current**
By default, the Electrical conductivity \( \sigma \) (SI unit: S/m) uses values From material.

- For User defined select Isotropic, Diagonal, Symmetric, or Anisotropic based on the characteristics of the current and enter values or expressions in the field or matrix.
- For Linearized resistivity the default values for the Reference temperature \( T_{\text{ref}} \) (SI unit: K), Resistivity temperature coefficient \( \alpha \) (SI unit: 1/K), and Reference resistivity \( \rho_0 \) (SI unit: \( \Omega \cdot \text{m} \)) are taken From material. For User defined enter other values or expressions for any of these variables.
- When Porous media is selected, the Porous Media subnode is available from the context menu (right-click the parent node) or from the Physics toolbar, Attributes menu.
- When Archie’s law is selected, the Archie’s Law subnode is available from the context menu (right-click the parent node) or from the Physics toolbar, Attributes menu.

For magnetic losses, beware of the time-harmonic sign convention requiring a lossy material having a negative imaginary part of the relative permeability (see Introducing Losses in the Frequency Domain).
**Divergence Constraint**

The **Divergence Constraint** subnode is available from the context menu (right-click the Wave Equation, Electric parent node) or from the **Physics** toolbar, **Attributes** menu. It is used for numerical stabilization when the frequency is low enough for the total electric current density related term in the wave equation to become numerically insignificant. For The Electromagnetic Waves, Frequency Domain Interface and Heat Transfer Interfaces the divergence condition is given by

$$\nabla \cdot \mathbf{J} = 0$$

and for The Electromagnetic Waves, Transient Interface it is

$$\nabla \cdot (\sigma \mathbf{A}) = 0$$

**DIVERGENCE CONSTRAINT**

Enter a value or expression for the **Divergence condition variable scaling** \(\psi_0\).

For the Electromagnetic Waves, Frequency Domain and Microwave Heating interfaces, the SI unit is kg/(m\(^3\)s\(^3\)A)). The default is 1 kg/(m\(^3\)s\(^3\)A).

For the Electromagnetic Waves, Transient interface (and the Microwave Plasma interface available with the Plasma Module) the SI unit is A/m and the default is 1 A/m.

**Initial Values**

The **Initial Values** node adds an initial value for the electric field that can serve as an initial guess for a nonlinear solver. Add additional **Initial Values** nodes from the **Physics** toolbar.

**INITIAL VALUES**

Enter values or expressions for the initial values of the components of the Electric field \(\mathbf{E}\) (SI unit: V/m). The default values are 0 V/m.

**External Current Density**

The **External Current Density** node adds an externally generated current density \(\mathbf{J}_e\), which appears in Ohm’s law

$$\mathbf{J} = \sigma \mathbf{E} + \mathbf{J}_e$$
and in the equation that the physics interface defines.

**EXTERNAL CURRENT DENSITY**

Based on space dimension, enter the components \((x, y, \text{ and } z\) for 3D components for example) of the External current density \(J_e\) (SI unit: \(A/m^2\)).

**Far-Field Domain**

To set up a far-field calculation, add a Far-Field Domain node and specify the far-field domains in its Settings window. Use Far-Field Calculation subnodes (one is added by default) to specify all other settings needed to define the far-field calculation. If a Perfectly Matched Layer (PML) node has been added before adding the Far-Field Domain, all of the domains in the Electromagnetic Waves, Frequency Domain interface adjacent to the PML are automatically selected by default. If there is no PML, all of the domains are selected. The selection can be modified. In that case, select only a homogeneous domain or domain group that is outside of all radiating and scattering objects and which has the material settings of the far-field medium.

| Far-Field Support in the Electromagnetic Waves, Frequency Domain Interface
|---|

**Far-Field Calculation**

A Far-Field Calculation subnode is added by default to the Far-Field Domain node and is used to select boundaries corresponding to a single closed surface surrounding all radiating and scattering objects. By default, all exterior boundaries of the Far-Field Domain are selected. If a Perfectly Matched Layer (PML) node has been added before adding the Far-Field Domain, all exterior boundaries of the Far-Field Domain adjacent to the PML are selected. Symmetry reduction of the geometry makes it relevant to select boundaries defining a nonclosed surface. Also use this feature to indicate symmetry planes and symmetry cuts applied to the geometry, and whether the selected...
boundaries are defining the inside or outside of the far field domain; that is, to say whether they are facing away from infinity or toward infinity.

**FAR-FIELD CALCULATION**

Enter a Far-field variable name. The default is \( E_{\text{far}} \).

Select as needed the **Symmetry in the \( x=0 \) plane**, **Symmetry in the \( y=0 \) plane**, or **Symmetry in the \( z=0 \) plane** check boxes to use it your model when calculating the far-field variable. The symmetry planes have to coincide with one of the Cartesian coordinate planes.

When a check box is selected, also choose the type of symmetry to use from the **Symmetry type** list that appears — **Symmetry in \( E \) (PMC)** or **Symmetry in \( H \) (PEC)**. The selection should match the boundary condition used for the symmetry boundary. Using these settings, include the parts of the geometry that are not in the model for symmetry reasons in the far-field analysis.

From the **Boundary relative to domain** list, select **Inside** or **Outside** (the default) to define if the selected boundaries are defining the inside or outside of the far-field domain (that is, whether facing away from infinity or toward infinity).

---

*If perfectly matched layers are added to the model after the Far-Field Domain is configured, then it is necessary to press the Reset Far-Field Boundaries button to reassign all exterior boundaries.*

---

*Dielectric Resonator Antenna: Application Library path RF_Module/Antennas/dielectric_resonator_antenna*

---

**Perfect Electric Conductor**

The **Perfect Electric Conductor** boundary condition

\[
\mathbf{n} \times \mathbf{E} = 0
\]

is a special case of the electric field boundary condition that sets the tangential component of the electric field to zero. It is used for modeling of a lossless metallic surface (for example, a ground plane) or as a symmetry type boundary condition. It imposes symmetry for magnetic fields and “magnetic currents” and antisymmetry for electric fields and electric currents. It supports induced electric surface currents and
thus any prescribed or induced electric currents (volume, surface, or edge currents) flowing into a perfect electric conductor boundary is automatically balanced by induced surface currents.

Figure 4-2: The perfect electric conductor boundary condition is used on exterior and interior boundaries representing the surface of a lossless metallic conductor or (on exterior boundaries) representing a symmetry cut. The shaded (metallic) region is not part of the model but still carries effective mirror images of the sources. Note also that any current flowing into the boundary is perfectly balanced by induced surface currents. The tangential electric field vanishes at the boundary.

**CONSTRAINT SETTINGS**

To display this section, click the Show button ( ) and select Advanced Physics Options.

---

**Perfect Magnetic Conductor**

The Perfect Magnetic Conductor boundary condition

\[ \mathbf{n} \times \mathbf{H} = 0 \]

is a special case of the surface current boundary condition that sets the tangential component of the magnetic field and thus also the surface current density to zero. On external boundaries, this can be interpreted as a “high surface impedance” boundary condition or used as a symmetry type boundary condition. It imposes symmetry for electric fields and electric currents. Electric currents (volume, surface, or edge currents) are not allowed to flow into a perfect magnetic conductor boundary as that would violate current conservation. On interior boundaries, the perfect magnetic conductor boundary condition literally sets the tangential magnetic field to zero,
which in addition to setting the surface current density to zero also makes the tangential electric field discontinuous.

Figure 4-3: The perfect magnetic conductor boundary condition is used on exterior boundaries representing the surface of a high impedance region or a symmetry cut. The shaded (high impedance) region is not part of the model but nevertheless carries effective mirror images of the sources. Note also that any electric current flowing into the boundary is forbidden as it cannot be balanced by induced electric surface currents. The tangential magnetic field vanishes at the boundary. On interior boundaries, the perfect magnetic conductor boundary condition literally sets the tangential magnetic field to zero which in addition to setting the surface current density to zero also makes the tangential electric field (and in dynamics the tangential electric field) discontinuous.

---

<table>
<thead>
<tr>
<th>Magnetic Frill: Application Library path</th>
</tr>
</thead>
<tbody>
<tr>
<td>RF_Module/Antennas/magnetic_frill</td>
</tr>
</tbody>
</table>

Port

Use the Port node where electromagnetic energy enters or exits the model. A port can launch and absorb specific modes. Use the boundary condition to specify wave type ports. Ports support S-parameter calculations but can be used just for exciting the model. This node is not available with the Electromagnetic Waves, Transient interface.
In 3D, also right-click these subnodes are available from the context menu (right-click the parent node) or from the Physics toolbar, Attributes menu:

- **Circular Port Reference Axis** to determine a reference direction for the modes. This subnode is selected from the Points submenu when Circular is selected as the type of port.
- **Periodic Port Reference Point** to uniquely determine reciprocal lattice vectors. This subnode is selected from the Points submenu when Periodic is selected as the type of port.

**PORT PROPERTIES**

Enter a unique Port name. Only nonnegative integer numbers can be used as Port name as it is used to define the elements of the S-parameter matrix and numeric port names are also required for port sweeps and Touchstone file export.

Select the **Type of Port** — User defined, Numeric, Rectangular, Coaxial, Circular, or Periodic.

Periodic ports are available in 3D and 2D. Circular and Coaxial ports are available in 3D and 2D axisymmetry.

Numeric ports require a Boundary Mode Analysis study type. It should appear before the frequency domain study node in the study branch of the model tree. If more than one numeric port is needed, use one Boundary Mode Analysis node per port and assign each to the appropriate port. Then, it is best to add all the studies; Boundary Mode Analysis 1, Boundary Mode Analysis 2,..., Frequency Domain 1, manually. Numeric ports are by default computed for the deformed mesh whereas other types of ports compute the mode shape using geometry information.

---

**Wave Excitation at this Port**

To set whether it is an import or a listener port, select On or Off from the Wave excitation at this port list. If On is selected, enter a Port input power $P_{in}$ (SI unit: W) or a Deposited power $P_{dep}$ (SI unit: W) if the Specify deposited power check box is marked. When a Deposited power is specified, the input power is adjusted so that the power that is not

---

It is only possible to excite one port at a time if the purpose is to compute S-parameters. In other cases (for example, when studying microwave heating) more than one import might be wanted, but the S-parameter variables cannot be correctly computed, so when several ports are excited, the S-parameter output is turned off.

---

**THE ELECTROMAGNETIC WAVES, FREQUENCY DOMAIN INTERFACE** | 95
reflected equals the specified Deposited power. If Specify deposited power is marked this must be the only import.

The Port Sweep Settings section in the Electromagnetic Waves, Frequency Domain interface cycles through the ports, computes the entire S-matrix and exports it to a Touchstone file. When using port sweeps, the local setting for Wave excitation at this port is overridden by the solver so only one port at a time is excited.

Activate Slit Condition
Select the Activate slit condition on interior port check box to use the Port boundary condition on interior boundaries.

Then select a Slit type — PEC-backed (the default) or Domain-backed. The PEC-backed type makes the port on interior boundaries perform as it does on exterior boundaries. The Domain-backed type can be combined with perfectly matched layers to absorb the excited mode from a source port and other higher order modes.

Select a Port orientation — Forward (the default) or Reverse to define the inward normal vector of the port. The Forward direction is visualized with a red arrow on the port boundary. The Reverse direction is opposite to the arrow.

Note that there can only be one domain-backed slit port per boundary.

When the Slit type is set to Domain-backed, there must be no waves reflected from the domain backing the port. Thus, the backing domain must have homogeneous material and geometric properties and it should be truncated by a PML domain or a non-reflecting boundary condition.

When an active slit port is adjacent to a periodic condition, the periodic condition needs to be split at the active port, as the field is discontinuous at the point (in 2D) or edge (in 3D) where the slit port and the periodic condition meet.

Analyze as a TEM Field
This check box is available for 3D components and when the Type of port is Numeric.

Select the Analyze as a TEM field check box to add Integration Line for Current and Integration Line for Voltage subnodes. These subnodes are available from the context.
menu (right-click the Port parent node) or from the **Physics** toolbar, **Attributes** menu. Enter a **Characteristic impedance** $Z_{\text{ref}}$ (SI unit: $\Omega$). The default is 50 $\Omega$. The characteristic impedance of a port is calculated using the ratio of the voltage and current and the mode field on a port boundary is scaled by the ratio between the characteristic impedance and $Z_{\text{ref}}$.

---

**PORT MODE SETTINGS**

The input is based on the **Type of Port** selected above — **User Defined**, **Rectangular**, **Circular**, or **Periodic**. No entry is required if **Numeric** or **Coaxial** are selected.

Set the **Mode phase** $\theta_m$ (SI unit: rad) for the port mode field. The default is 0 radians. For instance, if the inspected port mode field is polarized in the opposite direction compared to the expected direction, a **Mode phase** of $\pi$ (enter $\pi$ in the field) can be used for polarizing the mode field in the expected direction. Notice that a change of the **Mode phase**, either on the exciting or the listener port, changes also the S-parameter coupling the exciting and the listener port. However, a change of the **Mode phase** on the exciting port does not modify the reflection coefficient (normally denoted S11) associated with the exciting port.

**User Defined**

For **User defined** specify the eigenmode of the port. The mode field can be entered with an arbitrary amplitude and is normalized internally.

- Enter the components of the **Electric mode field** $E_0$ (SI unit: V/m) or the **Magnetic mode field** $H_0$ (SI unit: A/m). The entered expressions must be differentiable.
- Enter the **Propagation constant** $\beta$ (SI unit: rad/m). This is frequency dependent for all but TEM modes and a correct frequency-dependent expression must be used.

**Rectangular**

For **Rectangular** specify a unique rectangular mode.

---

**Notch Filter Using a Split Ring Resonator**: Application Library path

`RF_Module/Filters/notch_filter_srr`

Notice that expressions including coupling operators are not differentiable and cannot be used as mode fields.
For 3D components, select a **Mode type** — *Transverse electric (TE)* or *Transverse magnetic (TM)*. Enter the **Mode number**, for example, 10 for a TE\textsubscript{10} mode, or 11 for a TM\textsubscript{11} mode.

For 2D components, to excite the fundamental mode, select the mode type *Transverse electromagnetic (TEM)*, since the rectangular port represents a parallel-plate waveguide port that can support a TEM mode. Only TE modes are possible when solving for the out-of-plane vector component, and only TM and TEM modes are possible when solving for the in-plane vector components. There is only a single mode number, which is selected from a list.

**Coaxial**

In 2D axisymmetry, **Coaxial** does not support non-zero azimuthal mode number. The **Azimuthal mode number** in the **Physics interface** should be defined as zero.

**Circular**

For **Circular** specify a unique circular mode.

- Select a **Mode type** — *Transverse electric (TE)* or *Transverse magnetic (TM)*.
- Select the **Mode number** from the list.

For 3D components, enter the **Mode number**, for example, 11 for a TE\textsubscript{11} mode, or 01 for a TM\textsubscript{01} mode. When **Circular** is selected as the type of port in 3D, the **Circular Port Reference Axis** subnode is available from the context menu (right-click the parent node) or from the **Physics** toolbar, **Attributes** menu. It defines the orientation of fields on a port boundary.

For 2D axisymmetry components, select whether the **Azimuthal mode number** is defined in the **Physics interface** or if it is **User defined**. For **User defined** define an integer constant or an integer parameter expression for the **Azimuthal mode number**. Note that the absolute value of the **Azimuthal mode number** must be less than 11.

**Periodic**

For **Periodic** specify parameters for the mode field. When **Periodic** is selected, the **Diffraction Order** port subnode is available from the context menu (right-click the parent node) or from the **Physics** toolbar, **Attributes** menu.

Select a **Input quantity** — *Electric field* or *Magnetic field* and define the mode field amplitude.

- For 2D components and if the **Input quantity** is set to **Electric field**, define the **Electric mode field amplitude**. For example, for a TE wave set the \( x \), \( y \), and \( z \) components to
0, 0, 1. Similarly, if the Input quantity is set to Magnetic field, define the Magnetic mode field amplitude. For a TM wave set the x, y, and z components to 0, 0, and 1.

- Define the Angle of incidence, if Wave excitation at this port is On.

For 3D components, if Wave excitation at this port is On, define the Elevation angle of incidence and Azimuth angle of incidence. The Elevation angle of incidence $\alpha_1$ and Azimuth angle of incidence $\alpha_2$ are used in the relations

$$k = k_{\text{parallel}} + k_{\text{perpendicular}}$$

$$k_{\text{parallel}} = k_F = |k| \sin \alpha_1 (\hat{a}_1 \cos \alpha_2 + n \times \hat{a}_1 \sin \alpha_2)$$

where $k$ is the wave vector, $k_{\text{parallel}}$ is the projection of $k$ onto the port, $k_F$ is the $k$-vector for Floquet periodicity, $n$ is the outward unit normal vector to the boundary, and $\hat{a}_1$ is one of the normalized primitive unit cell vectors from the periodic structure defined from Periodic Port Reference Point.

The Elevation angle of incidence $\alpha_1$ is the angle between $n$ and $k$.

The Azimuth angle of incidence is the counterclockwise rotating angle from the primitive vector $a_1$ around the axis built with Periodic Port Reference Point and $n$.

For periodic ports with hexagonal port boundaries, the definition of the vector $a_1$ is slightly different from the default definition. In this case, the unit cell is actually a rhomboid, with primitive vectors pointing in other directions than the side vectors of the hexagon. Thus, for a hexagonal periodic port, the vector $a_1$ is defined along one of the sides of the hexagon, and it is not one of the primitive vectors of the hexagonal point lattice. The Azimuth angle of incidence $\alpha_2$ is still measured from the vector $a_1$, even though this vector now refers to a side vector of the hexagonal port boundary and not a primitive vector.

---

**Hexagonal Grating**: Application Library path

`RF_Module/Tutorials/hexagonal_grating`

For 2D components define the Angle of incidence. The Angle of incidence $\alpha$ is defined by the relation

$$k \times n = k \sin \alpha z$$
where $\mathbf{k}$ is the projection of the wave vector in the $xy$-plane, $\mathbf{n}$ is the normalized normal vector to the boundary, $\mathbf{k}$ is the magnitude of the projected wave vector in the $xy$-plane, and $\mathbf{z}$ is the unit vector in the $z$-direction.

Notice that the mode field defined for the Periodic port assumes isotropic material properties in the domain adjacent to the selected port boundary.

The propagation directions for listener Periodic ports are deduced from the angle setting(s) for the source Periodic port and the refractive indexes defined for the source and the listener ports. Thus, adding source Periodic ports with different propagation angles will give ambiguous propagation directions for the listener Periodic ports.

**AUTOMATIC DIFFRACTION ORDER CALCULATION**

This section is only available for Periodic ports to provide parameter settings that are used when automatically adding Diffraction Order subnodes to Periodic ports.

- Select the **Include in automatic diffraction order calculation** check box to add Diffraction Order subnodes to the selected Periodic port, when the **Compute Diffraction Order** button is clicked from the exciting Periodic port.

- Define the **Refractive index, real part** at the boundary.

- Define the **Maximum frequency — From study** (the default) or **User defined**. When **From study** is selected, the Maximum frequency is taken from the study step associated with the physics interface. For **User defined**, enter the maximum frequency $f_{\text{max}}$ (SI unit: Hz). The default value is 0 Hz. If a single frequency is used, insert the frequency, or if a frequency sweep is performed, insert the maximum frequency of the sweep. This parameter is only available when **Wave excitation at this port is On**.

When all parameters are defined, click the **Compute Diffraction Orders** button from the exciting Periodic port to automatically create Diffraction Order ports as subnodes to all Periodic ports having the **Include in automatic diffraction order calculation** check box selected.

- **S-Parameters and Ports**
- **S-Parameter Variables**
Integration Line for Current

The **Integration Line for Current** is available only in 3D from the context menu (right-click the Port node) when the Analyze as a TEM field check box is selected under the Port Properties section for the Port node.

Integration Line for Voltage

The **Integration Line for Voltage** is available only in 3D from the context menu (right-click the Port node) when the Analyze as a TEM field check box is selected under the Port Properties section for the Port node. The characteristic impedance of a Numeric port is defined by the ratio between the voltage and current.

Circular Port Reference Axis

The **Circular Port Reference Axis** is available only in 3D. When the Type of port is set to Circular under Port Properties, the Circular Port Reference Axis subnode is available from the context menu (right-click the Port parent node) or from the Physics toolbar, Attributes menu. Two points are used to define the orientation of fields on a port boundary. If there are more than two points on the selection list, the first and last points are used. For the fundamental TE_{11} mode, the direction of the reference axis corresponds to the polarization of the electric field at the port center.

Diffraction Order

The **Diffraction Order** port is available in 3D and 2D. When the Type of Port is set to Periodic under Port Properties, this subnode is available from the context menu.
Use the **Diffraction Order** port to define diffraction orders from a periodic structure. Normally a **Diffraction Order** node is added automatically during the **Periodic** port setup. Additional **Diffraction Order** ports subnodes are available from the context menu (right-click the parent node) or from the **Physics** toolbar, **Attributes** menu.

**PORT PROPERTIES**

Enter a unique **Port name.** Only non-negative integer numbers can be used as **Port name** as it is used to define the elements of the S-parameter matrix and numeric port names are also required for port sweeps and Touchstone file export.

The Diffraction Order port is a listener port feature.

**PORT MODE SETTINGS**

These settings define the diffracted plane wave.

**Components**

Select the **Components** setting for the port — **In-plane vector** (the default) or **Out-of-plane vector**. In 3D, when the diffraction order \( m \) and \( n \) are both set to 0, a third option is **Orthogonal polarization**, which represents a polarization orthogonal to the polarization of the incident wave.

**Diffraction Order**

Specify an integer constant or an integer parameter expression for the **Diffraction order** \( m \) (the default is 0) and in 3D \( n \) (the default is 0).

Note that **In-plane vector** and **Out-of-plane vector** are based on the plane of diffraction which is constructed with the diffraction wave vector and the outward normal vector of the port boundary. The diffraction wave vector is defined by

\[
\mathbf{k}_{\text{diffraction, parallel}} = \mathbf{k}_{\text{F}} + M \mathbf{G}_1 + N \mathbf{G}_2
\]

\[
\mathbf{k}_{\text{diffraction}} = \mathbf{k}_{\text{diffraction, parallel}} - \mathbf{n} k_{\text{diffraction, perpendicular}}
\]

\[
k_{\text{diffraction, perpendicular}} = \sqrt{k^2 - k_{\text{diffraction, parallel}}^2}
\]

where \( M \) and \( N \) are diffraction orders, \( k \geq k_{\text{diffraction, parallel}} \) \( k \) is the magnitude of the wave vector and \( k_{\text{diffraction, parallel}} \) is the magnitude of \( \mathbf{k}_{\text{diffraction, parallel}} \). Reciprocal lattice vectors, \( \mathbf{G}_1 \) and \( \mathbf{G}_2 \) are defined from **Periodic Port Reference Point**.
In-plane vector lies on the plane of diffraction while Out-of-plane vector is normal to the plane of diffraction.

For a 2D component, in-plane vector is available when the settings for the physics interface is set to either in-plane vector or Three-component vector under Electric Field Components Solved For. Out-of-plane vector is available when the settings for the physics interface is set to either Out-of-plane vector or Three-component vector under Electric Field Components Solved For.

Enter a value or expression for the Mode phase $\theta_m$ (SI unit: rad). The default is 0 radians. The Mode phase setting is further discussed for the Port feature.

Notice that the mode field defined for the Diffraction Order feature assumes isotropic material properties in the domain adjacent to the selected feature boundary.

- S-Parameters and Ports
- S-Parameter Variables

Plasmonic Wire Grating: Application Library path
RF_Module/Tutorials/plasmonic_wire_grating

Periodic Port Reference Point

The Periodic Port Reference Point subnode is available only in 3D. When the Type of Port is set to Periodic under Port Properties, this subnode is available from the context menu (right-click the Port parent node) or from the Physics toolbar, Attributes menu.

The Periodic Port Reference Point is used to uniquely identify two primitive unit cell vectors, $\mathbf{a}_1$ and $\mathbf{a}_2$ and two reciprocal lattice vectors, $\mathbf{G}_1$ and $\mathbf{G}_2$. These reciprocal vectors are defined in terms of the unit cell vectors, $\mathbf{a}_1$ and $\mathbf{a}_2$, tangent to the edges shared between the port and the adjacent periodic boundary conditions. $\mathbf{G}_1$ and $\mathbf{G}_2$ are defined by the relation

$$\frac{\mathbf{a}_1 \times \mathbf{a}_2}{|\mathbf{a}_1 \times \mathbf{a}_2|} = \mathbf{n}$$
where \( \mathbf{n} \) is the outward unit normal vector to the port boundary. If there are multiple points defined in the selection list, only the last point is used.

**POINT SELECTION**

The primitive unit cell vectors, \( \mathbf{a}_1 \) and \( \mathbf{a}_2 \) are defined from two edges sharing the **Periodic Port Reference Point** on a port boundary. The two vectors can have unequal lengths and are not necessarily orthogonal. They start from the **Periodic Port Reference Point**.

For listener (passive, observation, and not excited) ports, if the outward normal vector on the listener port boundary is opposite to that of the source port, the listener port reference point needs to be mirrored from the source port reference point based on the center coordinate of the model domain. For example, if the source port reference point is at \([-1,1,1]\) in a cubic domain around the origin, the mirrored listener port reference point is \([1,1,-1]\). In this case, if the **Azimuth angle of incidence** at the source port boundary is \( \alpha_2 \), the **Azimuth angle of incidence** at the listener port boundary \( \pi/2 - \alpha_2 \) and the signs of the diffraction order on the source and listener ports are opposite. See also **Periodic** for the angle definition.

If the lattice vectors are collinear with two Cartesian axes, then the lattice vectors can be defined without the **Periodic Port Reference Point**. For the port where \( n \) points along a positive Cartesian direction, \( \mathbf{a}_1 \) and \( \mathbf{a}_2 \) are also assigned to point along positive Cartesian directions. Conversely, for the port where \( n \) points along a negative Cartesian direction, \( \mathbf{a}_1 \) and \( \mathbf{a}_2 \) are assigned to point along negative Cartesian directions. The condition \( \mathbf{a}_1 \times \mathbf{a}_2 || \mathbf{n} \) is true on both ports. For example, if \( \mathbf{n} = \mathbf{z} \), then \( \mathbf{a}_1/|\mathbf{a}_1| = \mathbf{x} \) and \( \mathbf{a}_2/|\mathbf{a}_2| = \mathbf{y} \) and if \( \mathbf{n} = -\mathbf{z} \), then \( \mathbf{a}_1/|\mathbf{a}_1| = -\mathbf{x} \) and \( \mathbf{a}_2/|\mathbf{a}_2| = -\mathbf{y} \).

**Lumped Port**

Use the **Lumped Port** node to apply a voltage or current excitation of a model or to connect to a circuit. A lumped port is a simplification of the port boundary condition.
A Lumped Port condition can only be applied on boundaries that extend between two metallic boundaries — that is, boundaries where Perfect Electric Conductor, Impedance Boundary, or Transition Boundary (Electromagnetic Waves, Frequency Domain interface only) conditions apply — separated by a distance much smaller than the wavelength.

**LUMPED PORT PROPERTIES**

Enter a unique Lumped port name. It is recommended to use a numeric name as it is used to define the elements of the S-parameter matrix and numeric port names are also required for port sweeps and Touchstone file export (for the Electromagnetic Waves, Frequency Domain interface).

**Type of Lumped Port**
Select a Type of lumped port — Coaxial, Multielement uniform, Uniform, or User defined.

Select User defined for nonuniform ports, for example, a curved port and enter values or expressions in the fields — Height of lumped port $h_{\text{port}}$ (SI unit: m), Width of lumped port $w_{\text{port}}$ (SI unit: m), and Direction between lumped port terminals $a_{h}$. In 2D axisymmetry, Coaxial does not support a nonzero azimuthal mode number. The Azimuthal mode number in the Physics interface should be defined as zero.

Select Multielement uniform for multi-excitation or -termination of, for example, a coplanar waveguide port or a differential port. The direction of the field in each sub-element of the Multielement uniform lumped port is defined by the subnodes, Uniform element.

**Terminal Type**
Select a Terminal type — a Cable port for a voltage driven transmission line and S-parameter calculation, a Current driven port, or a Circuit port.

For Cable select On or Off from the Wave excitation at this port list to set whether it is an inport or a listener port. If On is selected, enter a Voltage $V_{0}$ (SI unit: V), and Port phase $\theta_{\text{in}}$ (SI unit: rad).
Note it is only possible to excite one Cable port at a time if the purpose is to compute S-parameters. In other cases, for example, when studying microwave heating, more than one input might be wanted, but the S-parameter variables cannot be correctly computed so if several ports are excited, the S-parameter output is turned off.

For the Electromagnetic Waves, Frequency Domain and Microwave Heating interfaces, the Port Sweep Settings cycles through the ports, computes the entire S-matrix, and exports it to a Touchstone file. When using port sweeps, the local setting for Wave excitation at this port is overridden by the solver so only one port at a time is excited.

**SETTINGS**

No entry is required if a Circuit terminal type is selected above.

- For a Cable terminal type enter the Characteristic impedance $Z_{\text{ref}}$ (SI unit: Ω).

- For a Current terminal type enter a Terminal current $I_0$ (SI unit: A).

**Calculate S-parameter**

The Calculate S-parameter check box needs to be activated for S-parameter calculation with the Electromagnetic Waves, Transient interface, while the Cable port in the Electromagnetic Wave, Frequency Domain calculates S-parameters automatically.

**VOLTAGE SOURCE TYPE**

When Calculate S-parameter is checked, select Voltage source type from the list (default Modulated Gaussian pulse). The Modulated Gaussian pulse is defined as

$$
\frac{2f_0}{\sqrt{2\pi}} e^{-\frac{(x - (0.97 \cdot 2/f_0))^2}{2/(2f_0)^2}} \cdot \sin(2\pi f_0 t)
$$

where the Center frequency $f_0$ defines the location as $0.97 \times 2/f_0$, the standard deviation as $1/(2f_0)$, and the modulation frequency $f_0$.

The Gaussian pulse location is slightly shifted from twice the period time by a factor of 0.97 to improve the frequency responses.
CENTER FREQUENCY

When Calculate S-parameter is checked, enter a Center frequency \( f_0 \) for the input modulated Gaussian pulse (SI unit: Hz).

- S-Parameters and Ports
- Lumped Ports with Voltage Input

As a multiport device example, Branch-Line Coupler: Application Library path RF_Module/Couplers_and_Power_Dividers/branch_line_coupler

For example of how to use the Multielement uniform lumped port, Coplanar Waveguide Bandpass Filter: Application Library path RF_Module/Filters/cpw_bandpass_filter

**Lumped Element**

Use a Lumped Element node to mimic the insertion of a capacitor, inductor, or general impedance between two metallic boundaries. A Lumped Element condition is a passive lumped port boundary condition which cannot be used as a source. Unlike a Lumped Port, it does not generate S-parameters. The sign of the current and power of a Lumped Element is opposite to that of a Lumped Port.

It can only be applied on boundaries that extend between two metallic boundaries — that is, boundaries where Perfect Electric Conductor, Impedance Boundary, or Transition Boundary (Electromagnetic Waves, Frequency Domain interface only) conditions apply — separated by a distance much smaller than the wavelength.

**LUMPED ELEMENT PROPERTIES**

Enter a unique Lumped element name. See Lumped Port for the rest of the settings.

**SETTINGS**

Select a Lumped element device — User defined (the default), Inductor, Capacitor, Parallel LC, Series LC, Parallel RLC, or Series RLC. Inductor, Capacitor, Parallel LC, Series LC, Parallel RLC, and Series RLC are available only in the frequency domain study type.

- For User defined enter a Lumped element impedance \( Z_{\text{element}} \) (SI unit: \( \Omega \)). The default is 50 \( \Omega \)
• For Inductor, Parallel LC, Series LC, Parallel RLC, or Series RLC enter a
Lumped element inductance $L_{\text{element}}$ (SI unit: H). The default is 1 nH.
• For Capacitor, Parallel LC, Series LC, Parallel RLC, or Series RLC enter a
Lumped element capacitance $C_{\text{element}}$ (SI unit: F). The default is 1 pF.
• For Parallel RLC, or Series RLC enter a Lumped element resistance $R_{\text{element}}$
(SI unit: $\Omega$). The default is 50 $\Omega$.

Uniform Element

The Uniform Element is available in 3D. When the Type of lumped port is set to
Multielement uniform under Lumped Port Properties, this subnode is available from the
context menu (right-click the Lumped Port parent node) or from the Physics toolbar,
Attributes menu.

PORT PROPERTIES

Enter a unique Uniform element name. Enter values or expressions in the fields —
Direction between uniform element terminals $a_h$. The Direction between uniform element
terminals defines the electric potential polarity, as $a_h$ points in the direction of the
electric field.

Electric Field

The Electric Field boundary condition

$$\mathbf{n} \times \mathbf{E} = \mathbf{n} \times \mathbf{E}_0$$

specifies the tangential component of the electric field. It should in general not be used
to excite a model. Consider using the Port, Lumped Port, or Scattering Boundary
Condition instead. It is provided mainly for completeness and for advanced users who
can recognize the special modeling situations when it is appropriate to use. The
commonly used special case of zero tangential electric field is described in the Perfect
Electric Conductor section.
**ELECTRIC FIELD**

Enter the value or expression for the components of the Electric field $E_0$ (SI unit: V/m).

**CONSTRAINT SETTINGS**

To display this section, click the **Show** button ( ) and select **Advanced Physics Options**.

**Magnetic Field**

The Magnetic Field node adds a boundary condition for specifying the tangential component of the magnetic field at the boundary:

$$\mathbf{n} \times \mathbf{H} = \mathbf{n} \times H_0$$

**MAGNETIC FIELD**

Enter the value or expression for the components of the Magnetic field $H_0$ (SI unit: A/m).

**Scattering Boundary Condition**

Use the Scattering Boundary Condition to make a boundary transparent for a scattered wave. The boundary condition is also transparent for an incoming plane wave. The scattered (outgoing) wave types for which the boundary condition is perfectly transparent are

$$\mathbf{E} = \mathbf{E}_{se} e^{-jk(\mathbf{n} \cdot \mathbf{r})} + \mathbf{E}_0 e^{-jk(\mathbf{k} \cdot \mathbf{r})}$$  Plane scattered wave

$$\mathbf{E} = \mathbf{E}_{sc} \frac{e^{-jk(\mathbf{n} \cdot \mathbf{r})}}{\sqrt{r}} + \mathbf{E}_0 e^{-jk(\mathbf{k} \cdot \mathbf{r})}$$  Cylindrical scattered wave

$$\mathbf{E} = \mathbf{E}_{sc} \frac{e^{-jk(\mathbf{n} \cdot \mathbf{r})}}{r_a} + \mathbf{E}_0 e^{-jk(\mathbf{k} \cdot \mathbf{r})}$$  Spherical scattered wave

The field $E_0$ is the incident plane wave that travels in the direction $\mathbf{k}$. The boundary condition is transparent for incoming (but not outgoing) plane waves with any angle of incidence.

The boundary is only perfectly transparent for scattered (outgoing) waves of the selected type at normal incidence to the boundary. That is, a plane wave at oblique incidence is partially reflected and so is a cylindrical wave or spherical wave unless the wave fronts are parallel to the boundary. For the Electromagnetic Waves, Frequency
Domain interface, the Perfectly Matched Layer feature is available as a general way of modeling an open boundary.

- For cylindrical waves, specify around which cylinder axis the waves are cylindrical. Do this by specifying one point at the cylinder axis and the axis direction.
- For spherical waves, specify the center of the sphere around which the wave is spherical.

The domain material adjacent to the boundary where the Scattering Boundary Condition is applied can be lossy.

If the problem is solved for the eigenfrequency or the scattered field, the boundary condition does not include the incident wave.

\[ E_{sc} = E_{sc}e^{-jk(r - r_0)} \]  
Plane scattered wave

\[ E_{sc} = E_{sc}e^{-jk(r - r_0)}/r \]  
Cylindrical scattered wave

\[ E_{sc} = E_{sc}e^{-jk(r - r_0)}/r_s \]  
Spherical scattered wave

**SCATTERING BOUNDARY CONDITION**

Select an **Incident field** — No incident field (the default), Wave given by E field, or Wave given by H field. Enter the expressions for the components for the **Incident electric field** \( E_0 \) or **Incident magnetic field** \( H_0 \).

If the **Incident field** is not set to No incident field, edit the **Incident wave direction** \( k_{dir} \) for the vector coordinates. The default direction is in the opposite direction to the boundary normal. For 2D axisymmetry, the **Incident wave direction** \( k_{dir} \) should be parallel or anti-parallel to the symmetry axis.

Select a **Scattered wave type** for which the boundary is absorbing — Plane wave (the default), Spherical wave, or Cylindrical wave.

- For the Electromagnetic Waves, Frequency Domain interface, select an **Order** — First order (the default) or Second order.
- For **Cylindrical wave** also enter coordinates for the **Source point** \( r_0 \) (SI unit: m) and **Source axis direction** \( r_{axis} \) (dimensionless). For 2D the **Source axis direction** is...
assumed to be in the $z$-direction, whereas in 2D axisymmetry it is assumed to be along the axis of rotation.

- For Spherical wave enter coordinates for the Source point $r_0$ (SI unit: m).

**INITIAL VALUES FOR INCIDENT WAVE**

For the Electromagnetic Waves, Transient interface enter the components for the initial value of the Magnetic vector potential $A_0$ (SI unit: Wb/m).

**DISPERSION AND ABSORPTION**

This section is only available for the Electromagnetic Waves, Transient interface. To display it, click the Show button ( ) and select Advanced Physics Options.

Select the Dispersion and absorption model that will be used when calculating the wave number and attenuation constant for the incident and scattered waves — Low loss.
approximation (the default), or High loss. For High loss also enter a Carrier frequency $f_0$
(SI unit: Hz). The default is 1 GHz.

When the Dispersion and absorption model is set to Low loss approximation
the refractive index is calculated from the relative permittivity and the
relative permeability as

$$n = \sqrt{\varepsilon_r \mu_r}.$$  

Similarly, the absorption coefficient is calculated as

$$\gamma = \frac{1}{2} \sigma \frac{\mu_0 \mu_r}{\varepsilon_0 \varepsilon_r} = \frac{1}{2} \sigma Z_c,$$

where $Z_c$ is the characteristic impedance.

When the Dispersion and absorption model is set to High loss, the real and
the imaginary parts of the complex refractive index is solved for from the
real and the imaginary parts of the relative permittivity, using the relations

$$n^2 - \kappa^2 = \varepsilon'_r \mu_r$$

and

$$2n\kappa = \varepsilon''_r \mu_r = \frac{\sigma \mu_r}{\omega \varepsilon_0}.$$  

The absorption coefficient is then given by

$$\gamma = \frac{\omega}{c} \kappa.$$  

Impedance Boundary Condition

The Impedance Boundary Condition

$$\left[\frac{\mu_0 \mathbf{H}}{\varepsilon_c} \mathbf{H} \times (\mathbf{n} \cdot \mathbf{E}) \mathbf{n} = (\mathbf{n} \cdot \mathbf{E}_a) \mathbf{n} - \mathbf{E}_a \right]$$

is used at boundaries where the field is known to penetrate only a short distance
outside the boundary. This penetration is approximated by a boundary condition to
avoid the need to include another domain in the model. Although the equation is identical to the one in the low-reflecting boundary condition, it has a different interpretation. The material properties are for the domain outside the boundary and not inside, as for low-reflecting boundaries. A requirement for this boundary condition to be a valid approximation is that the magnitude of the complex refractive index

\[ N = \left| \frac{\mu_1\varepsilon_\text{e}}{\mu_1\varepsilon_1} \right| \]

where \( \mu_1 \) and \( \varepsilon_1 \) are the material properties of the inner domain, is large; that is, \( |N| >> 1 \).

The source electric field \( \mathbf{E}_s \) can be used to specify a source surface current on the boundary.

The Surface Roughness subnode is available from the context menu (right-click the parent node) or from the Physics toolbar, Attributes menu.

**Figure 4-4:** The impedance boundary condition is used on exterior boundaries representing the surface of a lossy domain. The shaded (lossy) region is not part of the model. The effective induced image currents are of reduced magnitude due to losses. Any current flowing into the boundary is perfectly balanced by induced surface currents as for the perfect electric conductor boundary condition. The tangential electric field is generally small but non zero at the boundary.

**Impedance Boundary Condition**

Select an Electric displacement field model — Relative permittivity (the default), Refractive index, Loss tangent, loss angle, Loss tangent, dissipation factor, Dielectric loss, Drude-Lorentz dispersion model, or Debye dispersion model. See the Wave Equation, Electric node, Electric Displacement Field section, for all settings.
SOURCE ELECTRIC FIELD
To display this section, click the Show button ( ) and select Advanced Physics Options.

Enter a Source electric field $E_s$ (SI unit: V/m). The default is 0 V/m.

Surface Current Density
The Surface Current Density boundary condition

$$- \mathbf{n} \times \mathbf{H} = \mathbf{J}_s$$
$$\mathbf{n}_2 \times (\mathbf{H}_1 - \mathbf{H}_2) = \mathbf{J}_s$$

specifies a surface current density at both exterior and interior boundaries, respectively. The current density is specified as a three-dimensional vector, but because it needs to flow along the boundary surface, COMSOL Multiphysics projects it onto the boundary surface and neglects its normal component. This makes it easier to specify the current density and avoids unexpected results when a current density with a component normal to the surface is given.

For Transition Boundary Condition, the Surface Current Density subnode as an one-sided surface current density is available from the context menu (right-click the Transition Boundary Condition parent node) or from the Physics toolbar, Attributes menu.

SURFACE CURRENT DENSITY
Enter values or expressions for the components of the Surface current density $J_{s0}$ (SI unit: A/m).

For the Surface Current Density subnode, select Side — Upside (the default) or Downside to define on which side the Surface Current Density is applied. The red arrow visualized on the selected boundaries always indicates the upside.
Surface Magnetic Current Density

The Surface Magnetic Current Density boundary condition

\[ \mathbf{n} \times \mathbf{E} = J_{ms} \]

\[ \mathbf{n}_2 \times (\mathbf{E}_1 - \mathbf{E}_2) = -J_{ms} \]

specifies a surface magnetic current density at both exterior and interior boundaries, respectively. The magnetic current density is specified as a three-dimensional vector, but because it needs to flow along the boundary surface, COMSOL Multiphysics projects it onto the boundary surface and neglects its normal component. This makes it easier to specify the magnetic current density and avoids unexpected results when a magnetic current density with a component normal to the surface is given.

**Surface Magnetic Current Density**

Enter values or expressions for the components of the Surface magnetic current density \( J_{ms0} \) (SI unit: V/m).

Surface Roughness

The Surface Roughness subnode is available from the context menu (right-click the Impedance Boundary Condition or Transition Boundary Condition parent node) or from the Physics toolbar, Attributes menu.

**Surface Roughness**

Select a Surface roughness model — Sawtooth (the default) or Snowball. For Sawtooth, enter a value or expression for the Surface roughness \( \Delta_{RMS} \) (SI unit: m). For Snowball, enter a Snowball hexagon cell area \( A_{hex} \) (SI unit: m\(^2\)). Then enter values for the Snowball radius (SI unit: m) and Number of snowballs in the table, adding as many rows as you need.

Select the Skin depth type to be Physics-controlled (the default) or User defined. For User defined, enter the Skin depth \( \delta \) (SI unit: m).

The Surface Roughness increases the impedance, and consequently it decreases the surface current density of Impedance Boundary Condition or Transition Boundary Condition, proportional to the ratio between the impedance of a rough surface and that of a smooth surface:
SAWTOOTH MODEL

\[ Z_{\text{rough}} = Z_{\text{smooth}} \left[ 1 + \frac{2}{\pi} \arctan \left( 1.4 \left( \frac{\Delta_{\text{RMS}}}{\delta} \right)^2 \right) \right] \]

where \( \Delta_{\text{RMS}} \) is the root mean square of the surface roughness, and \( \delta \) is the skip depth of the material (Ref. 1).

SNOWBALL MODEL

\[ Z_{\text{rough}} = Z_{\text{smooth}} \left[ 1 + \frac{3}{2} \sum_i N_i \left( \frac{4\pi a_i^2}{A_{\text{hex}}} \right) \left( 1 + \frac{\delta}{a_i} + \frac{\delta^2}{a_i^2} \right) \right] \]

where \( A_{\text{hex}} \) is the hexagonal area of a unit cell, \( N_i \) is the number of snowballs, \( a_i \) is the radius of a snowball, and \( \delta \) is the skip depth of the material (Ref. 2).

For Transition Boundary Condition, select **Side — Upside** (the default) or **Downside** to define on which side the **Surface Roughness** is applied. The red arrow visualized on the selected boundaries always indicates the upside.

**References**


**Transition Boundary Condition**

The *Transition Boundary Condition* is used on interior boundaries to model a sheet of a medium that should be geometrically thin but does not have to be electrically thin. It represents a discontinuity in the tangential electric field. Mathematically it is described by a relation between the electric field discontinuity and the induced surface current density:
where indices 1 and 2 refer to the different sides of the layer. This feature is not available with the Electromagnetic Waves, Transient interface.

The Surface Roughness subnode is available from the context menu (right-click the parent node) or from the Physics toolbar, Attributes menu.

TRANSITION BOUNDARY CONDITION

Select an Electric displacement field model — Relative permittivity (the default), Refractive index, Loss tangent, loss angle, Loss tangent, dissipation factor, Dielectric loss, Drude-Lorentz dispersion model, or Debye dispersion model. See the Wave Equation, Electric node, Electric Displacement Field section, for all settings.

Enter a Thickness \( d \) (SI unit: m). The default is 0.01 m.

Periodic Condition

The Periodic Condition sets up a periodicity between the selected boundaries. The Destination Selection subnode is available from the context menu (right-click the parent node) or from the Physics toolbar, Attributes menu.

BOUNDARY SELECTION

The software automatically identifies the boundaries as either source boundaries or destination boundaries. This works fine for cases like opposing parallel boundaries. To control the destination, add a Destination Selection subnode. By default it contains the selection that COMSOL Multiphysics has identified.
PERIODICITY SETTINGS
Select a Type of periodicity — Continuity (the default), Antiperiodicity, or Floquet periodicity. Select:

- **Continuity** to make the electric field periodic (equal on the source and destination),
- **Antiperiodicity** to make it antiperiodic, or
- **Floquet periodicity** (The Electromagnetic Waves, Frequency Domain Interface only) to use a Floquet periodicity (Bloch-Floquet periodicity).
  - For **Floquet periodicity** also enter the source for the k-vector for Floquet periodicity.
  - For **User defined** specify the components of the k-vector for Floquet periodicity \( k_F \) (SI unit: rad/m).
  - For **From periodic port** the k-vector for Floquet periodicity \( k_F \) is obtained from the Periodic Port settings.

CONSTRAINT SETTINGS
To display this section, click the Show button ( ) and select Advanced Physics Options.

For information about the Orientation of Source section, see Orientation of Source and Destination in the COMSOL Multiphysics Reference Manual.

---

### Magnetic Current

The **Magnetic Current** node specifies a magnetic line current along one or more edges. For a single Magnetic Current source, the electric field is orthogonal to both the line and the distance vector from the line to the field point. For 2D and 2D axisymmetric models the Magnetic Current node is applied to Points, representing magnetic currents directed out of the model plane. For 3D models, the **Magnetic Current** is applied to Edges.

<table>
<thead>
<tr>
<th>Periodic Boundary Conditions</th>
</tr>
</thead>
<tbody>
<tr>
<td>• Fresnel Equations: Application Library path&lt;br&gt;RF_Module/Verification_Examples/fresnel_equations</td>
</tr>
<tr>
<td>• Plasmonic Wire Grating: Application Library path:&lt;br&gt;RF_Module/Tutorials/plasmonic_wire_grating</td>
</tr>
</tbody>
</table>
MAGNETIC CURRENT
Enter a value for the Magnetic current $I_m$ (SI unit: V).

Two-Port Network
Use the Two-Port Network node to characterize the response of a two-port network system such as reflection and transmission using S-parameters.

A Two-Port Network can only be applied on boundaries that extend between two metallic boundaries — that is, boundaries where Perfect Electric Conductor, Impedance Boundary, or Transition Boundary conditions apply — separated by a distance much smaller than the wavelength.

TWO-PORT NETWORK PROPERTIES
Type of Port
Select a Type of Port — Coaxial or Uniform.

Coaxial does not support nonzero azimuthal mode number. The Azimuthal mode number in the Physics interface should be defined as zero.

Type of S-parameter Definition
Select a Type of S-parameter Definition — Matrix or Touchstone file.

Matrix defines the S-parameter by a constant matrix input. Touchstone file imports a two-port Touchstone file to characterize the two-port boundaries as a function of frequency.

SETTINGS
Enter the Characteristic impedance $Z_{ref}$ (SI unit: $\Omega$) for Matrix.

INTERPOLATION AND EXTRAPOLATION
Select Interpolation and Extrapolation types to obtain S-parameter values from the imported Touchstone file corresponding to the simulation frequencies.

Interpolation
Select a Interpolation — Nearest neighbor, Linear, Piecewise cubic, or Cubic spline (the default).

The S-parameter values are interpolated within the frequency range specified in the Touchstone file.
**Extrapolation**

Select an **Extrapolation** — **Constant** (the default), **Linear**, **Nearest function**, or **Specific value**.

The S-parameter values are extrapolated outside the frequency range specified in the Touchstone file.

---

**Two-Port Network Port**

A pair of **Two-Port Network Port** subnodes is added by default to the **Two-Port Network** node and is used to select boundaries corresponding to port 1 and port 2 in the S-parameter input, respectively.

---

**Edge Current**

The **Edge Current** node specifies an electric line current along one or more edges.

**EDGE CURRENT**

Enter an **Edge current** $I_0$ (SI unit: A).

---

**Electric Point Dipole**

Add **Electric Point Dipole** nodes to 3D and 2D models. This represents the limiting case of when the length $d$ of a current filament carrying uniform current $I$ approaches zero while maintaining the product between $I$ and $d$. The dipole moment is a vector entity with the positive direction set by the current flow.

**DIPOLE SPECIFICATION**

Select a **Dipole specification** — **Magnitude and direction** or **Dipole moment**.

**DIPOLE PARAMETERS**

Based on the **Dipole specification** selection:

- For **Magnitude and direction** enter coordinates for the **Electric current dipole moment direction** $\mathbf{n}_p$ and **Electric current dipole moment, magnitude** $p$ (SI unit: A·m).
- For **Dipole moment** enter coordinates for the **Electric current dipole moment** $\mathbf{p}$ (SI unit: A·m).
**Magnetic Point Dipole**

Add a Magnetic Point Dipole to 3D and 2D models. The point dipole source represents a small circular current loop $I$ in the limit of zero loop area $a$ at a fixed product $I^*a$.

**Dipole Specification**

Select a Dipole specification — **Magnitude and direction** or **Dipole moment**.

**Dipole Parameters**

Based on the Dipole specification selection:

- For **Magnitude and direction** enter coordinates for the Magnetic dipole moment direction $n_m$ and Magnetic dipole moment, magnitude $m$ (SI unit: m²·A).
- For **Dipole moment** enter coordinates for the Magnetic dipole moment $m$ (SI unit: m²·A).

**Line Current (Out-of-Plane)**

Add a Line Current (Out-of-Plane) node to 2D or 2D axisymmetric models. This specifies a line current out of the modeling plane. In axially symmetric geometries this is the rotational direction, in 2D geometries it is the $z$ direction.

**Line Current (Out-of-Plane)**

Enter an Out-of-plane current $I_0$ (SI unit: A).

**Archie’s Law**

This subfeature is available only when Archie’s law is selected as the Electrical conductivity material parameter in the parent feature (for example, the Wave Equation, Electric node). Then the subnodes are made available from the context menu (right-click the parent node) as well as from the Physics toolbar, Attributes menu.

Use the Archie’s Law subnode to provide an electrical conductivity computed using Archie’s Law. This subnode can be used to model nonconductive porous media saturated (or variably saturated) by conductive liquids, using the relation:
CONDUCTION CURRENTS

By default, the Electrical conductivity $\sigma_L$ (SI unit: S/m) for the fluid is defined From material. This uses the value of the conductivity of the material domain.

For User defined enter a value or expression. If another type of temperature dependence is used other than a linear temperature relation, enter any expression for the conductivity as a function of temperature.

Enter these dimensionless parameters as needed:

- Cementation exponent $m$
- Saturation exponent $n$
- Fluid saturation $S_L$
- Porosity $\varepsilon_p$ to set up the volume fraction of the fluid.

Archie’s Law Theory

POROUS MEDIA

This subfeature is available only when Porous media is selected as the material parameter (for example, Relative permeability or Relative permittivity) in the parent feature node when it is available with the physics interface (for example, the Wave Equation, Electric node). Then the subnodes are made available from the context menu (right-click the parent node) as well as from the Physics toolbar, Attributes menu.

Use the Porous Media subfeature to specify the material properties of a domain consisting of a porous medium using a mixture model. Depending on the specific physics interface being used, the subfeature can be used to provide a mixture model for the electric conductivity $\sigma$, the relative dielectric permittivity $\varepsilon_r$, or the relative magnetic permeability $\mu_r$.

POROUS MEDIA

This section is always available and is used to define the mixture model for the domain.

Select the Number of materials (up to 5) to be included in the mixture model.
For each material (Material 1, Material 2, and so on), select either Domain material, to use the material specified for the domain, or one of the other materials specified in the Materials node. For each material, enter a Volume fraction \( \theta_1, \theta_2, \) and so on.

The Volume fractions specified for the materials should be fractional (between 0 and 1) and should add to 1 in normal cases.

The availability of the Effective Electrical Conductivity, Effective Relative Permittivity, and Effective Relative Permeability sections depend on the material properties used in the physics interface. In addition, these sections are only active if Porous media is selected in the corresponding material property for the parent feature node.

**Effective Electrical Conductivity, Effective Relative Permittivity, or Effective Relative Permeability**

Select the averaging method to use in the mixture model between the Volume average of the material property (for example, conductivity or permittivity), the volume average of its inverse (for example, the resistivity), or the Power law. For each material, specify either From material, to take the value from the corresponding material specified in the Porous Media section, or User defined to manually input a value.
The Electromagnetic Waves, Transient Interface

The Electromagnetic Waves, Transient (temw) interface, found under the Radio Frequency branch when adding a physics interface, is used to solve a time-domain wave equation for the magnetic vector potential. The sources can be in the form of point dipoles, line currents, or incident fields on boundaries or domains. It is primarily used to model electromagnetic wave propagation in different media and structures when a time-domain solution is required—for example, for non-sinusoidal waveforms or for nonlinear media. Typical applications involve the propagation of electromagnetic pulses.

When this physics interface is added, these default nodes are also added to the Model Builder—Wave Equation, Electric, Perfect Electric Conductor, and Initial Values. Then, from the Physics toolbar, add other nodes that implement, for example, boundary conditions and mass sources. You can also right-click Electromagnetic Waves, Transient to select physics features from the context menu.

Except where indicated, most of the settings are the same as for The Electromagnetic Waves, Frequency Domain Interface.

**SETTINGS**

The Label is the default physics interface name.

The Name is used primarily as a scope prefix for variables defined by the physics interface. Refer to such physics interface variables in expressions using the pattern `<name>_<variable_name>`. In order to distinguish between variables belonging to different physics interfaces, the name string must be unique. Only letters, numbers and underscores (_) are permitted in the Name field. The first character must be a letter.

The default Name (for the first physics interface in the model) is `temw`.

**COMPONENTS**

This section is available for 2D and 2D axisymmetric components.

Select the Electric field components solved for. Select:

-Three-component vector (the default) to solve using a full three-component vector for the electric field $E$. 
• **Out-of-plane vector** to solve for the electric field vector component perpendicular to the modeling plane, assuming that there is no electric field in the plane.

• **In-plane vector** to solve for the electric field vector components in the modeling plane assuming that there is no electric field perpendicular to the plane.

**DEPENDENT VARIABLES**

The dependent variable (field variable) is for the **Magnetic vector potential** \( \mathbf{A} \). The name can be changed but the names of fields and dependent variables must be unique within a model.

**DISCRETIZATION**

To display this section, click the **Show** button ( ) and select **Discretization**.

- Domain, Boundary, Edge, Point, and Pair Nodes for the Electromagnetic Waves, Transient Interface
- Theory for the Electromagnetic Waves Interfaces

---

**Transient Modeling of a Coaxial Cable**

Application Library path

`RF_Module/Verification_Examples/coaxial_cable_transient`

The Electromagnetic Waves, Transient Interface shares most of its nodes with The Electromagnetic Waves, Frequency Domain Interface.

The domain, boundary, edge, point, and pair nodes are available from the **Physics** ribbon toolbar (Windows users), **Physics** context menu (Mac or Linux users), or right-click to access the context menu (all users).

In general, to add a node, go to the **Physics** toolbar, no matter what operating system you are using. Subnodes are available by clicking the parent node and selecting it from the **Attributes** menu.
**Domain**

These nodes are unique for this physics interface and described in this section:

- Wave Equation, Electric
- Initial Values
- Drude-Lorentz Polarization

**Boundary Conditions**

With no surface currents present the boundary conditions need to be fulfilled. Depending on the field being solved for, it is necessary to analyze these conditions differently. When solving for $A$, the first condition can be formulated in the following way.

\[
\mathbf{n}_2 \times (\mathbf{E}_1 - \mathbf{E}_2) = 0 \\
\mathbf{n}_2 \times (\mathbf{H}_1 - \mathbf{H}_2) = 0
\]

The tangential component of the magnetic vector potential is always continuous and thus the first condition is fulfilled. The second condition is equivalent to the natural boundary condition.

\[
-\mathbf{n} \times (\mu_r^{-1} \nabla \times \mathbf{A}_1 - \mu_r^{-1} \nabla \times \mathbf{A}_2) = -\mathbf{n} \times \mu_r^{-1} (\mathbf{H}_1 - \mathbf{H}_2) = 0
\]

These nodes and subnodes are available and described for the Electromagnetic Waves, Frequency Domain interface (listed in alphabetical order):

- Archie’s Law
- Lumped Port
- Magnetic Field
- Perfect Electric Conductor
- Perfect Magnetic Conductor
- Periodic Condition
- Porous Media
- Scattering Boundary Condition
- Surface Current Density
**EDGE, POINT, AND PAIR**

These edge, point, and pair nodes are available and described for the Electromagnetic Waves, Frequency Domain interface (listed in alphabetical order):

- Edge Current
- Electric Point Dipole (2D and 3D components)
- Line Current (Out-of-Plane) (2D and 2D axisymmetric components)
- Magnetic Point Dipole (2D and 3D components)
- Perfect Electric Conductor
- Perfect Magnetic Conductor
- Surface Current Density

---

For axisymmetric components, COMSOL Multiphysics takes the axial symmetry boundaries (at \( r = 0 \)) into account and automatically adds an **Axial Symmetry** node to the component that is valid on the axial symmetry boundaries only.

---

In the *COMSOL Multiphysics Reference Manual* see Table 2-3 for links to common sections and Table 2-4 to common feature nodes. You can also search for information: press F1 to open the **Help** window or Ctrl+F1 to open the **Documentation** window.

---

**Wave Equation, Electric**

The **Wave Equation, Electric** node is the main node for the Electromagnetic Waves, Transient interface. The governing equation can be written in the form

\[
\mu_0 \sigma \frac{\partial A}{\partial t} + \mu_0 \varepsilon_0 \nabla \cdot \left( \varepsilon \frac{\partial A}{\partial t} \right) + \nabla \times (\mu_r^{-1} \nabla \times A) = 0
\]

for transient problems with the constitutive relations \( \mathbf{B} = \mu_0 \mu_r \mathbf{H} \) and \( \mathbf{D} = \varepsilon_0 \varepsilon_r \mathbf{E} \). Other constitutive relations can also be handled for transient problems. The **Divergence Constraint** subnode is available from the context menu (right-click the parent node) or from the **Physics** toolbar, **Attributes** menu.
**Electric Displacement Field**

Select an electric displacement field model—Relative permittivity (the default), Refractive index, Polarization, Remanent electric displacement, or Drude-Lorentz dispersion model.

**Relative Permittivity**

When **Relative permittivity** is selected, the default Relative permittivity $\varepsilon_r$ (dimensionless) takes values From material. For **User defined** select Isotropic, Diagonal, Symmetric, or Anisotropic and enter values or expressions in the field or matrix. If **Porous media** is selected, the Porous Media subnode is available from the context menu (right-click the parent node) or from the Physics toolbar, Attributes menu.

**Refractive Index**

When **Refractive index** is selected, the default Refractive index $n$ (dimensionless) takes the value From material. To specify the refractive index and assume a relative permeability of unity and zero conductivity, for one or both of the options, select **User defined** then choose Isotropic, Diagonal, Symmetric, or Anisotropic. Enter values or expressions in the field or matrix.

**Polarization**

For Polarization enter coordinates for the Polarization $\mathbf{P}$ (SI unit: C/m$^2$).

**Remanent Electric Displacement**

For Remanent electric displacement enter coordinates for the Remanent electric displacement $\mathbf{D}_r$ (SI unit: C/m$^2$). Then select **User defined** or **From Material** as above for the Relative permittivity $\varepsilon_r$.

**Drude-Lorentz Dispersion Model**

For Drude-Lorentz Dispersion Model select **User defined** or **From Material** for the Relative permittivity, high-frequency $\varepsilon_\infty$ and enter a value for the Plasma frequency $\omega_p$ (SI unit: rad/s).

When Drude-Lorentz Dispersion Model is selected, the Drude-Lorentz Polarization subnode is available from the context menu (right-click the parent node) or from the Physics toolbar, Attributes menu. Each Drude-Lorentz Polarization subnode adds another polarization term $\mathbf{P}_n$ to the electric displacement field $\mathbf{D}$, defined by
where the polarization is the solution to the ordinary differential equation

\[
\left( \frac{\partial^2}{\partial t^2} + \Gamma_n \frac{\partial}{\partial t} + \omega_n^2 \right) P_n = \varepsilon_0 \rho_n \omega_p^2 E.
\]

For more information, see the Drude-Lorentz Polarization feature.

**Magnetic Field**

This section is available if **Relative permittivity, Polarization, or Remanent electric displacement** are chosen as the **Electric displacement field model**.

Select the **Constitutive relation—Relative permeability** (the default), **Remanent flux density**, or **Magnetization**.

**Relative Permeability**

For **Relative permeability** the relative permeability \( \mu_r \) uses values **From material**. For **User defined** select **Isotropic, Diagonal, Symmetric, or Anisotropic** based on the characteristics of the magnetic field, and then enter values or expressions in the field or matrix. If **Porous media** is selected, the **Porous Media** subnode is available from the context menu (right-click the parent node) or from the **Physics toolbar, Attributes menu**.

**Remanent Flux Density**

For **Remanent flux density** the relative permeability \( \mu_r \) uses values **From material**. For **User defined** select **Isotropic, Diagonal, Symmetric, or Anisotropic** based on the characteristics of the magnetic field, and then enter values or expressions in the field or matrix. Then enter coordinates for the **Remanent flux density** \( B_r \) (SI unit: T).

**Magnetization**

For **Magnetization** enter coordinates for \( \mathbf{M} \) (SI unit: A/m).

**Conduction Current**

This section is available if **Relative permittivity, Polarization, or Remanent electric displacement** are chosen as the **Electric displacement field model**.

By default, the **Electrical conductivity** \( \sigma \) (SI unit: S/m) uses values **From material**.

- For **User defined** select **Isotropic, Diagonal, Symmetric, or Anisotropic** based on the characteristics of the current and enter values or expressions in the field or matrix.
• For Linearized resistivity the default values for the Reference temperature \( T_{\text{ref}} \) (SI unit: K), Resistivity temperature coefficient \( \alpha \) (SI unit: 1/K), and Reference resistivity \( \rho_0 \) (SI unit: \( \Omega \cdot \text{m} \)) use values from material. For User defined enter other values or expressions for any of these variables.

• If Porous media is selected, the Porous Media subnode is available from the context menu (right-click the parent node) or from the Physics toolbar, Attributes menu.

• If Archie’s Law is selected, the Archie’s Law subnode is available from the context menu (right-click the parent node) or from the Physics toolbar, Attributes menu.

**Initial Values**

The Initial Values node adds an initial value for the magnetic vector potential and its time derivative that serves as initial conditions for the transient simulation.

**INITIAL VALUES**

Enter values or expressions for the initial values of the components of the magnetic vector potential \( \mathbf{A} \) (SI unit: Wb/m) and its time derivative \( \partial \mathbf{A}/\partial t \) (SI unit: V/m). The default values are 0 Wb/m and 0 V/m, respectively.

**Drude-Lorentz Polarization**

This subfeature is available only when Drude-Lorentz Dispersion Model is selected as the Electric displacement field model in the Wave Equation, Electric feature node. Then the subnodes are made available from the context menu (right-click the parent node) as well as from the Physics toolbar, Attributes menu.

Each Drude-Lorentz Polarization subnode adds another polarization term \( \mathbf{P}_n \) to the electric displacement field \( \mathbf{D} \), defined by

\[
\mathbf{D} = \varepsilon_0 \varepsilon_\infty \mathbf{E} + \sum_{n=1}^{N} \mathbf{P}_n,
\]

where the polarization is the solution to the ordinary differential equation

\[
\left( \frac{\partial^2}{\partial t^2} + \Gamma_n \frac{\partial}{\partial t} + \omega_n^2 \right) \mathbf{P}_n = \varepsilon_0 \varepsilon'_n \omega_p^2 \mathbf{E}.
\]

Here \( \Gamma_n \) is a damping coefficient, \( \omega_n \) is a resonance frequency, \( f_n \) is an oscillator strength, and \( \omega_p \) is the plasma frequency.
Enter values or expressions for the Oscillator strength $f_n$ (SI unit: 1), the Resonance frequency $\omega_n$ (SI unit: rad/s), and the Damping in time coefficient $\Gamma_n$ (SI unit: rad/s).

**INITIAL VALUES**

Enter values or expressions for the initial values of the components of the Drude-Lorentz polarization $P_n$ (SI unit: C/m$^2$) and its time derivative $\partial P_n/\partial t$ (SI unit: A/m$^2$). The default values are 0 C/m$^2$ and 0 A/m$^2$, respectively.

**DISCRETIZATION**

To display this section, click the Show button ( ) and select Discretization. Select the element order from the list box for the Drude-Lorentz polarization $P_n$.
The Transmission Line Interface

The Transmission Line (tl) interface, found under the Radio Frequency branch when adding a physics interface, is used to study propagation of waves along one-dimensional transmission lines. The physics interface solves the time-harmonic transmission line equation for the electric potential.

The physics interface is used when solving for electromagnetic wave propagation along one-dimensional transmission lines and is available in 1D, 2D, and 3D. The physics interface has Eigenfrequency and Frequency Domain study types available. The Frequency Domain study is used for source driven simulations for a single frequency or a sequence of frequencies.

When this physics interface is added, these default nodes are also added to the Model Builder—Transmission Line Equation, Absorbing Boundary, and Initial Values. Then, from the Physics toolbar, add other nodes that implement, for example, boundary conditions. You can also right-click Transmission Line to select physics features from the context menu.

**SETTINGS**

The Label is the default physics interface name.

The Name is used primarily as a scope prefix for variables defined by the physics interface. Refer to such physics interface variables in expressions using the pattern `<name>.<variable_name>`. In order to distinguish between variables belonging to different physics interfaces, the name string must be unique. Only letters, numbers, and underscores (_) are permitted in the Name field. The first character must be a letter.

The default Name (for the first physics interface in the model) is `tl`.

**PORT SWEEP SETTINGS**

Select the Activate port sweep check box to switch on the port sweep. When selected, this invokes a parametric sweep over the lumped ports in addition to the automatically generated frequency sweep. The generated lumped parameters are in the form of an S-parameter matrix. For Activate port sweep enter a Sweep parameter name (the default is PortName) to assign a specific name to the variable that controls the port number solved for during the sweep.
For this physics interface, the S-parameters are subject to Touchstone file export. Click **Browse** to locate the file, or enter a file name and path. Select an **Output format**—Magnitude angle, Magnitude (dB) angle, or Real imaginary.

**DEPENDENT VARIABLES**
The dependent variable (field variable) is the **Electric potential** \( V \) (SI unit: V). The name can be changed but the names of fields and dependent variables must be unique within a model.

**DISCRETIZATION**
To display this section, click the **Show** button ( \( \text{Show} \) ) and select **Discretization**.

- Domain, Boundary, Edge, Point, and Pair Nodes for the Transmission Line Equation Interface
- Theory for the Transmission Line Interface
- Visualization and Selection Tools in the COMSOL Multiphysics Reference Manual

**Quarter-Wave Transformer**: Application Library path

`RF_Module/Transmission_Lines_and_Waveguides/quarter_wave_transformer`

*Domain, Boundary, Edge, Point, and Pair Nodes for the Transmission Line Equation Interface*

The Transmission Line Interface has these domain, boundary, edge, point, and pair nodes available from the **Physics** ribbon toolbar (Windows users), **Physics** context menu (Mac or Linux users), or right-click to access the context menu (all users).

In general, to add a node, go to the **Physics** toolbar, no matter what operating system you are using. Subnodes are available by clicking the parent node and selecting it from the **Attributes** menu.
Transmission Line Equation

The Transmission Line Equation node is the main feature of the Transmission Line interface. It defines the 1D wave equation for the electric potential. The wave equation is written in the form

\[
\frac{\partial}{\partial x} \left( \frac{1}{R + i\omega L} \frac{\partial V}{\partial x} \right) - (G + i\omega C)V = 0
\]

where \( R, L, G, \) and \( C \) are the distributed resistance, inductance, conductance, and capacitance, respectively.

**Transmission Line Equation**

Enter the values for the following:

- **Distributed resistance** \( R \) (SI unit: m·kg/(s^3·A^2)). The default is 0 m·kg/(s^3·A^2).
- **Distributed inductance** \( L \) (SI unit: H/m). The default is 2.5e-6 H/m.
- **Distributed conductance** \( G \) (SI unit: S/m). The default is 0 S/m.
- **Distributed capacitance** \( C \) (SI unit: F/m). The default is 1e-9 F/m.

The default values give a characteristic impedance for the transmission line of 50 Ω.
**Initial Values**

The *Initial Values* node adds an initial value for the electric potential that can serve as an initial guess for a nonlinear solver.

**INITIAL VALUES**

Enter values or expressions for the initial values of the **Electric potential** \( V \) (SI unit: V).

**Absorbing Boundary**

The *Absorbing Boundary* condition is stated as

\[
\frac{n \cdot \nabla V}{R + j\omega L} + \frac{V}{Z_0} = 0
\]

where \( \gamma \) is the complex propagation constant defined by

\[
\gamma = \sqrt{(R + j\omega L)(G + j\omega C)}
\]

and \( n \) is the normal pointing out of the domain. The *absorbing boundary* condition prescribes that propagating waves are absorbed at the boundary and, thus, that there is no reflection at the boundary. The Absorbing Boundary condition is only available on external boundaries.

**Incoming Wave**

The *Incoming Wave* boundary condition

\[
\frac{n \cdot \nabla V}{R + j\omega L} + \frac{V - 2V_0}{Z_0} = 0
\]

lets a wave of complex amplitude \( V_{in} \) enter the domain. The complex propagation constant \( \gamma \) and the outwards-pointing normal \( n \) are defined in the section describing the *Absorbing Boundary* node. The Incoming Wave boundary condition is only available on external boundaries.
**VOLTAGE**

Enter the value or expression for the input **Electric potential** \( V_0 \) (SI unit: V). The default is 1 V.

---

**Open Circuit**

The **Open Circuit** boundary condition is a special case of the **Terminating Impedance** boundary condition, assuming an infinite impedance, and, thus, zero current at the boundary. The condition is thus

\[
\mathbf{n} \cdot \nabla V = 0
\]

The Open Circuit boundary condition is only available on external boundaries.

---

**Terminating Impedance**

The **Terminating Impedance** boundary condition

\[
\frac{\mathbf{n} \cdot \nabla V}{R + j\omega L} + \frac{V}{Z_L} = 0
\]

specifies the terminating impedance to be \( Z_L \). Notice that the **Absorbing Boundary** condition is a special case of this boundary condition for the case when

\[ Z_L = Z_0 = \sqrt{\frac{R + j\omega L}{G + j\omega C}} \]

The **Open Circuit** and **Short Circuit** boundary conditions are also special cases of this condition. The Terminating Impedance boundary condition is only available on external boundaries.
**Impedance**
Enter the value or expression for the Impedance $Z_L$ (SI unit: $\Omega$). The default is 50 $\Omega$.

**Short Circuit**
The Short Circuit node is a special case of the Terminating Impedance boundary condition, assuming that impedance is zero and, thus, the electric potential is zero. The constraint at this boundary is, thus, $V = 0$.

**Constraint Settings**
To display this section, click the Show button ( ) and select Advanced Physics Options.

**Lumped Port**
Use the Lumped Port node to apply a voltage or current excitation of a model or to connect to a circuit. The Lumped Port node also defines S-parameters (reflection and transmission coefficients) that can be used in later postprocessing steps.

**Port Properties**
Enter a unique Port Name. It is recommended to use a numeric name as it is used to define the elements of the S-parameter matrix and numeric port names are also required for port sweeps and Touchstone file export.

Select a Type of Port — Cable (the default), Current, or Circuit.

**Settings**
If a Circuit port type is selected under Port Properties, this section does not require any selection.
• If a Cable port type is selected under Port Properties, enter the Characteristic impedance \( Z_{\text{ref}} \) (SI unit: \( \Omega \)). The default is 50 \( \Omega \).

• If a Current terminal type is selected under Port Properties, enter a Terminal current \( I_0 \) (SI unit: A). The default is 1 A.

If Cable is selected as the port type, select the Wave excitation at this port check box to enter values or expressions for the:

• Electric potential \( V_0 \) (SI unit: V). The default is 1 V.

• Port phase \( \theta_{\text{in}} \) (SI unit: rad). The default is 0 radians.
The Electromagnetic Waves, Time Explicit Interface

The Electromagnetic Waves, Time Explicit (ewte) interface ( ), found under the Radio Frequency branch ( ) when adding a physics interface, is used to model time-dependent electromagnetic wave propagation in linear media. The sources can be in the form of volumetric electric or magnetic currents, or electric surface currents or fields on boundaries.

This physics interface solves two first-order partial differential equations (Faraday’s law and Maxwell-Ampère’s law) for the electric and magnetic fields using the time explicit discontinuous Galerkin method.

When this physics interface is added, these default nodes are also added to the Model Builder—Wave Equations, Perfect Electric Conductor, and Initial Values. Then, from the Physics toolbar, add other nodes that implement, for example, boundary conditions. You can also right-click Electromagnetic Waves, Time Explicit to select physics features from the context menu.

The interface includes absorbing layers that are used to set up effective nonreflecting like boundary conditions. These features are added from the Definitions toolbar, by clicking Absorbing Layer. If COMSOL Multiphysics is not running in full-screen mode nor in a large window, Absorbing Layer is accessible on the Definitions toolbar by first clicking Coordinate Systems and then Absorbing Layer. You can also right-click Definitions in the Model Builder and select Absorbing Layer from the context menu.

SETTINGS
The Label is the default physics interface name.

The Name is used primarily as a scope prefix for variables defined by the physics interface. Refer to such physics interface variables in expressions using the pattern <name>.<variable_name>. In order to distinguish between variables belonging to different physics interfaces, the name string must be unique. Only letters, numbers, and underscores (_) are permitted in the Name field. The first character must be a letter.

The default Name (for the first physics interface in the model) is ewte.

COMPONENTS
This section is available for 2D and 2D axisymmetric components.
Select the **Field components solved for**:

- **Full wave** (the default) to solve using a full three-component vector for the electric field \( E \) and the magnetic field \( H \).
- **E in plane (TM wave)** to solve for the electric field vector components in the modeling plane and one magnetic field vector component perpendicular to the plane, assuming that there is no electric field perpendicular to the plane and no magnetic field components in the plane.
- **H in plane (TE wave)** to solve for the magnetic field vector components in the modeling plane and one electric field vector component perpendicular to the plane.

**DEPENDENT VARIABLES**
The dependent variables (field variables) are for the **Electric field vector \( E \)** and for the **Magnetic field vector \( H \)**. The name can be changed but the names of fields and dependent variables must be unique within a model.

**DISCRETIZATION**
To display this section, click the **Show** button ( ) and select **Discretization**.

---

**Domain, Boundary, and Pair Nodes for the Electromagnetic Waves, Time Explicit Interface**

The Electromagnetic Waves, Time Explicit Interface has these domain and boundary nodes, listed in alphabetical order, available from the **Physics** ribbon toolbar (Windows users), **Physics** context menu (Mac or Linux users), or right-click to access the context menu (all users).

- **Domain, Boundary, and Pair Nodes for the Electromagnetic Waves, Time Explicit Interface**
- **Theory for the Electromagnetic Waves, Time Explicit Interface**

**In general, to add a node, go to the Physics toolbar, no matter what operating system you are using. Subnodes are available by clicking the parent node and selecting it from the Attributes menu.**
Wave Equations

The Wave Equations node is the main node for the Electromagnetic Waves, Time Explicit interface. The governing transient equations can be written in the form

\[
\nabla \times \mathbf{H} = \sigma \mathbf{E} + \frac{\partial \mathbf{D}}{\partial t}
\]

\[
\nabla \times \mathbf{E} = -\frac{\partial \mathbf{B}}{\partial t}
\]

with the constitutive relations \( \mathbf{B} = \mu_0 \mu_r \mathbf{H} \) and \( \mathbf{D} = \varepsilon_0 \varepsilon_r \mathbf{E} \), which reads

\[
\varepsilon_0 \varepsilon_r \frac{\partial \mathbf{E}}{\partial t} - \nabla \times \mathbf{H} + \sigma \mathbf{E} = 0
\]

\[
\mu_0 \mu_r \frac{\partial \mathbf{H}}{\partial t} + \nabla \times \mathbf{E} = 0
\]

**Material Properties**

The default **Relative permittivity** \( \varepsilon_r \) (dimensionless), **Relative permeability** \( \mu_r \) (dimensionless), and **Electrical conductivity** \( \sigma \) (SI unit: S/m) take values **From material.**
For **User defined** select **Isotropic**, **Diagonal**, **Symmetric**, or **Anisotropic** and enter values or expressions in the field or matrix.

**NUMERICAL PARAMETERS**

The defaults for each parameter are as follows:

- **Lax-Friedrichs flux parameter for E field** $\tau_E$ (SI unit: S), the default is $0.5/Z$ for Ampere’s law.
- **Lax-Friedrichs flux parameter for H field** $\tau_H$ (SI unit: $\Omega$), the default is $0.5Z$ for Faraday’s law, where $Z$ is the impedance of vacuum.
- **Estimate of maximum wave speed** $c_{\text{max}}$ (SI unit: m/s) the default is taken from the speed of light in a vacuum $c_{\text{const}}$.

**FILTER PARAMETERS**

The filter provides higher-order smoothing of nodal discontinuous Galerkin formulations and is intended to be used for absorbing layers, but you can also use it to stabilize linear wave problems with highly varying coefficients. The filter is constructed by transforming the solution (in each global time step) to an orthogonal polynomial representation, multiplying with a damping factor and then transforming back to the (Lagrange) nodal basis. Select the **Activate** check box to use this filter.

The exponential filter can be described by the matrix formula

$$VAV^{-1}$$

where $V$ is a Vandermonde matrix induced by the node points, and $A$ is a diagonal matrix with the exponential damping factors on the diagonal:

$$\Lambda_{mm} = \sigma(\eta) = \begin{cases} 
1, & 0 \leq \eta \leq \eta_c \\
\exp\left(-\alpha \frac{(\eta - \eta_c)^m}{1 - \eta_c}\right), & \eta_c \leq \eta \leq 1 
\end{cases}$$

where

$$\eta = \eta(m) = \frac{i_m}{N_p}$$

and $N_p$ is the basis function and $i_m$ the polynomial order for coefficient $m$. $\alpha$ (default value: 36), $\eta_c$ (default value: 1), and $s$ (default value: 3) are the filter parameters that
you specify in the corresponding text fields. The damping is derived from an a spatial
dissipation operator of order 2s. For s = 1, you obtain a damping that is related to the
classical 2nd-order Laplacian. Higher order (larger s) gives less damping for the
lower-order polynomial coefficients (a more pronounced low-pass filter), while
keeping the damping property for the highest values of η, which is controlled by α.
The default values 36 for α correspond to maximal damping for η = 1. It is important
to realize that the effect of the filter is influenced by how much of the solution (energy)
is represented by the higher-order polynomial coefficients. For a well resolved solution
this is a smaller part than for a poorly resolved solution. The effect is stronger for
poorly resolved solutions than for well resolved ones. This is one of the reasons why
this filter is useful in an absorbing layer where the energy is transferred to the
higher-order coefficients through a coordinate transformation. See Ref. 1 (Chapter 5)
for more information.

α must be positive; α = 0 means no dissipation, and the maximum value is related to
the machine precision, −log(ε), which is approximately 36. ηc should be between 0 and
1, where ηc = 0 means maximum filtering, and ηc = 1 means no filtering, even if
filtering is active.

When using Absorbing Layer features, add an additional Wave Equations feature for the
 corresponding domain selection. Select the Activate check box and add filter
parameters. An example of a filter parameter combination that can be used for a Wave
Equations feature active on an Absorbing Layer domain selection is α = 0.1, ηc = 0.01,
and s = 2. However, other combinations could work better, depending on the
particular application.

Reference

Initial Values

The Initial Values node adds the initial values for the Electric field and Magnetic field
variables that serve as an initial condition for the transient simulation.
**DOMAIN SELECTION**

If there is more than one type of domain, each with different initial values defined, it might be necessary to remove these domains from the selection. These are then defined in an additional **Initial Values** node.

**INITIAL VALUES**

Enter values or expressions for the initial values of the components of the **Electric field** \( \mathbf{E} \) (SI unit: V/m) and **Magnetic field** \( \mathbf{H} \) (SI unit: A/m). The default values are 0 for all vector components.

**Electric Current Density**

The **Electric Current Density** node adds an external current density to the specified domains, which appears on the right-hand side of Ampere’s law

\[
\varepsilon_0 \varepsilon_r \frac{\partial \mathbf{E}}{\partial t} - \nabla \times \mathbf{H} + \sigma \mathbf{E} = -\mathbf{J}_e
\]

**MAGNETIC CURRENT DENSITY**

Based on space dimension, enter the coordinates \((x, y, z)\) for 3D components for example) of the **Magnetic current density** \( \mathbf{J}_m \) (SI unit: A/m²).

**Magnetic Current Density**

The **Magnetic Current Density** node adds an external current density to the specified domains, which appears on the right-hand side of Faraday’s law

\[
\mu_0 \mu_r \frac{\partial \mathbf{H}}{\partial t} + \nabla \times \mathbf{E} = -\mathbf{J}_m
\]

**Electric Field**

The **Electric Field** boundary condition

\[
\mathbf{n} \times \mathbf{E} = \mathbf{n} \times \mathbf{E}_0
\]
specifies the tangential component of the electric field. The commonly used special case of zero tangential electric field (perfect electric conductor) is described in the next section.

**ELECTRIC FIELD**

Enter values or expressions for the components of the Electric field $\mathbf{E}_0$ (SI unit: V/m).

**Perfect Electric Conductor**

The Perfect Electric Conductor boundary condition

$$\mathbf{n} \times \mathbf{E} = 0$$

is a special case of the electric field boundary condition that sets the tangential component of the electric field to zero. It is used for the modeling of a lossless metallic surface, e.g., a ground plane or as a symmetry type boundary condition.

It imposes symmetry for magnetic fields and antisymmetry for electric fields and electric currents. It supports induced electric surface currents and thus any prescribed or induced electric currents (volume, surface or edge currents) flowing into a perfect electric conductor boundary is automatically balanced by induced surface currents.

**Magnetic Field**

The Magnetic Field node adds a boundary condition for specifying the tangential component of the magnetic field at the boundary:

$$\mathbf{n} \times \mathbf{H} = \mathbf{n} \times \mathbf{H}_0$$

**MAGNETIC FIELD**

Enter values or expressions for the components of the Magnetic field $\mathbf{H}_0$ (SI unit: A/m).

**Perfect Magnetic Conductor**

The Perfect Magnetic Conductor boundary condition

$$\mathbf{n} \times \mathbf{H} = 0$$

is a special case of the surface current density boundary condition that sets the tangential component of the magnetic field and thus also the surface current density.
to zero. On external boundaries, this can be interpreted as a “high surface impedance”
boundary condition or used as a symmetry type boundary condition. It imposes
symmetry for electric fields and electric currents. Electric currents (volume, surface, or
edge currents) are not allowed to flow into a perfect magnetic conductor boundary as
that would violate current conservation. On interior boundaries, the perfect magnetic
conductor boundary condition literally sets the tangential magnetic field to zero which
in addition to setting the surface current density to zero also makes the tangential
electric field discontinuous.

Surface Current Density

The Surface Current Density boundary condition

\[-\mathbf{n} \times \mathbf{H} = \mathbf{J}_s\]

\[\mathbf{n} \times (\mathbf{H}_1 - \mathbf{H}_2) = \mathbf{J}_s\]

specifies a surface current density at both exterior and interior boundaries. The current
density is specified as a three-dimensional vector, but because it needs to flow along
the boundary surface, COMSOL Multiphysics projects it onto the boundary surface
and neglects its normal component. This makes it easier to specify the current density
and avoids unexpected results when a current density with a component normal to the
surface is given.

**Surface Current Density**

Enter values or expressions for the components of the Surface current density \( \mathbf{J}_{s0} \)
(SI unit: A/m). The defaults are 0 A/m for all vector components.

Low-Reflecting Boundary

The Low-Reflecting Boundary condition

\[\mathbf{n} \times \mathbf{E} = Z_0 \mathbf{H}\]

specifies the tangential component of both electric and magnetic fields.

**Impedance**

Enter the value or expression for the medium Impedance \( Z_0 \) (SI unit: \( \Omega \)). By default,
the \( Z_0 \) uses the value of the vacuum’s impedance. Then select Isotropic, Diagonal,
Symmetric, or Anisotropic based on the material characteristics and enter values or
expressions in the field or matrix.
Flux/Source

The Flux/Source boundary condition

\[ \mathbf{n} \times \mathbf{E} = \mathbf{E}_0 \]
\[ \mathbf{n} \times \mathbf{H} = \mathbf{H}_0 \]

specifies the tangential component of both electric and magnetic fields.

BOUNDARY FLUX/SOURCE

Enter values or expressions for the components of the tangential Electric field \( \mathbf{E}_0 \) (SI unit: V/m) and the tangential Magnetic field \( \mathbf{H}_0 \) (SI unit: A/m).
Theory for the Electromagnetic Waves Interfaces

The Electromagnetic Waves, Frequency Domain Interface and The Electromagnetic Waves, Transient Interface theory is described in this section:

- Introduction to the Physics Interface Equations
- Frequency Domain Equation
- Time Domain Equation
- Vector Elements
- Eigenfrequency Calculations
- Gaussian Beams as Background Fields
- Effective Material Properties in Porous Media and Mixtures
- Effective Conductivity in Porous Media and Mixtures
- Effective Relative Permittivity in Porous Media and Mixtures
- Effective Relative Permeability in Porous Media and Mixtures
- Archie’s Law Theory

Introduction to the Physics Interface Equations

Formulations for high-frequency waves can be derived from Maxwell-Ampère’s and Faraday’s laws,

\[ \nabla \times \mathbf{H} = \mathbf{J} + \frac{\partial \mathbf{D}}{\partial t} \]

\[ \nabla \times \mathbf{E} = -\frac{\partial \mathbf{B}}{\partial t} \]

Using the constitutive relations for linear materials \( \mathbf{D} = \varepsilon \mathbf{E} \) and \( \mathbf{B} = \mu \mathbf{H} \) as well as a current \( \mathbf{J} = \sigma \mathbf{E} \), these two equations become

\[ \nabla \times \mathbf{H} = \sigma \mathbf{E} + \frac{\partial \mathbf{E}}{\partial t} \]

\[ \nabla \times \mathbf{E} = -\mu \frac{\partial \mathbf{H}}{\partial t} \]
**Frequency Domain Equation**

Writing the fields on a time-harmonic form, assuming a sinusoidal excitation and linear media,

\[
E(x, y, z, t) = E(x, y, z)e^{j\omega t}
\]

\[
H(x, y, z, t) = H(x, y, z)e^{j\omega t}
\]

the two laws can be combined into a time harmonic equation for the electric field, or a similar equation for the magnetic field

\[
\nabla \times (\mu - 1 \nabla \times E) - \omega^2 \varepsilon E = 0
\]

\[
\nabla \times (\varepsilon - 1 \nabla \times H) - \omega^2 \mu H = 0
\]

The first of these, based on the electric field is used in The Electromagnetic Waves, Frequency Domain Interface.

Using the relation \( \varepsilon = n^2 \), where \( n \) is the refractive index, the equation can alternatively be written

\[
\nabla \times (\nabla \times E) - k_0^2 n^2 E = 0
\]

The wave number in vacuum \( k_0 \) is defined by

\[
k_0 = \frac{\omega \sqrt{\mu_0 \varepsilon_0}}{c_0} = \frac{\omega}{c_0}
\]

where \( c_0 \) is the speed of light in vacuum.

When the equation is written using the refractive index, the assumption is that \( \mu = 1 \) and \( \sigma = 0 \) and only the constitutive relations for linear materials are available. When solving for the scattered field the same equations are used but \( E = E_{\text{sc}} + E_i \) and \( E_{\text{sc}} \) is the dependent variable.

**Eigenfrequency Analysis**

When solving the frequency domain equation as an eigenfrequency problem the eigenvalue is the complex eigenfrequency \( \lambda = j\omega + \delta \), where \( \delta \) is the damping of the solution. The Q-factor is given from the eigenvalue by the formula
\[ Q_{fa} = \frac{\omega}{2|\beta|} \]

**Mode Analysis and Boundary Mode Analysis**

In mode analysis and boundary mode analysis COMSOL Multiphysics solves for the propagation constant. The time-harmonic representation is almost the same as for the eigenfrequency analysis, but with a known propagation in the out-of-plane direction

\[ \mathbf{E}(\mathbf{r}, t) = \text{Re}(\mathbf{E}(\mathbf{r}) e^{i(\omega t - \beta z)}) = \text{Re}(\mathbf{E}(\mathbf{r}) e^{i(\omega t - \delta z)}) \]

The spatial parameter, \( \alpha = \delta_z + j\beta = -\lambda \), can have a real part and an imaginary part. The propagation constant is equal to the imaginary part, and the real part, \( \delta_z \), represents the damping along the propagation direction. When solving for all three electric field components the allowed anisotropy of the optionally complex relative permittivity and relative permeability is limited to:

\[
\begin{bmatrix}
\varepsilon_{xx} & \varepsilon_{xy} & 0 \\
\varepsilon_{yx} & \varepsilon_{yy} & 0 \\
0 & 0 & \varepsilon_{zz}
\end{bmatrix}
\]

\[
\begin{bmatrix}
\mu_{xx} & \mu_{xy} & 0 \\
\mu_{yx} & \mu_{yy} & 0 \\
0 & 0 & \mu_{zz}
\end{bmatrix}
\]

Limiting the electric field component solved for to the out-of-plane component for TE modes requires that the medium is homogeneous; that is, \( \mu \) and \( \varepsilon \) are constant. When solving for the in-plane electric field components for TM modes, \( \mu \) can vary but \( \varepsilon \) must be constant. It is strongly recommended to use the most general approach, that is solving for all three components which is sometimes referred to as “perpendicular hybrid-mode waves”.

### Variables Influenced by Mode Analysis

The following table lists the variables that are influenced by the mode analysis:

<table>
<thead>
<tr>
<th>NAME</th>
<th>EXPRESSION</th>
<th>CAN BE COMPLEX</th>
<th>DESCRIPTION</th>
</tr>
</thead>
<tbody>
<tr>
<td>beta</td>
<td>\text{imag}(-\lambda)</td>
<td>No</td>
<td>Propagation constant</td>
</tr>
<tr>
<td>dampz</td>
<td>\text{real}(-\lambda)</td>
<td>No</td>
<td>Attenuation constant</td>
</tr>
<tr>
<td>dampzdB</td>
<td>\text{20*log10(exp(1))*dampz}</td>
<td>No</td>
<td>Attenuation per meter in dB</td>
</tr>
<tr>
<td>neff</td>
<td>\text{j*lambda/k0}</td>
<td>Yes</td>
<td>Effective mode index</td>
</tr>
</tbody>
</table>
PROPAGATING WAVES IN 2D

In 2D, different polarizations can be chosen by selecting to solve for a subset of the 3D vector components. When selecting all three components, the 3D equation applies with the addition that out-of-plane spatial derivatives are evaluated for the prescribed out-of-plane wave vector dependence of the electric field.

In 2D, the electric field varies with the out-of-plane wave number $k_z$ as

$$E(x, y, z) = \tilde{E}(x, y)\exp(-ik_z z).$$

The wave equation is thereby rewritten as

$$(\nabla - ik_z \hat{z}) \times [\mu_r^{-1}(\nabla - ik_z \hat{z}) \times \tilde{E}] - k_0^2\varepsilon_{re}\tilde{E} = 0,$$

where $\hat{z}$ is the unit vector in the out-of-plane $z$-direction.

Similarly, in 2D axisymmetry, the electric field varies with the azimuthal mode number $m$ as

$$E(r, \varphi, z) = \tilde{E}(r, z)\exp(-im\varphi)$$

and the wave equation is expressed as

$$(\nabla - i\frac{m}{r}\hat{\varphi}) \times [\mu_r^{-1}(\nabla - i\frac{m}{r}\hat{\varphi}) \times \tilde{E}] - k_0^2\varepsilon_{re}\tilde{E} = 0,$$

where $\hat{\varphi}$ is the unit vector in the out-of-plane $\varphi$-direction.

In-plane Hybrid-Mode Waves

Solving for all three components in 2D is referred to as “hybrid-mode waves”. The equation is formally the same as in 3D with the addition that out-of-plane spatial derivatives are evaluated for the prescribed out-of-plane wave vector dependence of the electric field.

In-plane TM Waves

The TM waves polarization has only one magnetic field component in the $z$ direction, and the electric field lies in the modeling plane. Thus the time-harmonic fields can be obtained by solving for the in-plane electric field components only. The equation is formally the same as in 3D, the only difference being that the out-of-plane electric field component is zero everywhere and that out-of-plane spatial derivatives are evaluated for the prescribed out-of-plane wave vector dependence of the electric field.
In-plane TE Waves

As the field propagates in the modeling $xy$-plane a TE wave has only one non zero electric field component, namely in the $z$ direction. The magnetic field lies in the modeling plane. Thus the time-harmonic fields can be simplified to a scalar equation for $E_z$,

$$-\nabla \cdot (\hat{\mu}_r \nabla E_z) - \epsilon_{zzz} \epsilon_0 \frac{\partial^2 E_z}{\partial t^2} = 0$$

where

$$\hat{\mu}_r = \frac{\mu_r^T}{\det(\mu_r)}$$

To be able to write the fields in this form, it is also required that $\epsilon_r$, $\sigma$, and $\mu_r$ are non diagonal only in the $xy$-plane. $\mu_r$ denotes a 2-by-2 tensor, and $\epsilon_{zzz}$ and $\sigma_{zz}$ are the relative permittivity and conductivity in the $z$ direction.

Axisymmetric Hybrid-Mode Waves

Solving for all three components in 2D is referred to as “hybrid-mode waves”. The equation is formally the same as in 3D with the addition that spatial derivatives with respect to $\phi$ are evaluated for the prescribed azimuthal mode number dependence of the electric field.

Axisymmetric TM Waves

A TM wave has a magnetic field with only a $\phi$ component and thus an electric field with components in the $rz$-plane only. The equation is formally the same as in 3D, the only difference being that the $\phi$ component is zero everywhere and that spatial derivatives with respect to $\phi$ are evaluated for the prescribed azimuthal mode number dependence of the electric field.

Axisymmetric TE Waves

A TE wave has only an electric field component in the $\phi$ direction, and the magnetic field lies in the modeling plane. Given these constraints, the 3D equation can be simplified to a scalar equation for $E_\phi$. To write the fields in this form, it is also required that $\epsilon_r$ and $\mu_r$ are non diagonal only in the $rz$-plane. $\mu_r$ denotes a 2-by-2 tensor, and $\epsilon_{\phi\phi}$ and $\sigma_{\phi\phi}$ are the relative permittivity and conductivity in the $\phi$ direction.
INTRODUCING LOSSES IN THE FREQUENCY DOMAIN

Electric Losses
The frequency domain equations allow for several ways of introducing electric losses. Finite conductivity results in a complex permittivity,

\[ \varepsilon_c = \varepsilon - j\frac{\sigma}{\omega \varepsilon_0} \]

The conductivity gives rise to ohmic losses in the medium.

A more general approach is to use a complex permittivity,

\[ \varepsilon_c = \varepsilon_0(\varepsilon' - j\varepsilon'') \]

where \( \varepsilon' \) is the real part of \( \varepsilon_r \), and all losses are given by \( \varepsilon'' \). This dielectric loss model can be combined with a finite conductivity resulting in:

\[ \varepsilon_c = \varepsilon_0\left(\varepsilon' - j\frac{\sigma}{\omega \varepsilon_0} + \varepsilon''\right) \]

The complex permittivity can also be introduced as a loss tangent:

\[ \varepsilon_c = \varepsilon_0\varepsilon'(1 - j\tan\delta) \]

When specifying losses through a loss tangent, conductivity is not allowed as an input.

In optics and photonics applications, the refractive index is often used instead of the permittivity. In materials where \( \mu_r \) is 1, the relation between the complex refractive index

\[ \bar{n} = n - j\kappa \]

and the complex relative permittivity is

\[ \varepsilon_{rc} = \bar{n}^2 \]

that is

\[ \varepsilon'_r = n^2 - \kappa^2 \]
\[ \varepsilon''_r = 2n\kappa \]
The inverse relations are

\[ n^2 = \frac{1}{2}(\varepsilon'_r + \sqrt{\varepsilon'^2_r + \varepsilon''^2_r}) \]
\[ \kappa^2 = \frac{1}{2}(-\varepsilon'_r + \sqrt{\varepsilon'^2_r + \varepsilon''^2_r}) \]

The parameter \( \kappa \) represents a damping of the electromagnetic wave. When specifying the refractive index, conductivity is not allowed as an input.

In the physics and optics literature, the time harmonic form is often written with a minus sign (and \( "i" \) instead of \( "j" \)):

\[ E(x, y, z, t) = E(x, y, z)e^{-i\omega t} \]

This makes an important difference in how loss is represented by complex material coefficients like permittivity and refractive index, that is, by having a positive imaginary part rather than a negative one. Therefore, material data taken from the literature might have to be conjugated before using it in a model.

**Magnetic Losses**

The frequency domain equations allow for magnetic losses to be introduced as a complex relative permeability.

\[ \mu_r = (\mu' - j\mu'') \]

The complex relative permeability can be combined with any electric loss model except refractive index.

**Time Domain Equation**

The relations \( \mu \mathbf{H} = \nabla \times \mathbf{A} \) and \( \mathbf{E} = -\partial \mathbf{A}/\partial t \) make it possible to rewrite Maxwell-Ampère’s law using the magnetic potential.

\[ \mu_0 \frac{\partial \mathbf{A}}{\partial t} + \mu_0 \frac{\partial}{\partial t} \frac{\partial \mathbf{A}}{\partial t} + \nabla \times (\mu_r^{-1} \nabla \times \mathbf{A}) = 0 \]

This is the equation used by *The Electromagnetic Waves, Transient Interface*. It is suitable for the simulation of non-sinusoidal waveforms or non linear media.

Using the relation \( \varepsilon_r = n^2 \), where \( n \) is the refractive index, the equations can alternatively be written.
WAVES IN 2D

In 2D, different polarizations can be chosen by selecting to solve for a subset of the 3D vector components. When selecting all three components, the 3D equation applies with the addition that out-of-plane spatial derivatives are set to zero.

**In-plane Hybrid-Mode Waves**

Solving for all three components in 2D is referred to as “hybrid-mode waves”. The equation form is formally the same as in 3D with the addition that out-of-plane spatial derivatives are set to zero.

**In-plane TM Waves**

The TM waves polarization has only one magnetic field component in the $z$ direction, and thus the electric field and vector potential lie in the modeling plane. Hence it is obtained by solving only for the in-plane vector potential components. The equation is formally the same as in 3D, the only difference being that the out-of-plane vector potential component is zero everywhere and that out-of-plane spatial derivatives are set to zero.

**In-plane TE Waves**

As the field propagates in the modeling $xy$-plane a TE wave has only one non zero vector potential component, namely in the $z$ direction. The magnetic field lies in the modeling plane. Thus the equation in the time domain can be simplified to a scalar equation for $A_z$:

$$\mu_0 \varepsilon_0 \frac{\partial}{\partial t} \left( n^2 \frac{\partial A_z}{\partial t} \right) + \nabla \times (\nabla \times A_z) = 0$$

Using the relation $\varepsilon_r = n^2$, where $n$ is the refractive index, the equation can alternatively be written

$$\mu_0 \varepsilon_0 \frac{\partial}{\partial t} \left( n^2 \frac{\partial A_z}{\partial t} \right) + \nabla \cdot (\nabla A_z) = 0$$

When using the refractive index, the assumption is that $\mu_r = 1$ and $\sigma = 0$ and only the constitutive relations for linear materials can be used.
Chapter 4: Radio Frequency Interfaces

**Axisymmetric Hybrid-Mode Waves**
Solving for all three components in 2D is referred to as “hybrid-mode waves”. The equation form is formally the same as in 3D with the addition that spatial derivatives with respect to $\phi$ are set to zero.

**Axisymmetric TM Waves**
TM waves have a magnetic field with only a $\phi$ component and thus an electric field and a magnetic vector potential with components in the $rz$-plane only. The equation is formally the same as in 3D, the only difference being that the $\phi$ component is zero everywhere and that spatial derivatives with respect to $\phi$ are set to zero.

**Axisymmetric TE Waves**
A TE wave has only a vector potential component in the $\phi$ direction, and the magnetic field lies in the modeling plane. Given these constraints, the 3D equation can be simplified to a scalar equation for $A_\phi$. To write the fields in this form, it is also required that $\varepsilon_r$ and $\mu_r$ are non diagonal only in the $rz$-plane. $\mu_r$ denotes a 2-by-2 tensor, and $\varepsilon_r\phi$ and $\sigma_r\phi$ are the relative permittivity and conductivity in the $\phi$ direction.

**Vector Elements**
Whenever solving for more than a single vector component, it is not possible to use Lagrange elements for electromagnetic wave modeling. The reason is that they force the fields to be continuous everywhere. This implies that the physics interface conditions, which specify that the normal components of the electric and magnetic fields are discontinuous across interior boundaries between media with different permittivity and permeability, cannot be fulfilled. To overcome this problem, the Electromagnetic Waves, Frequency Domain interface uses *vector elements*, which do not have this limitation.

The solution obtained when using vector elements also better fulfills the divergence conditions $\nabla \cdot \mathbf{D} = 0$ and $\nabla \cdot \mathbf{B} = 0$ than when using Lagrange elements.
Eigenfrequency Calculations

When making eigenfrequency calculations, there are a few important things to note:

- Nonlinear eigenvalue problems appear for impedance boundary conditions with nonzero conductivity and for scattering boundary conditions adjacent to domains with nonzero conductivity. Such problems have to be treated specially.

- Some of the boundary conditions, such as the surface current density condition and the electric field condition, can specify a source in the eigenvalue problem. These conditions are available as a general tool to specify arbitrary expressions between the $\mathbf{H}$ field and the $\mathbf{E}$ field. Avoid specifying solution-independent sources for these conditions because the eigenvalue solver ignores them anyway.

Using the default parameters for the eigenfrequency study, it might find a large number of false eigenfrequencies, which are almost zero. This is a known consequence of using vector elements. To avoid these eigenfrequencies, change the parameters for the eigenvalue solver in the Study Settings. Adjust the settings so that the solver searches for eigenfrequencies closer to the lowest eigenfrequency than to zero.

Gaussian Beams as Background Fields

When solving for the scattered field, the background wave type can be set to a predefined Gaussian beam from within the Settings of The Electromagnetic Waves, Frequency Domain Interface. The background field for a Gaussian beam propagating along the $z$-axis is defined below,

$$
\mathbf{E}_{bg}(x, y, z) = \mathbf{E}_{bg0}(x, y) \frac{w_0}{w(z)} \exp \left[ -\frac{\rho^2}{w^2(z)} - jkz - jk\frac{\rho^2}{2R(z)} + j\eta(z) \right],
$$

where $w_0$ is the beam radius, $p_0$ is the focal plane on the $z$-axis, $\mathbf{E}_{bg0}$ is the background electric field amplitude and the spot radius for different positions along the propagation axis is given by

$$
w(z) = w_0 \sqrt{1 + \left( \frac{z - p_0}{z_0} \right)^2}.
$$

$$
R(z) = (z - p_0) \left[ 1 + \left( \frac{z_0}{z - p_0} \right)^2 \right].
$$
defines the radius of curvature for the phase of the field and the so called Gouy phase
shift is given by

$$\eta(z) = \text{atan} \left( \frac{z - p_0}{z_0} \right).$$

The equations above are expressed using the Rayleigh range $z_0$ and the transverse
coordinate $\rho$, defined by

$$z_0 = \frac{k_0 w_0^2}{2}, \quad \rho^2 = x^2 + y^2.$$

Note that the time-harmonic ansatz in COMSOL is $e^{\text{j} \omega t}$ and with this convention, the
beam above propagates in the $+z$-direction. The equations are modified accordingly
for beams propagating along the other coordinate axes.

The background field for a Gaussian beam is defined in a similar way for 2D
components. In the particular case where the beam propagates along the $x$-axis, the
background field is defined as

$$E_b(x, y, z) = E_{bg0} \frac{w_0}{w(x)} \exp \left[ -\frac{2}{w^2(x)} \left( -j k x - jk \frac{y^2}{2R(x)} + j \eta(x) \right) \right].$$

For a beam propagating along the $y$-axis, the coordinates $x$ and $y$ are interchanged.

Notice that the expressions above for Gaussian beams are not solutions to the
Helmholtz equation, but to the so called paraxial approximation of the Helmholtz
equation. This means that these equations become less accurate the smaller the spot
radius is and should not be used when the spot radius is of the same size as or smaller
than the wavelength.

**Effective Material Properties in Porous Media and Mixtures**

One way of dealing with porous media or mixtures of solids in electromagnetic models
is to replace them with an homogenized medium. The electric and magnetic properties
of this medium are computed from the properties of each phase by means of an
averaging formula.

There are several possible approaches to compute an average material property starting
from the material properties and the volume fraction of each material.
The following sections illustrate the different formulas available to compute the effective electrical conductivity, the effective relative permittivity and the effective relative permeability of a homogenized medium. In the following, volume fractions of the materials are indicated with $\theta_i$, where $i$ is the material index, and they are assumed to be fractional (between 0 and 1). Up to five different materials can be specified as phases of the mixture. Typically, their volume fractions should add up to 1.

**Effective Conductivity in Porous Media and Mixtures**

Three methods are available to compute the averaged electrical conductivity of the mixture.

**VOLUME AVERAGE, CONDUCTIVITY**

If the electric conductivities of the two materials are not so different from each other, a simple form of averaging can be used, such as a volume average:

$$\sigma = \sum_{i=1}^{n} \theta_i \sigma_i = \theta_1 \sigma_1 + \theta_2 \sigma_2 + \ldots$$

where $\sigma_i$ is the conductivity of the material $i$. This is equivalent to a “parallel” system of resistivities.

If the conductivities are defined by second order tensors (such as for anisotropic materials), the volume average is applied element by element.

**VOLUME AVERAGE, RESISTIVITY**

A similar expression for the effective conductivity can be used, which mimics a “series” connection of resistivities. Equivalently, the effective conductivity is obtained from

$$\frac{1}{\sigma} = \sum_{i=0}^{n} \frac{\theta_i}{\sigma_i} = \frac{\theta_1}{\sigma_1} + \frac{\theta_2}{\sigma_2} + \ldots$$

If the conductivities are defined by second order tensors, the inverse of the tensors are used.

**POWER LAW**

A power law gives the following expression for the equivalent conductivity:
The effective conductivity calculated by Volume Average, Conductivity is the upper bound, the effective conductivity calculated by Volume Average, Resistivity is the lower bound, and the Power Law average is somewhere in between these two.

**Effective Relative Permittivity in Porous Media and Mixtures**

Three methods are available to compute the averaged electrical conductivity of the mixture.

**Volume Average, Permittivity**

If the relative permittivity of the two materials is not so different from each other, the effective relative permittivity \( \varepsilon_r \) is calculated by simple volume average:

\[
\varepsilon = \sum_{i=1}^{n} \theta_i \varepsilon_i = \theta_1 \varepsilon_1 + \theta_2 \varepsilon_2 + \ldots
\]

where \( \varepsilon_i \) is the relative permeability of the material \( i \).

If the permittivity is defined by second-order tensors (such as for anisotropic materials), the volume average is applied element by element.

**Volume Average, Reciprocal Permittivity**

The second method is the volume average of the inverse of the permittivities:

\[
\frac{1}{\varepsilon} = \sum_{i=0}^{n} \frac{\theta_i}{\varepsilon_i} = \frac{\theta_1}{\varepsilon_1} + \frac{\theta_2}{\varepsilon_2} + \ldots
\]

If the permittivity is defined by a second-order tensor, the inverse of the tensor is used.

**Power Law**

A power law gives the following expression for the equivalent permittivity:

\[
\varepsilon = \prod_{i=0}^{n} \varepsilon_i^{\theta_i} = \varepsilon_1^{\theta_1} \varepsilon_2^{\theta_2} \ldots
\]
The effective permeability calculated by Volume Average, Permeability is the upper bound, the effective permeability calculated by Volume Average, Reciprocal Permeability is the lower bound, and the Power Law average gives a value somewhere in between these two.

**Effective Relative Permeability in Porous Media and Mixtures**

Three methods are available to compute the averaged electrical conductivity of the mixture.

**Volume Average, Permeability**

If the relative permeability of the two materials is not so different from each other, the effective relative permeability $\mu_r$ is calculated by simple volume average:

$$\mu = \sum_{i = 1}^{n} \theta_i \mu_i = \theta_1 \mu_1 + \theta_2 \mu_2 + \ldots$$

where $\mu_i$ is the relative permeability of the material $i$.

If the permeability is defined by second-order tensors (such as for anisotropic materials), the volume average is applied element by element.

**Volume Average, Reciprocal Permeability**

The second method is the volume average of the inverse of the permeabilities:

$$\frac{1}{\mu_i} \sum_{i = 0}^{n} \theta_i \frac{1}{\mu_i} = \frac{\theta_1}{\mu_1} + \frac{\theta_2}{\mu_2} + \ldots$$

If the permeability is defined by a second-order tensor, the inverse of the tensor is used.

**Power Law**

A power law gives the following expression for the equivalent permeability:

$$\mu = \prod_{i = 0}^{n} \mu_i^{\theta_i} = \mu_1^{\theta_1} \mu_2^{\theta_2} \ldots$$

The effective permeability calculated by Volume Average, Permeability is the upper bound, the effective permeability calculated by Volume Average, Reciprocal Permeability is the lower bound, and the Power Law average gives a value somewhere in between these two.
Permeability is the lower bound, and the Power Law average gives a value somewhere in between these two.

Archie’s Law Theory

The electrical conductivity of the materials composing saturated rocks and soils can vary over many orders of magnitude. For instance, in the petroleum reservoirs, normal sea water (or brine) has a typical conductivity of around 3 S/m, whereas hydrocarbons are typically much more resistive and have conductivities in the range 0.1–0.01 S/m.

The porous rocks and sediments can have even lower conductivities. In variably saturated soils, the conductivity of air is roughly ten orders of magnitude lower than the ground water. A simple volume average (of either conductivity or resistivity) in rocks or soils might give different results compared to experimental data.

Since most crustal rocks, sedimentary rocks, and soils are formed by nonconducting materials, Archie (Ref. 2) assumed that electric current are mainly caused by ion fluxes through the pore network. Originally, Archie’s law is an empirical law for the effective conductivity of a fully saturated rock or soil, but it can be extended to variably saturated porous media.

Archie’s law relates the effective conductivity to the fluid conductivity $\sigma_L$, fluid saturation $s_L$, and porosity $\varepsilon_p$:

$$\sigma = s_L^{n} \varepsilon_p^{m} \sigma_L$$

here, $m$ is the cementation exponent, a parameter that describes the connectivity of the pores. The cementation exponent normally varies between 1.3 and 2.5 for most sedimentary rocks and is close to 2 for sandstones. The lower limit $m = 1$ represents a volume average of the conductivities of a fully saturated, insulating (zero conductivity) porous matrix, and a conducting fluid. The saturation coefficient $n$ is normally close to 2. The ratio $F = \sigma_L/\sigma$ is called the formation factor.

Archie’s law does not take care of the relative permittivity of either fluids or solids, so the effective relative permittivity of the porous medium is normally consider as $\varepsilon_r = 1$.

Reference for Archie’s Law

Theory for the Transmission Line Interface

The Transmission Line Interface theory is described in this section.

- Introduction to Transmission Line Theory
- Theory for the Transmission Line Boundary Conditions

Introduction to Transmission Line Theory

Figure 4-5 is an illustration of a transmission line of length \( L \). The distributed resistance \( R \), inductance \( L \), conductance \( G \), and capacitance \( C \), characterize the properties of the transmission line.

![Figure 4-5: Schematic of a transmission line with a load impedance.](image)

The distribution of the electric potential \( V \) and the current \( I \) describes the propagation of the signal wave along the line. The following equations relate the current and the electric potential

\[
\frac{\partial V}{\partial x} = -(R + j\omega L)I \quad (4-1)
\]

\[
\frac{\partial I}{\partial x} = -(G + j\omega C)V \quad (4-2)
\]

Equation 4-1 and Equation 4-2 can be combined to the second-order partial differential equation
\[ \frac{\partial^2 V}{\partial x^2} = \gamma^2 V \]  

(4-3)

where

\[ \gamma = \sqrt{(R + j\omega L)(G + j\omega C)} = \alpha + j\beta \]

Here \( \gamma \), \( \alpha \), and \( \beta \) are called the complex propagation constant, the attenuation constant, and the (real) propagation constant, respectively.

The solution to Equation 4-3 represents a forward- and a backward-propagating wave

\[ V(x) = V_+ e^{-\gamma x} + V_- e^{\gamma x} \]  

(4-4)

By inserting Equation 4-4 in Equation 4-1 you get the current distribution

\[ I(x) = \frac{\gamma}{R + j\omega L}(V_+ e^{-\gamma x} - V_- e^{\gamma x}) \]

If only a forward-propagating wave is present in the transmission line (no reflections), dividing the voltage by the current gives the characteristic impedance of the transmission line

\[ Z_0 = \frac{V}{I} = \frac{R + j\omega L}{\gamma} = \sqrt{\frac{R + j\omega L}{G + j\omega C}} \]

To make sure that the current is conserved across internal boundaries, COMSOL Multiphysics solves the following wave equation (instead of Equation 4-3)

\[ \frac{\partial}{\partial x} \left( \frac{1}{R + j\omega L} \frac{\partial V}{\partial x} \right) - (G + j\omega C)V = 0 \]  

(4-5)

**Theory for the Transmission Line Boundary Conditions**

The Transmission Line Interface has these boundary conditions:

\[ V_1 = V_2 \]  

(4-6)
and

\[ I_1 = I_2 \]  \hspace{1cm} (4-7)

In Equation 4-6 and Equation 4-7, the indices 1 and 2 denote the domains on the two sides of the boundary. The currents flowing out of a boundary are given by

\[ I_i = -\frac{n_i \cdot \nabla V_i}{R_i + j\omega L_i}, \quad i = 1, 2 \]

where \( n_i \) are the normals pointing out of the domain.

Because \( V \) is solved for, the electric potential is always continuous, and thus Equation 4-6 is automatically fulfilled. Equation 4-7 is equivalent to the natural boundary condition

\[ \left. \frac{1}{R_2 + j\omega L_2} \frac{\partial V}{\partial x} \right|_2 - \left. \frac{1}{R_1 + j\omega L_1} \frac{\partial V}{\partial x} \right|_1 = 0 \]

which is fulfilled with the wave-equation formulation in Equation 4-5.

When the transmission line is terminated by a load impedance, as Figure 4-5 shows, the current through the load impedance is given by

\[ I(L) = \frac{V(L)}{Z_L} \]  \hspace{1cm} (4-8)

Inserting Equation 4-1 into Equation 4-8, results in the Terminating Impedance boundary condition

\[ \frac{1}{R + j\omega L} \frac{\partial V}{\partial x} + \frac{V}{Z_L} = 0 \]  \hspace{1cm} (4-9)

If the arbitrary load impedance \( Z_L \) is replaced by the characteristic impedance of the transmission line \( Z_0 \), you get the Absorbing Boundary condition. By inserting the voltage, defined in Equation 4-4, in Equation 4-9 you can verify that the boundary condition does not allow any reflected wave (that is, \( V \) is zero).

The Open Circuit boundary condition is obtained by letting the load impedance become infinitely large, that is, no current flows through the load impedance.

On the other hand, the Short Circuit boundary condition specifies that the voltage at the load is zero. In COMSOL Multiphysics this is implemented as a constraint on the electric potential.
To excite the transmission line, use the **Incoming Wave** boundary condition. Referring to the left (input) end of the transmission line in Figure 4-5, the forward propagating wave has a voltage amplitude of $V_0$. Thus, the total voltage at this boundary is given by

$$V(0) = V = V_0 + V.$$  

Thereby, the current can be written as

$$I(0) = -\left. \frac{1}{R+j\omega L} \frac{\partial V}{\partial x} \right|_{x=0} = \frac{1}{Z_0} (V_0 - V) = \frac{2V_0 - V}{Z_0}$$

resulting in the boundary condition

$$-\frac{1}{R+j\omega L} \frac{\partial V}{\partial x} + \frac{V - 2V_0}{Z_0} = 0$$

For the **Lumped Port** boundary condition, the port current (positive when entering the transmission line) defines the boundary condition as

$$-\frac{1}{R+j\omega L} \frac{\partial V}{\partial x} I_{\text{port}} = 0$$

where the port current $I_{\text{port}}$ is given by

$$I_{\text{port}} = \frac{2V_0 - V}{Z_0}$$

for a Cable lumped port (see the **Lumped Port** section for a description of the lumped port settings).

For a Current-controlled lumped port, you provide $I_{\text{port}}$ as an input parameter, whereas it is part of an electrical circuit equation for a Circuit-based lumped port.
Theory for the Electromagnetic Waves, Time Explicit Interface

The Electromagnetic Waves, Time Explicit Interface theory is described in this section:

- **The Equations**
- **In-plane E Field or In-plane H Field**
- **Fluxes as Dirichlet Boundary Conditions**
- **Absorbing Layers**

**The Equations**

Maxwell’s equations are a set of equations, written in differential or integral form, stating the relationships between the fundamental electromagnetic quantities. These quantities are the:

- Electric field intensity $\mathbf{E}$
- Electric displacement or electric flux density $\mathbf{D}$
- Magnetic field intensity $\mathbf{H}$
- Magnetic flux density $\mathbf{B}$
- Current density $\mathbf{J}$
- Electric charge density $\rho$

For general time-varying fields, the differential form of Maxwell’s equations can be written as

\[
\nabla \times \mathbf{H} = \mathbf{J} + \frac{\partial \mathbf{D}}{\partial t} \\
\nabla \times \mathbf{E} = -\frac{\partial \mathbf{B}}{\partial t} \\
\n\nabla \cdot \mathbf{D} = \rho \\
\n\nabla \cdot \mathbf{B} = 0
\]

(4-10)

The first two equations are also called Maxwell-Ampere’s law and Faraday’s law, respectively. Equation three and four are two forms of Gauss’ law, the electric and magnetic form, respectively.
CONSTITUTIVE RELATIONS

To obtain a closed system of equations, the constitutive relations describing the macroscopic properties of the medium are included. These are given as

\[ \mathbf{D} = \varepsilon_0 \mathbf{E} + \mathbf{P} \]
\[ \mathbf{B} = \mu_0 (\mathbf{H} + \mathbf{M}) \]
\[ \mathbf{J} = \sigma \mathbf{E} \]

(4-11)

Here \( \varepsilon_0 \) is the permittivity of a vacuum, \( \mu_0 \) is the permeability of a vacuum, and \( \sigma \) the electric conductivity of the medium. In the SI system, the permeability of a vacuum is chosen to be \( 4\pi \times 10^{-7} \) H/m. The velocity of an electromagnetic wave in a vacuum is given as \( c_0 \) and the permittivity of a vacuum is derived from the relation

\[ \varepsilon_0 = \frac{1}{\varepsilon_0 \mu_0} = 8.854 \times 10^{-12} \text{ F/m} = \frac{1}{36\pi} \times 10^{-9} \text{ F/m} \]

The electric polarization vector \( \mathbf{P} \) describes how the material is polarized when an electric field \( \mathbf{E} \) is present. It can be interpreted as the volume density of electric dipole moments. \( \mathbf{P} \) is generally a function of \( \mathbf{E} \). Some materials might have a nonzero \( \mathbf{P} \) also when there is no electric field present.

The magnetization vector \( \mathbf{M} \) similarly describes how the material is magnetized when a magnetic field \( \mathbf{H} \) is present. It can be interpreted as the volume density of magnetic dipole moments. \( \mathbf{M} \) is generally a function of \( \mathbf{H} \). Permanent magnets, for example, have a nonzero \( \mathbf{M} \) also when there is no magnetic field present.

To get a wave equation for the \( \mathbf{E} \) field, for example, take the curl of the second equation in Equation 4-10 (previously divided by \( \mu_0 \)), and insert it into the time derivative of the first row in Equation 4-10

\[ -\nabla \times \left( \frac{1}{\mu_0} \nabla \times \mathbf{E} + \frac{\partial \mathbf{M}}{\partial t} \right) = \sigma \frac{\partial \mathbf{E}}{\partial t} + \varepsilon_0 \frac{\partial^2 \mathbf{E}}{\partial t^2} + \frac{\partial^2 \mathbf{P}}{\partial t^2} \]

this is referred as curl-curl formulation in the literature (second order time derivatives and second order space derivatives).

LINEAR MATERIALS

In the simplest case linear materials, the polarization is directly proportional to the electric field, that is

\[ \frac{\partial \mathbf{P}}{\partial \mathbf{E}} = \varepsilon_0 \chi_e \text{ and } \mathbf{P} = \varepsilon_0 \chi_e \mathbf{E} \]
where $\chi_e$ is the electric susceptibility (which can be a scalar or a second-rank tensor). Similarly, the magnetization is directly proportional to the magnetic field, or
\[
\frac{\partial \mathbf{M}}{\partial \mathbf{H}} = \chi_m \text{ and } \mathbf{M} = \chi_m \mathbf{H}
\]
where $\chi_m$ is the magnetic susceptibility.

As a consequence, for linear materials, the constitutive relations in Equation 4-11 can be written as
\[
\begin{align*}
\mathbf{D} &= \varepsilon_0 \mathbf{E} + \mathbf{P} = \varepsilon_0 (1 + \chi_e) \mathbf{E} = \varepsilon_0 \varepsilon_r \mathbf{E} \\
\mathbf{B} &= \mu_0 \left( \mathbf{H} + \mathbf{M} \right) = \mu_0 (1 + \chi_m) \mathbf{H} = \mu_0 \mu_r \mathbf{H}
\end{align*}
\]

Here, $\varepsilon = \varepsilon_0 \varepsilon_r$ and $\mu = \mu_0 \mu_r$ are the permittivity and permeability of the material. The relative permittivity $\varepsilon_r$ and the relative permeability $\mu_r$ are usually scalar properties but these can be second-rank symmetric (Hermitian) tensors for a general anisotropic material.

For general time-varying fields, Maxwell’s equations in linear materials described in Equation 4-10 can be simplified to Maxwell-Ampere’s law and Faraday’s law:
\[
\nabla \times \mathbf{H} = \sigma \mathbf{E} + \varepsilon_0 \varepsilon_r \frac{\partial \mathbf{E}}{\partial t} \\
\nabla \times \mathbf{E} = -\mu_0 \mu_r \frac{\partial \mathbf{H}}{\partial t}
\]
(4-12)

The electric conductivity $\sigma$ can also be a scalar or a second-rank tensor. Another important assumption is that the relative permittivity $\varepsilon_r$, the relative permeability $\mu_r$ and the electric conductivity $\sigma$ might change with position and orientation (inhomogeneous or anisotropic materials) but not with time.

**First Order Implementation of Maxwell Equations**

In order to accommodate Maxwell’s equations in the coefficients for the Wave Form PDE interface in the form
\[
d^a \frac{\partial \mathbf{u}}{\partial t} + \nabla \cdot \Gamma(\mathbf{u}) = \mathbf{f}
\]
the curl of a vector is written in divergence form as
\[
\n\nabla \times \mathbf{u} = \nabla \times \begin{bmatrix}
0 & u_3 & -u_2 \\
-u_3 & 0 & u_1 \\
u_2 & -u_1 & 0
\end{bmatrix}
\]

(4-13)

where the divergence is applied on each row of the flux \( \Gamma(\mathbf{u}) \).

Maxwell’s equations in 3D

\[
\begin{align*}
\varepsilon_0 \mu_r \frac{\partial \mathbf{E}}{\partial t} + \nabla \times \mathbf{H} & = -\mathbf{\sigma} \mathbf{E} \\
\mu_0 \varepsilon_r \frac{\partial \mathbf{H}}{\partial t} + \nabla \times \mathbf{E} & = 0
\end{align*}
\]

are then accommodated to the Wave Form PDE as

\[
\begin{align*}
\frac{d \mathbf{E}}{dt} + \nabla \cdot \Gamma_E(\mathbf{H}) & = \mathbf{f} \\
\frac{d \mathbf{H}}{dt} + \nabla \cdot \Gamma_H(\mathbf{E}) & = 0
\end{align*}
\]

with the “mass” coefficients

\[d_E = \varepsilon_0 \varepsilon_r \text{ and } d_H = \mu_0 \mu_r\]

the “flux” terms

\[
\Gamma_E(\mathbf{H}) = \begin{bmatrix}
0 & h_3 & -h_2 \\
-h_3 & 0 & h_1 \\
h_2 & -h_1 & 0
\end{bmatrix}
\]

and \( \Gamma_H(\mathbf{E}) = \begin{bmatrix} 0 & e_3 & -e_2 \\
-e_3 & 0 & e_1 \\
e_2 & -e_1 & 0 \end{bmatrix} \)

and the “source” term \( \mathbf{f} = -\mathbf{\sigma} \mathbf{E} \).

**THE LAX-FRIEDRICHS FLUX PARAMETERS**

When using SI units (or other) for the electromagnetic fields and material properties, the Lax-Friedrichs flux parameter is not dimensionless and must have units of \( \tau_E = 1/(2Z) \) for Ampere’s law and \( \tau_H = Z/2 \) for Faraday’s law, where \( Z \) is the impedance of the medium.
In-plane E Field or In-plane H Field

In the general case, in 2D and 2D axisymmetric, solving for three variables for each field is still required. The “in-plane H” or “in-plane E” assumption simplifies the problem to only three dependent variables.

TM WAVES IN 2D

For TM waves in 2D, solve for an in-plane electric field vector and one out-of-plane variable for the magnetic field. Maxwell’s equations then read

\[
\begin{align*}
\varepsilon_0 \varepsilon_r \frac{\partial \mathbf{E}}{\partial t} + \nabla \cdot \mathbf{H}(\mathbf{H}) &= -\sigma \cdot \mathbf{E} \\
\mu_0 \mu_r \frac{\partial \mathbf{H}}{\partial t} + \nabla \cdot \mathbf{E}(\mathbf{E}) &= 0
\end{align*}
\]  
(4-14)

with the flux terms

\[
\Gamma_{E}(\mathbf{H}) = \begin{bmatrix} 0 & -h_3 \\ h_3 & 0 \end{bmatrix} \quad \text{and} \quad \Gamma_{H}(\mathbf{E}) = \begin{bmatrix} \varepsilon_2 & -\varepsilon_3 \\ -\varepsilon_3 & e \end{bmatrix}
\]  
(4-15)

The divergence on \( \Gamma_{E}(\mathbf{H}) \) is applied row-wise. The conductivity and permittivity tensors \( \sigma \) and \( \varepsilon_r \) represent in-plane material properties, while the relative permeability \( \mu_r \) is an out-of-plane scalar property.

The default Lax-Friedrichs flux parameters are \( \tau_\varepsilon = 1/(2Z) \) for Ampere law, and the scalar \( \tau_H = Z/2 \) for Faraday’s law, where \( Z \) is the impedance of a vacuum.

TE WAVES IN 2D

For TE waves in 2D, solve for an in-plane magnetic field vector and one out-of-plane variable for the electric field. Maxwell’s equations then read

\[
\begin{align*}
\varepsilon_0 \varepsilon_r \frac{\partial \mathbf{E}}{\partial t} + \nabla \cdot \mathbf{H}(\mathbf{H}) &= -\sigma \mathbf{E} \\
\mu_0 \mu_r \frac{\partial \mathbf{H}}{\partial t} + \nabla \cdot \mathbf{E}(\mathbf{E}) &= 0
\end{align*}
\]  
(4-16)

with the flux terms

\[
\Gamma_{E}(\mathbf{H}) = \begin{bmatrix} -h_2 & h_1 \\ h_1 & 0 \end{bmatrix} \quad \text{and} \quad \Gamma_{H}(\mathbf{E}) = \begin{bmatrix} 0 & e_3 \\ -e_3 & 0 \end{bmatrix}
\]  
(4-17)

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The divergence of $\Gamma_H(E)$ is applied row-wise. The tensor of relative permeability $\mu_r$ represents in-plane material properties, while the relative permittivity $\varepsilon_r$ and conductivity $\sigma$ are out-of-plane scalar properties.

The default Lax-Friedrichs flux parameters are $\tau_E = 1/(2Z)$ for Ampere law, and two scalar $\tau_H = Z/2$ for Faraday’s law, where $Z$ is the impedance of a vacuum.

**Fluxes as Dirichlet Boundary Conditions**

Consider Maxwell’s equations in 3D

$$\varepsilon_0 \varepsilon_r \frac{\partial E}{\partial t} + \nabla \cdot \Gamma_E(H) = -\sigma E$$

$$\mu_0 \mu_r \frac{\partial H}{\partial t} + \nabla \cdot \Gamma_H(E) = 0$$

with the flux terms

$$\Gamma_E(H) = \begin{bmatrix} 0 & -h_3 & h_2 \\ h_3 & 0 & -h_1 \\ -h_2 & h_1 & 0 \end{bmatrix} \text{ and } \Gamma_H(E) = \begin{bmatrix} 0 & e_3 & -e_2 \\ -e_3 & 0 & e_1 \\ e_2 & -e_1 & 0 \end{bmatrix}$$

and the divergence on $\Gamma_E(H)$ and $\Gamma_H(E)$ applied row-wise.

For Ampere’s law, the normal to the flux term on exterior boundaries reads

$$\mathbf{n} \cdot \Gamma_E(H) = -\mathbf{n} \times \mathbf{H}$$

and for Faraday’s law

$$\mathbf{n} \cdot \Gamma_H(E) = \mathbf{n} \times \mathbf{E}$$

which means that normal fluxes on external boundaries can only prescribe tangential components for the fields.

**Boundary Conditions**

The boundary conditions for outer boundaries are computed from the normal fluxes $\mathbf{n} \cdot \Gamma_H(E)$ and $\mathbf{n} \cdot \Gamma_E(H)$.

- Perfect electric conductor $\mathbf{n} \times \mathbf{E} = \mathbf{0}$, or zero tangential components for $\mathbf{E}$, is obtained by setting $\mathbf{n} \cdot \Gamma_H(E) = 0$. 

• Perfect magnetic conductor \( \mathbf{n} \times \mathbf{H} = 0 \), or zero tangential components for \( \mathbf{H} \), is obtained by prescribing \( \mathbf{n} \cdot \Gamma_{E}(\mathbf{H}) = 0 \).

• Electric field \( \mathbf{n} \times \mathbf{E} = \mathbf{n} \times \mathbf{E}_0 \), or \( \mathbf{n} \cdot \Gamma_{H}(\mathbf{E}) = \mathbf{n} \times \mathbf{E}_0 \).

• Magnetic field \( \mathbf{n} \times \mathbf{H} = \mathbf{n} \times \mathbf{H}_0 \), or \( -\mathbf{n} \cdot \Gamma_{E}(\mathbf{H}) = \mathbf{n} \times \mathbf{H}_0 \).

• For external boundaries, the surface currents \( BC \) means \( \mathbf{n} \times \mathbf{H} = \mathbf{J}_s \), or \( -\mathbf{n} \cdot \Gamma_{E}(\mathbf{H}) = \mathbf{J}_s \).

**Absorbing Boundary Condition**

A simple absorbing boundary can be implemented by setting \( \mathbf{n} \times \mathbf{E} = \mathbf{Z} \mathbf{H} \).

**Absorbing Layers**

The Electromagnetic Waves, Time Explicit Interface includes so-called *absorbing layers*, also often referred to as *sponge layers*. The layers work by combining three techniques: a scaling system, filtering, and simple nonreflecting conditions. For a review of the method see, for example, Ref. 1.

The layers are set up by adding the Absorbing Layer under the Definitions node. This adds a special scaled system. The scaling effectively slows down the propagating waves and ensures that they hit the outer boundary in the normal direction. For the Absorbing Layer domain selection, add an additional Wave Equations feature, mark the Activate check box under the Filter Parameters section, and enter filter parameters. Filtering attenuates and filters out high-frequency components of the wave. Finally, at the outer boundary of the layer add a simple Low-Reflecting Boundary condition, which will work well to remove all remaining waves as normal incidence has been ensured.

For more detailed information about the filter see the Filter Parameters section under Wave Form PDE in the COMSOL Multiphysics Reference Manual.
For the Absorbing Layers select the Type (Cartesian, cylindrical, spherical, or user defined) under the Geometry section. Enter values for the Physical Width and Pole Distance under the Scaling section.

For more detailed on the Geometry and Scaling see the Infinite Elements, Perfectly Matched Layers, and Absorbing Layers in the COMSOL Multiphysics Reference Manual.

For the layers to work optimally the filter should not be too aggressive. Moreover, the scaled coordinates in the layer domain should also vary smoothly. To inspect the scaled system you can plot the coordinate variables \( x_{\text{absorb\_ab1}} \), \( y_{\text{absorb\_ab1}} \), and \( z_{\text{absorb\_ab1}} \). Using the absorbing layers with the three combined techniques will enable the reduction of spurious reflections by a factor between 100 and 1000 compared to the incident amplitude.

For an example of a filter parameter combination that can be used for a Wave Equations feature on an Absorbing Layer domain selection see the Filter Parameters section for the Wave Equations feature.

Reference
This chapter summarizes the functionality of the electrical circuit interface found under the AC/DC branch (fault) when adding a physics interface.

In this chapter:

- The Electrical Circuit Interface
- Theory for the Electrical Circuit Interface

See The AC/DC Interfaces in the COMSOL Multiphysics Reference Manual for other AC/DC interface and feature node settings.
The Electrical Circuit Interface

The Electrical Circuit (cir) interface ( ), found under the AC/DC branch ( ) when adding a physics interface, is used to model currents and voltages in circuits including voltage and current sources, resistors, capacitors, inductors, and semiconductor devices. Models created with the Electrical Circuit interface can include connections to distributed field models. The physics interface supports stationary, frequency-domain and time-domain modeling and solves Kirchhoff’s conservation laws for the voltages, currents and charges associated with the circuit elements.

When this physics interface is added, it adds a default Ground Node feature and associates that with node zero in the electrical circuit.

| Circuit nodes are nodes in the electrical circuit (electrical nodes) and should not be confused with nodes in the Model Builder tree of the COMSOL Multiphysics software. Circuit node names are not restricted to numerical values but can contain alphanumeric characters. |

DEVICE NAMES

Each circuit component has an associated Device name, which is constructed from a prefix identifying the type of the device and a string. The string can be specified in the feature’s Settings window. The Device name is used to identify variables defined by the component, and for the SPICE Import and Export functionality.

SETTINGS

The Label is the default physics interface name.

The Name is used primarily as a scope prefix for variables defined by the physics interface. Refer to such physics interface variables in expressions using the pattern <name>.<variable_name>. In order to distinguish between variables belonging to different physics interfaces, the name string must be unique. Only letters, numbers and underscores (_) are permitted in the Name field. The first character must be a letter.

The default Name (for the first physics interface in the model) is cir.
**RESISTANCE IN PARALLEL TO PN JUNCTIONS**

For numerical stability, a large resistance is added automatically in parallel to the pn junctions in diodes and BJT devices. Enter a default value for the **Resistance in parallel to pn junctions** $R_j$ (SI unit: $\Omega$). The default value is $1 \times 10^{12} \Omega$.

**CREATE UNIQUE NODES FOR NEW DEVICES**

When this setting is selected (the default), newly added devices will be assigned unused node names. The devices will be disconnected from the rest of the circuit and the nodes should be updated to reflect the actual circuit connections. When the this setting is deselected, new devices will be connected to the lowest-numbered nodes starting from 0.

---

**ELECTRICAL CIRCUIT TOOLBAR**

The following nodes are available from the **Electrical Circuit** ribbon toolbar (Windows users), **Electrical Circuit** context menu (Mac or Linux users), or right-click to access the context menu (all users):

- Theory for the Electrical Circuit Interface
- Connecting to Electrical Circuits

---

For step-by-step instructions and general documentation descriptions, this is the **Electrical Circuit** toolbar.
**Ground Node**

The Ground Node ( ) feature adds a ground node with the default node number zero to the electrical circuit. This is the default node in the Electrical Circuit interface. More ground nodes can be added but those must have unique node numbers and are by default given higher node numbers.

**GROUND CONNECTION**

Set the Node name for the ground node in the circuit. The convention is to use 0 (zero) for the ground node. If adding more ground nodes, each must have a unique node name (number).

**Resistor**

The Resistor ( ) feature connects a resistor between two nodes in the electrical circuit.

**DEVICE NAME**

Enter a Device name for the resistor. The prefix is R.
NODE CONNECTIONS
Set the two Node names for the connecting nodes for the resistor.

DEVICE PARAMETERS
Enter the Resistance of the resistor.

Capacitor

The Capacitor ( ) feature connects a capacitor between two nodes in the electrical circuit.

DEVICE NAME
Enter a Device name for the capacitor. The prefix is C.

NODE CONNECTIONS
Set the two Node names for the connecting nodes for the capacitor.

DEVICE PARAMETERS
Enter the Capacitance of the capacitor.

Inductor

The Inductor ( ) feature connects an inductor between two nodes in the electrical circuit.

DEVICE NAME
Enter a Device name for the inductor. The prefix is L.

NODE CONNECTIONS
Set the two Node names for the connecting nodes for the inductor.

DEVICE PARAMETERS
Enter the Inductance of the inductor.

Voltage Source

The Voltage Source ( ) feature connects a voltage source between two nodes in the electrical circuit.
**DEVICE NAME**

Enter a **Device name** for the voltage source. The prefix is V.

**NODE CONNECTIONS**

Set the two **Node names** for the connecting nodes for the voltage source. The first node represents the positive reference terminal.

**DEVICE PARAMETERS**

Enter the **Source type** that should be adapted to the selected study type. It can be **DC-source**, **AC-source**, or a time-dependent **Sine source** or **Pulse source**. Depending on the choice of source, also specify the following parameters:

- For a DC-source, the **Voltage** $V_{\text{src}}$ (default value: 1 V). DC-sources are active in Stationary and Time-Dependent studies.
- For an AC-source: the **Voltage** $V_{\text{src}}$ (default value: 1 V) and the **Phase** $\Theta$ (default value: 0 rad). AC-sources are active in Frequency Domain studies only.
- For a sine source: the **Voltage** $V_{\text{src}}$ (default value: 1 V), the **Offset** $V_{\text{off}}$ (default value: 0 V), the **Frequency** (default value: 1 kHz), and the **Phase** $\Theta$ (default value: 0 rad). The sine sources are active in Time-Dependent studies and also in Stationary studies, providing that a value for $t$ has been provided as a model parameter or global variable.
- For a pulse source: the **Voltage** $V_{\text{src}}$ (default value: 1 V), the **Offset** $V_{\text{off}}$ (default value: 0 V), the **Delay** $t_d$ (default value: 0s), the **Rise time** $t_r$, and the **Fall time** $t_f$ (default values: 0 s), the **Pulse width** $p_w$ (default value: 1 $\mu$s), and the **Period** $T_{\text{per}}$ (default value: 2 $\mu$s). The pulse sources are active in Time-Dependent studies and also in Stationary studies, providing that a value for $t$ has been provided as a model parameter or global variable.

All values are peak values rather than RMS.

---

For the AC source, the frequency is a global input set by the solver. AC sources should be used in Frequency-domain studies only. Do not use the Sine source unless the model is time-dependent.

---

**Current Source**

The **Current Source** ( ) feature connects a current source between two nodes in the electrical circuit.
**DEVICE NAME**

Enter a **Device name** for the current source. The prefix is I.

**NODE CONNECTIONS**

Set the two **Node names** for the connecting nodes for the current source. The first node represents the positive reference terminal from where the current flows through the source to the second node.

**DEVICE PARAMETERS**

Enter the **Source type** that should be adapted to the selected study type. It can be **DC-source**, **AC-source**, or a time-dependent **Sine source** or **Pulse source**. Depending on the choice of source, also specify the following parameters:

- For a DC-source, the **Current** $i_{src}$ (default value: 1 A). DC-sources are active in Stationary and Time-Dependent studies.
- For an AC-source: the **Current** $i_{src}$ (default value: 1 A) and the **Phase** $\Theta$ (default value: 0 rad). AC-sources are active in Frequency Domain studies only.
- For a sine source: the **Current** $i_{src}$ (default value: 1 A), the **Offset** $i_{off}$ (default value: 0 A), the **Frequency** (default value: 1 kHz), and the **Phase** $\Theta$ (default value: 0 rad). The sine sources are active in Time-Dependent studies and also in Stationary studies, providing that a value for $t$ has been provided as a model parameter or global variable.
- For a pulse source: the **Current** $i_{src}$ (default value: 1 A), the **Offset** $i_{off}$ (default value: 0 A), the **Delay** $t_d$ (default value: 0 s), the **Rise time** $t_r$ and **Fall time** $t_f$ (default values: 0 s), the **Pulse width** $p_w$ (default value: 1 $\mu$s), and the **Period** $T_{per}$ (default value: 2 $\mu$s). The pulse sources are active in Time-Dependent studies and also in Stationary studies, providing that a value for $t$ has been provided as a model parameter or global variable.

All values are peak values rather than RMS.

---

For the AC source, the frequency is a global input set by the solver. AC sources should be used in Frequency-domain studies only. Do not use the **Sine source** unless the model is time-dependent.
Voltage-Controlled Voltage Source

The Voltage-Controlled Voltage Source \( \text{(VCCS)} \) feature connects a voltage-controlled voltage source between two nodes in the electrical circuit. A second pair of nodes define the input control voltage.

**DEVICE NAME**

Enter a Device name for the voltage-controlled voltage source. The prefix is E.

**NODE CONNECTIONS**

Specify four Node names: the first pair for the connection nodes for the voltage source and the second pair defining the input control voltage. The first node in a pair represents the positive reference terminal.

**DEVICE PARAMETERS**

Enter the voltage Gain. The resulting voltage is this number multiplied by the control voltage.

Voltage-Controlled Current Source

The Voltage-Controlled Current Source \( \text{(VCCS)} \) feature connects a voltage-controlled current source between two nodes in the electrical circuit. A second pair of nodes define the input control voltage.

**DEVICE NAME**

Enter a Device name for the voltage-controlled current source. The prefix is G.

**NODE CONNECTIONS**

Specify four Node names: the first pair for the connection nodes for the current source and the second pair defining the input control voltage. The first node in a pair represents the positive voltage reference terminal or the one from where the current flows through the source to the second node.

**DEVICE PARAMETERS**

Enter the source Gain (SI units: S). The resulting current is this number multiplied by the control voltage. It represents the transconductance of the source.
Current-Controlled Voltage Source

The Current-Controlled Voltage Source (\( \text{\textcopyright} \)) feature connects a current-controlled voltage source between two nodes in the electrical circuit. The input control current is the one flowing through a two-pin device.

**DEVICE NAME**
Enter a Device name for the current-controlled voltage source. The prefix is H.

**NODE CONNECTIONS**
Set two Node names for the connection nodes for the voltage source. The first node in a pair represents the positive reference terminal.

**DEVICE PARAMETERS**
Enter the voltage Gain and select the Device whose current is taken as the control current. The resulting voltage is this number multiplied by the control current through the named Device (any two-pin device). Thus it formally has the unit of resistance.

Current-Controlled Current Source

The Current-Controlled Current Source (\( \text{\textcopyright} \)) feature connects a current-controlled current source between two nodes in the electrical circuit. The input control current is the one flowing through a named device that must be a two-pin device.

**DEVICE NAME**
Enter a Device name for the current-controlled current source. The prefix is F.

**NODE CONNECTIONS**
Specify two Node names for the connection nodes for the current source. The first node in a pair represents the positive reference terminal from where the current flows through the source to the second node.

**DEVICE PARAMETERS**
Enter the current Gain and select the Device whose current is taken as the control current. The resulting current is this number multiplied by the control current through the Device.
Subcircuit Definition

The Subcircuit Definition ( ) feature is used to define subcircuits, which can be inserted as devices into the main circuit using Subcircuit Instance nodes. Create the subcircuit by adding subnodes to the Subcircuit Definition node, either by using the Physics toolbar, or by right-clicking the Subcircuit Definition.

SUBCIRCUIT PINS

Define the Pin names at which the subcircuit connects to the main circuit or to other subcircuits when referenced by a Subcircuit Instance node. The Pin names refer to circuit nodes in the subcircuit. The order in which the Pin names are defined is the order in which they are referenced by a Subcircuit Instance node. The devices constituting the subcircuit should be connected only to the subcircuit’s pins and to themselves.

Subcircuit Instance

The Subcircuit Instance ( ) feature represents an instance of a subcircuits defined by a Subcircuit Definition feature.

DEVICE NAME

Enter a Device name for the subcircuit instance. The prefix is X.

NODE CONNECTIONS

Select the Name of subcircuit link from the list of defined subcircuits in the circuit model and the circuit Node names at which the subcircuit instance connects to the main circuit or to another subcircuit if used therein.

NPN BJT and PNP BJT

The NPN BJT ( ) and the PNP BJT ( ) device models are large-signal models for bipolar junction transistors (BJT). It is an advanced device model and no thorough description and motivation of the many input parameters are attempted here. Many device manufacturers provide model input parameters for this BJT model. For any particular make of BJT, the device manufacturer should be the primary source of information.

DEVICE NAME

Enter a Device name for the BJT. The prefix is Q.
**NODE CONNECTIONS**

Specify three *Node names* for the connection nodes for the **BJT** device. These represent the *collector*, *base*, and *emitter* nodes for the **NPN** transistor, and the *emitter*, *base*, and *collector* nodes for the **PNP** transistor.

**MODEL PARAMETERS**

Specify the *Model Parameters*. Reasonable defaults are provided but for any particular BJT, the device manufacturer should be the primary source of information.

The interested reader is referred to Ref. 1 for more details on semiconductor modeling within circuits.

For an explanation of the *Model Parameters* see Bipolar Transistors.

---

**n-Channel MOSFET and p-Channel MOSFET**

The **n-Channel MOSFET** ( ) and the **p-Channel MOSFET** ( ) device models are large-signal models for, respectively, an n-Channel MOS field-effect transistor (MOSFET) and p-Channel MOSFET. These are advanced device models and no thorough description and motivation of the many input parameters are attempted here. Many device manufacturers provide model parameters for the MOSFET models. For any particular make of MOSFET, the device manufacturer should be the primary source of information.

**DEVICE NAME**

Enter a *Device name* for the MOSFET. The prefix is M.

**NODE CONNECTIONS**

Specify four *Node names* for the connection nodes for the **n-Channel MOSFET** or **p-Channel MOSFET** device. These represent the *drain*, *gate*, *source*, and *bulk* nodes, respectively.
**MODEL PARAMETERS**

Specify the Model Parameters. Reasonable defaults are provided but for any particular MOSFET, the device manufacturer should be the primary source of information.

The interested reader is referred to Ref. 1 for more details on semiconductor modeling within circuits.

For an explanation of the Model Parameters see MOSFET Transistors.

---

**Mutual Inductance**

The Mutual Inductance allows specifying a coupling between two existing Inductor features in the circuit. The mutual inductance of the coupling is

\[ M = k \sqrt{L_1 L_2} \]

where \( k \) is the coupling factor and \( L_1 \) and \( L_2 \) are the inductances of the inductors.

---

**DEVICE PARAMETERS**

Enter values or expressions for the:

- **Coupling factor** \( k \) (dimensionless). The value must be between 0 and 1, and the default is 0.98.
- **First inductance** \( L_1 \) (SI unit: H) and **Second inductance** \( L_2 \) (SI unit: H). These must be set to two different Inductor features in the circuit.

---

**Transformer**

The Transformer feature represents either a combination of two Inductor and a Mutual Inductance features, or an ideal transformer.

---

**NODE CONNECTIONS**

Enter or edit the table in the Node names column for the primary and secondary node connections.

---

**DEVICE PARAMETERS**

Chose a Transformer model—Specify inductors (the default) or Ideal transformer.

For Specify inductors enter values or expressions for the:

- **Coupling factor** \( k \) (dimensionless). The default is 0.98.
• First inductance \( L_1 \) (SI unit: H). The default is 1 mH.
• Second inductance \( L_2 \) (SI unit: H). The default is 1 mH.

For Ideal transformer enter values or expressions for the Winding ratio \( N_1/N_2 \) (dimensionless). The default is 10.

**Diode**

The Diode device model is a large-signal model for a diode. It is an advanced device model and no thorough description and motivation of the many input parameters are attempted here. The interested reader is referred to Ref. 1 for more details on semiconductor modeling within circuits. Many device manufacturers provide model parameters for this diode model. For any particular make of diode, the device manufacturer should be the primary source of information.

**DEVICE NAME**
Enter a Device name for the diode. The prefix is D.

**NODE CONNECTIONS**
Specify two Node names for the positive and negative nodes for the Diode device.

**MODEL PARAMETERS**
Specify the Model Parameters. Reasonable defaults are provided but for any particular diode, the device manufacturer should be the primary source of information.

For an explanation of the Model Parameters see Diode.

**External I vs. U**

The External I vs. U feature connects an arbitrary voltage measurement (for example, a circuit terminal or circuit port boundary or a coil domain from another physics interface) as a voltage source between two nodes in the electrical circuit. The resulting circuit current from the first node to the second node is typically coupled back as a prescribed current source in the context of the voltage measurement.

**DEVICE NAME**
Enter a Device name for the External I vs. U node.
NODE CONNECTIONS

Specify the two Node names for the connecting nodes for the voltage source. The first node represents the positive reference terminal.

EXTERNAL DEVICE

Enter the source of the Voltage. If circuit or current excited terminals or circuit ports are defined on boundaries or domains or a multiturn coil domains is defined in other physics interfaces, these display as options in the Voltage list. Also select the User defined option and enter your own voltage variable, for example, using a suitable coupling operator. For inductive or electromagnetic wave propagation models, the voltage measurement must be performed as an integral of the electric field because the electric potential only does not capture induced EMF. Also the integration must be performed over a distance that is short compared to the local wavelength.

Except when coupling to a circuit terminal, circuit port, or coil, the current flow variable must be manually coupled back in the electrical circuit to the context of the voltage measurement. This applies also when coupling to a current excited terminal. The name of this current variable follows the convention cirn.IvsUm_i, where cirn is the tag of the Electrical Circuit interface node and IvtsUm is the tag of the External I vs. U node. The tags are typically displayed within curly brackets {} in the Model Builder.

Component Couplings and Coupling Operators in the COMSOL Multiphysics Reference Manual

External U vs. I

The External U vs. I ( ) feature connects an arbitrary current measurement (for example, a coil domain from another physics interface) as a current source between two nodes in the electrical circuit. The resulting circuit voltage between the first node and the second node is typically coupled back as a prescribed voltage source in the context of the current measurement.

DEVICE NAME

Enter a Device name for the External U vs. I node.
**NODE CONNECTIONS**
Specify the two **Node names** for the connecting nodes for the current source. The current flows from the first node to the second node.

**EXTERNAL DEVICE**
Enter the source of the **Current**. Voltage excited terminals or lumped ports defined on boundaries in other physics interfaces are natural candidates but do not appear as options in the **Voltage** list because those do not have an accurate built-in current measurement variable. A **User defined** option must be selected and a current variable entered, for example, using a suitable coupling operator.

The voltage variable must be manually coupled back in the electrical circuit to the context of the current measurement. This applies also when coupling to a voltage excited terminal or lumped port. The name of this voltage variable follows the convention cirn.UvsIm_v, where cirn is the tag of the Electrical Circuit interface node and UvsIm is the tag of the **External U vs. I** node. The tags are typically displayed within curly brackets {} in the Model Builder.

**External I-Terminal**
The **External I-Terminal** feature connects an arbitrary voltage-to-ground measurement (for example, a circuit terminal from another physics interface) as a voltage-to-ground assignment to a node in the electrical circuit. The resulting circuit current from the node is typically coupled back as a prescribed current source in the context of the voltage measurement. This node does not apply when coupling to inductive or electromagnetic wave propagation models because then voltage must be defined as a line integral between two points rather than a single point measurement of electric potential. For such couplings, use the **External I vs. U** node instead.

**DEVICE NAME**
Enter a **Device name** for the External I-terminal.
NODE CONNECTIONS

Set the **Node name** for the connecting node for the voltage assignment.

EXTERNAL TERMINAL

Enter the source of the **Voltage**. If circuit- or current-excited terminals are defined on boundaries in other physics interfaces, these display as options in the **Voltage** list. Also select the **User defined** option and enter a voltage variable, for example, using a suitable coupling operator.

---

Except when coupling to a circuit terminal, the current flow variable must be manually coupled back in the electrical circuit to the context of the voltage measurement. This applies also when coupling to a current excited terminal. The name of this current variable follows the convention `cirn.termIm_i`, where `cirn` is the tag of the Electrical Circuit interface node and `termIm` is the tag of the **External I-Terminal** node. The tags are typically displayed within curly brackets `{}` in the Model Builder.

---

**SPICE Circuit Import**

Right-click the **Electrical Circuit** feature node to import an existing SPICE netlist (select **Import Spice Netlist**). A window opens—enter a file location or browse your directories to find one. The default file extension for a SPICE netlist is `.cir`. The SPICE circuit import translates the imported netlist into Electrical Circuit interface nodes so these define the subset of SPICE features that can be imported.

---

See **Spice Import** about the supported SPICE commands.

---

**SPICE Circuit Export**

Right-click the **Electrical Circuit** feature node to export the current circuit to the SPICE netlist file format (select **Export Spice Netlist**). A window opens—enter a file location or browse your directories to find one. The default file extension for a
SPICE netlist is `.cir`. The compatible circuit nodes are exported as SPICE devices.

See SPICE Export for more details on the supported SPICE commands.
Theory for the Electrical Circuit Interface

The Electrical Circuit Interface theory is discussed in this section:

- Electric Circuit Modeling and the Semiconductor Device Models
- Bipolar Transistors
- MOSFET Transistors
- Diode
- Reference for the Electrical Circuit Interface

Connecting to Electrical Circuits

Electric Circuit Modeling and the Semiconductor Device Models

Electrical circuit modeling capabilities are useful when simulating all sorts of electrical and electromechanical devices ranging from heaters and motors to advanced plasma reactors in the semiconductor industry. There are two fundamental ways that an electrical circuit model relates to a physical field model.

- The field model is used to get a better, more accurate description of a single device in the electrical circuit model.
- The electrical circuit is used to drive or terminate the device in the field model in such a way that it makes more sense to simulate both as a tightly coupled system.

The Electrical Circuit interface makes it possible to add nodes representing circuit elements directly to the Model Builder tree in a COMSOL Multiphysics model. The circuit variables can then be connected to a physical device model to perform co-simulations of circuits and multiphysics. The model acts as a device connected to the circuit so that its behavior is analyzed in larger systems.

The fundamental equations solved by the Electrical Circuit interface are Kirchhoff’s circuit laws, which in turn can be deduced from Maxwell’s equations. The supported study types are Stationary, Frequency Domain, and Time Dependent.
There are three more advanced large-signal semiconductor device features available in the Electrical Circuit interface. The equivalent circuits and the equations defining their non-ideal circuit elements are described in this section. For a more detailed account on semiconductor device modeling, see Ref. 1.

**Bipolar Transistors**

Figure 5-1 illustrates the equivalent circuit for the npn bipolar junction transistor.

![Bipolar Transistor Circuit](image)

**Figure 5-1: A circuit for the bipolar transistor.**

The pnp transistor model is similar in all regards to the npn transistor, with the difference that the polarities of the currents and voltages involved are reversed. The following equations are used to compute the relations between currents and voltages in the circuit.
There are also two capacitances that use the same formula as the junction capacitance of the diode model. In the parameter names below, replace \( x \) with \( C \) for the base-collector capacitance and \( E \) for the base-emitter capacitance.

The model parameters are listed in the table below.

<table>
<thead>
<tr>
<th>TABLE 5-1: BIPOLAR TRANSISTOR MODEL PARAMETERS</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>PARAMETER</strong></td>
</tr>
<tr>
<td>( B_F )</td>
</tr>
<tr>
<td>( B_R )</td>
</tr>
<tr>
<td>( C_{JC} )</td>
</tr>
<tr>
<td>( C_{JE} )</td>
</tr>
<tr>
<td>( F_C )</td>
</tr>
<tr>
<td>( I_{KF} )</td>
</tr>
<tr>
<td>( I_{KR} )</td>
</tr>
</tbody>
</table>
### TABLE 5-1: BIPOLAR TRANSISTOR MODEL PARAMETERS

<table>
<thead>
<tr>
<th>PARAMETER</th>
<th>DEFAULT</th>
<th>DESCRIPTION</th>
</tr>
</thead>
<tbody>
<tr>
<td>$I_S$</td>
<td>$1e^{-15}$ A/m$^2$</td>
<td>Saturation current</td>
</tr>
<tr>
<td>$I_{SC}$</td>
<td>0 A/m$^2$</td>
<td>Base-collector leakage saturation current</td>
</tr>
<tr>
<td>$I_{SE}$</td>
<td>0 A/m$^2$</td>
<td>Base-emitter leakage saturation current</td>
</tr>
<tr>
<td>$M_{JC}$</td>
<td>$1/3$</td>
<td>Base-collector grading coefficient</td>
</tr>
<tr>
<td>$M_{JE}$</td>
<td>$1/3$</td>
<td>Base-emitter grading coefficient</td>
</tr>
<tr>
<td>$N_C$</td>
<td>2</td>
<td>Base-collector ideality factor</td>
</tr>
<tr>
<td>$N_E$</td>
<td>1.4</td>
<td>Base-emitter ideality factor</td>
</tr>
<tr>
<td>$N_F$</td>
<td>1</td>
<td>Forward ideality factor</td>
</tr>
<tr>
<td>$N_R$</td>
<td>1</td>
<td>Reverse ideality factor</td>
</tr>
<tr>
<td>$R_B$</td>
<td>0 $\Omega$m$^2$</td>
<td>Base resistance</td>
</tr>
<tr>
<td>$R_{BM}$</td>
<td>0 $\Omega$m$^2$</td>
<td>Minimum base resistance</td>
</tr>
<tr>
<td>$R_C$</td>
<td>0 $\Omega$m$^2$</td>
<td>Collector resistance</td>
</tr>
<tr>
<td>$R_E$</td>
<td>0 $\Omega$m$^2$</td>
<td>Emitter resistance</td>
</tr>
<tr>
<td>$T_{NOM}$</td>
<td>298.15 K</td>
<td>Device temperature</td>
</tr>
<tr>
<td>$V_{AF}$</td>
<td>$\text{Inf}$ (V)</td>
<td>Forward Early voltage</td>
</tr>
<tr>
<td>$V_{AR}$</td>
<td>$\text{Inf}$ (V)</td>
<td>Reverse Early voltage</td>
</tr>
<tr>
<td>$V_{JC}$</td>
<td>0.71 V</td>
<td>Base-collector built-in potential</td>
</tr>
<tr>
<td>$V_{JE}$</td>
<td>0.71 V</td>
<td>Base-emitter built-in potential</td>
</tr>
</tbody>
</table>
**MOSFET Transistors**

Figure 5-2 illustrates an equivalent circuit for the n-channel MOSFET transistor. The p-channel MOSFET transistor is treated similarly, but the polarities of the involved voltages are reversed.

Figure 5-2: A circuit for the MOSFET transistor.

The following equations are used to compute the relations between currents and voltages in the circuit.
There are several capacitances between the terminals:

\[ C_{gd} = C_{gd0}W \]
\[ C_{gs} = C_{gs0}W \]

\[ C_{jbd} = C_{BD} \times \left\{ \begin{array}{ll}
(1 - v_{bd})^{-1} - M_r - 1 & v_{bx} < FCP_B \\
(1 - F_C)(1 + M_J + M_J v_{bx} / F_B) & v_{bx} \geq FCP_B
\end{array} \right. \]

The model parameters are as follows:

<table>
<thead>
<tr>
<th>PARAMETER</th>
<th>DEFAULT</th>
<th>DESCRIPTION</th>
</tr>
</thead>
<tbody>
<tr>
<td>CBD</td>
<td>0 F/m</td>
<td>Bulk-drain zero-bias capacitance</td>
</tr>
<tr>
<td>CGDO</td>
<td>0 F/m</td>
<td>Gate-drain overlap capacitance</td>
</tr>
<tr>
<td>CGSO</td>
<td>0 F/m</td>
<td>Gate-source overlap capacitance</td>
</tr>
<tr>
<td>FC</td>
<td>0.5</td>
<td>Capacitance factor</td>
</tr>
<tr>
<td>IS</td>
<td>1e-13 A</td>
<td>Bulk junction saturation current</td>
</tr>
<tr>
<td>KP</td>
<td>2e-5 A/V^2</td>
<td>Transconductance parameter</td>
</tr>
<tr>
<td>L</td>
<td>50e-6 m</td>
<td>Gate length</td>
</tr>
<tr>
<td>MJ</td>
<td>0.5</td>
<td>Bulk junction grading coefficient</td>
</tr>
<tr>
<td>N</td>
<td>1</td>
<td>Bulk junction ideality factor</td>
</tr>
</tbody>
</table>
### TABLE 5-2: MOSFET TRANSISTOR MODEL PARAMETERS

<table>
<thead>
<tr>
<th>PARAMETER</th>
<th>DEFAULT</th>
<th>DESCRIPTION</th>
</tr>
</thead>
<tbody>
<tr>
<td>$P_B$</td>
<td>0.75 V</td>
<td>Bulk junction potential</td>
</tr>
<tr>
<td>$R_B$</td>
<td>0 Ω</td>
<td>Bulk resistance</td>
</tr>
<tr>
<td>$R_D$</td>
<td>0 Ω</td>
<td>Drain resistance</td>
</tr>
<tr>
<td>$R_{DS}$</td>
<td>Inf (Ω)</td>
<td>Drain-source resistance</td>
</tr>
<tr>
<td>$R_G$</td>
<td>0 Ω</td>
<td>Gate resistance</td>
</tr>
<tr>
<td>$R_S$</td>
<td>0 Ω</td>
<td>Source resistance</td>
</tr>
<tr>
<td>$T_{NOM}$</td>
<td>298.15 K</td>
<td>Device temperature</td>
</tr>
<tr>
<td>$V_{TO}$</td>
<td>0 V</td>
<td>Zero-bias threshold voltage</td>
</tr>
<tr>
<td>$W$</td>
<td>50e-6 m</td>
<td>Gate width</td>
</tr>
<tr>
<td>$\Gamma$ (GAMMA)</td>
<td>0 $\sqrt{0.5}$</td>
<td>Bulk threshold parameter</td>
</tr>
<tr>
<td>$\Phi$ (PHI)</td>
<td>0.5 V</td>
<td>Surface potential</td>
</tr>
<tr>
<td>$\Lambda$ (LAMBDA)</td>
<td>0 1/V</td>
<td>Channel-length modulation</td>
</tr>
</tbody>
</table>
**Diode**

Figure 5-3 illustrates equivalent circuit for the diode.

The following equations are used to compute the relations between currents and voltages in the circuit.
where the following model parameters are required

\[
\begin{align*}
    i_d &= i_{dhl} + i_{drec} + i_{db} + i_c \\
    i_{dhl} &= IS \left( \frac{e^{-\frac{V}{NVT}} - 1}{1 + \frac{IS}{IKF} \left( e^{-\frac{V}{NVT}} - 1 \right)} \right) \\
    i_{drec} &= ISR \left( \frac{1 - \frac{V}{CTJ}^{1-M}}{1 - FC(1+M) + M \frac{V_{J}}{V_{T}}} \right) \\
    i_{db} &= IBV e^{\frac{V}{NVT}} \\
    C_j &= C_{J0} \times \left\{ \begin{array}{ll}
        \left(1 - \frac{V}{V_{J}} \right)^{-M} & v_d < FCV_J \\
        \left(1 - FC \right)^{-1-M} \left(1 - FC(1+M) + M \frac{V_{d}}{V_{J}} \right) & v_d \geq FCV_J \\
    \end{array} \right. \\
    V_T &= \frac{kBT_{NOM}}{q}
\end{align*}
\]

<table>
<thead>
<tr>
<th>PARAMETER</th>
<th>DEFAULT</th>
<th>DESCRIPTION</th>
</tr>
</thead>
<tbody>
<tr>
<td>BV</td>
<td>Inf (V)</td>
<td>Reverse breakdown voltage</td>
</tr>
<tr>
<td>CJ0</td>
<td>0 F</td>
<td>Zero-bias junction capacitance</td>
</tr>
<tr>
<td>FC</td>
<td>0.5</td>
<td>Forward-bias capacitance coefficient</td>
</tr>
<tr>
<td>IBV</td>
<td>1e-09 A</td>
<td>Current at breakdown voltage</td>
</tr>
<tr>
<td>IKF</td>
<td>Inf (A)</td>
<td>Corner for high-current roll-off</td>
</tr>
<tr>
<td>IS</td>
<td>1e-13 A</td>
<td>Saturation current</td>
</tr>
<tr>
<td>M</td>
<td>0.5</td>
<td>Grading coefficient</td>
</tr>
<tr>
<td>N</td>
<td>1</td>
<td>Ideality factor</td>
</tr>
<tr>
<td>NBV</td>
<td>1</td>
<td>Breakdown ideality factor</td>
</tr>
<tr>
<td>NR</td>
<td>2</td>
<td>Recombination ideality factor</td>
</tr>
<tr>
<td>RS</td>
<td>0 Ω</td>
<td>Series resistance</td>
</tr>
<tr>
<td>T_{NOM}</td>
<td>298.15 K</td>
<td>Device temperature</td>
</tr>
<tr>
<td>V_{J}</td>
<td>1.0 V</td>
<td>Junction potential</td>
</tr>
</tbody>
</table>
Reference for the Electrical Circuit Interface

This chapter describes The Microwave Heating Interface found under the Heat Transfer > Electromagnetic Heating branch when adding a physics interface.

The Microwave Heating Interface

The Microwave Heating interface is used to model electromagnetic heating for systems and devices that are on a scale ranging from 1/10 of a wavelength up to, depending on available computer memory, about 10 wavelengths. This multiphysics interface adds an Electromagnetic Waves, Frequency Domain interface and a Heat Transfer in Solids interface. The multiphysics couplings add the electromagnetic losses from the electromagnetic waves as a heat source, and the electromagnetic material properties can depend on the temperature. The modeling approach is based on the assumption that the electromagnetic cycle time is short compared to the thermal time scale.

The following table shows what study step combinations of Electromagnetic Waves, Frequency Domain interface and Heat Transfer in Solids interface are supported from the Microwave Heating interface.

<table>
<thead>
<tr>
<th>PRESET STUDIES</th>
<th>ELECTROMAGNETIC WAVES, FREQUENCY DOMAIN INTERFACE</th>
<th>HEAT TRANSFER IN SOLIDS INTERFACE</th>
</tr>
</thead>
<tbody>
<tr>
<td>Frequency-Stationary</td>
<td>Frequency Domain</td>
<td>Stationary</td>
</tr>
<tr>
<td>Frequency-Transient</td>
<td>Frequency Domain</td>
<td>Transient</td>
</tr>
</tbody>
</table>

When a predefined Microwave Heating interface is added from the Heat Transfer>Electromagnetic Heating branch of the Model Wizard or Add Physics windows, Electromagnetic Waves, Frequency Domain and Heat Transfer in Solids interfaces are added to the Model Builder.

In addition, a Multiphysics node is added, which automatically includes the multiphysics coupling features Electromagnetic Heat Source, Boundary Electromagnetic Heat Source, and Temperature Coupling.

On the Constituent Physics Interfaces

The Electromagnetic Waves, Frequency Domain interface computes time-harmonic electromagnetic field distributions. To use this physics interface, the maximum mesh element size should be limited to a fraction of the wavelength. Thus, the domain size that can be simulated scales with the amount of available computer memory and the wavelength. The physics interface solves the time-harmonic wave equation for the electric field.
The Heat Transfer in Solids interface provides features for modeling heat transfer by conduction, convection, and radiation. A Heat Transfer in Solids model is active by default on all domains. All functionality for including other domain types, such as a fluid domain, is also available. The temperature equation defined in solid domains corresponds to the differential form of Fourier’s law that may contain additional contributions like heat sources.

In previous versions of COMSOL Multiphysics, a specific physics interface called Microwave Heating was added to the Model Builder. Now, a predefined multiphysics coupling approach is used, improving the flexibility and design options for your modeling. For specific details, see Multiphysics Modeling Approaches in the COMSOL Multiphysics Reference Manual.

### Settings for Physics Interfaces and Coupling Features

When physics interfaces are added using the predefined couplings, for example Microwave Heating, specific settings are included with the physics interfaces and the coupling features.

However, if physics interfaces are added one at a time, followed by the coupling features, these modified settings are not automatically included.

For example, if single Electromagnetic Waves, Frequency Domain and Heat Transfer in Solids interfaces are added, COMSOL adds an empty Multiphysics node. You can choose from the available coupling features, Electromagnetic Heat Source, Boundary Electromagnetic Heat Source, and Temperature Coupling, but the modified settings are not included.

Coupling features are available from the context menu (right-click the Multiphysics node) or from the Physics toolbar, Multiphysics menu.

**TABLE 6-2: MODIFIED SETTINGS FOR A MICROWAVE HEATING INTERFACE**

<table>
<thead>
<tr>
<th>PHYSICS INTERFACE OR COUPLING FEATURE</th>
<th>MODIFIED SETTINGS (IF ANY)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Electromagnetic Waves, Frequency Domain</td>
<td>No changes.</td>
</tr>
<tr>
<td>Heat Transfer in Solids</td>
<td>No changes.</td>
</tr>
</tbody>
</table>
CHAPTER 6: HEAT TRANSFER INTERFACES

PHYSICS INTERFACES AND COUPLING FEATURES

Coupling Features

- The Electromagnetic Heat Source coupling feature node is described in this section.
- The Boundary Electromagnetic Heat Source and Temperature Coupling coupling feature nodes are described for The Joule Heating Interface in the COMSOL Multiphysics Reference Manual.

TABLE 6-2: MODIFIED SETTINGS FOR A MICROWAVE HEATING INTERFACE

<table>
<thead>
<tr>
<th>PHYSICS INTERFACE OR COUPLING FEATURE</th>
<th>MODIFIED SETTINGS (IF ANY)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Electromagnetic Heat Source</td>
<td>The Domain Selection is the same as that of the participating physics interfaces. The corresponding Electromagnetic Waves, Frequency Domain and Heat Transfer in Solids interfaces are preselected in the Electromagnetic Heat Source section (described in the COMSOL Multiphysics Reference Manual).</td>
</tr>
<tr>
<td>Boundary Electromagnetic Heat Source</td>
<td>The Boundary Selection is the same as the exterior and interior boundaries of the Domain Selection of the participating physics interfaces. The corresponding Electromagnetic Waves, Frequency Domain and Heat Transfer in Solids interfaces are preselected in the Boundary Electromagnetic Heat Source section (described in the COMSOL Multiphysics Reference Manual).</td>
</tr>
</tbody>
</table>

A side effect of adding physics interfaces one at a time is that two study types—Frequency-Stationary and Frequency-Transient—are not available for selection until after at least one coupling feature is added. In this case, it is better to first add an Empty Study, then add the coupling features to the Multiphysics node, and lastly, right-click the Study node to add the study steps as needed.

Use the online help in COMSOL Multiphysics to locate and search all the documentation. All these links also work directly in COMSOL Multiphysics when using the Help system.
Physics Interface Features

Physics nodes are available from the Physics ribbon toolbar (Windows users), Physics context menu (Mac or Linux users), or right-click to access the context menu (all users).

In general, to add a node, go to the Physics toolbar, no matter what operating system you are using. Subnodes are available by clicking the parent node and selecting it from the Attributes menu.

• The available physics features for The Electromagnetic Waves, Frequency Domain Interface are listed in the section Domain, Boundary, Edge, Point, and Pair Nodes for the Electromagnetic Waves, Frequency Domain Interface.

• See The Heat Transfer Interfaces in the COMSOL Multiphysics Reference Manual for information about the available physics features for heat transfer.

If you have an add-on module, such as the Heat Transfer Module, there are additional specialized physics nodes available and described in the individual module documentation.

Electromagnetic Heat Source

The Electromagnetic Heat Source node represents the electromagnetic losses, $Q_e$ (SI unit: W/m$^3$), as a heat source in the heat transfer part of the model. It is given by

\[ Q_e = Q_{rh} + Q_{ml} \]

where the resistive losses are

\[ Q_{rh} = \frac{1}{2} \text{Re}(J \cdot E^*) \]

and the magnetic losses are

\[ Q_{ml} = \frac{1}{2} \text{Re}(i\omega B \cdot H^*) \]
**SETTINGS**

The **Label** is the default multiphysics coupling feature name.

The **Name** is used primarily as a scope prefix for variables defined by the coupling node. Refer to such variables in expressions using the pattern `<name>.<variable_name>`. In order to distinguish between variables belonging to different coupling nodes or physics interfaces, the `name` string must be unique. Only letters, numbers and underscores (_) are permitted in the **Name** field. The first character must be a letter.

The default **Name** (for the first multiphysics coupling feature in the model) is emh.

**ELECTROMAGNETIC HEAT SOURCE**

This section defines the physics involved in the electromagnetic heat source multiphysics coupling. By default, the applicable physics interface is selected in the Electromagnetic list to apply the Heat transfer to its physics interface to establish the coupling.

You can also select **None** from either list to uncouple the Electromagnetic Heat Source node from a physics interface. If the physics interface is removed from the Model Builder, for example Heat Transfer in Solids is deleted, then the Heat transfer list defaults to **None** as there is nothing to couple to.

If a physics interface is deleted and then added to the model again, and in order to re-establish the coupling, you need to choose the physics interface again from the Heat transfer or Electromagnetic lists. This is applicable to all multiphysics coupling nodes that would normally default to the once present physics interface. See Multiphysics Modeling Approaches in the **COMSOL Multiphysics Reference Manual**.
This Glossary of Terms contains finite element modeling terms in an electromagnetic waves context. For mathematical terms as well as geometry and CAD terms specific to the COMSOL Multiphysics® software and documentation, see the glossary in the COMSOL Multiphysics Reference Manual. For references to more information about a term, see the index.
Glossary of Terms

absorbing boundary A boundary that lets an electromagnetic wave propagate through the boundary without reflections.

anisotropy Variation of material properties with direction.

constitutive relation The relation between the $\mathbf{D}$ and $\mathbf{E}$ fields and between the $\mathbf{B}$ and $\mathbf{H}$ fields. These relations depend on the material properties.

cutoff frequency The lowest frequency for which a given mode can propagate through, for example, a waveguide or optical fiber.

d grid element See vector element.

eigenmode A possible propagating mode of, for example, a waveguide or optical fiber.

electric dipole Two equal and opposite charges $+q$ and $-q$ separated a short distance $d$. The electric dipole moment is given by $\mathbf{p} = q\mathbf{d}$, where $\mathbf{d}$ is a vector going from $-q$ to $+q$.

gauge transformation A variable transformation of the electric and magnetic potentials that leaves Maxwell’s equations invariant.

lumped port A type of port feature. Use the lumped port to excite the model with a voltage, current, or circuit input. The lumped port must be applied between two metallic objects, separated by much less than a wavelength.

magnetic dipole A small circular loop carrying a current. The magnetic dipole moment is $\mathbf{m} = I\mathbf{A}\mathbf{e}$, where $I$ is the current carried by the loop, $\mathbf{A}$ its area, and $\mathbf{e}$ a unit vector along the central axis of the loop.

Maxwell's equations A set of equations, written in differential or integral form, stating the relationships between the fundamental electromagnetic quantities.

Nedelec's edge element See vector element.

perfect electric conductor (PEC) A material with high electrical conductivity, modeled as a boundary where the electric field is zero.
**perfect magnetic conductor**  A material with high permeability, modeled as a boundary where the magnetic field is zero.

**phasor**  A complex function of space representing a sinusoidally varying quantity.

**quasi-static approximation**  The electromagnetic fields are assumed to vary slowly, so that the retardation effects can be neglected. This approximation is valid when the geometry under study is considerably smaller than the wavelength.

**surface current density**  Current density defined on the surface. The component normal to the surface is zero. The unit is A/m.

**vector element**  A finite element often used for electromagnetic vector fields. The tangential component of the vector field at the mesh edges is used as a degree of freedom. Also called *Nedelec’s edge element* or just *edge element*. 
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